Introduction to Game Theory

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Basic Information

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Office hours: Tue 13:00-14:00 339 NB

Textbooks: Osborne – Introduction to Game Theory, 2004

Gibbons, R.: A Primer in Game Theory, 1992

Cerge library – politických vězňů 7

Exams: Midterm exam: 40%

- Final exam: 60%
- Extra points from several experiments – max 5%

Economic modeling

- Model: an abstraction we use to understand our observations and experiences
  - Assumptions – key part of every model – some of them are not visible from the first sight
  - Conclusions → insights to the observed reality
- Model is unlikely to enhance our understanding if its assumptions are wrong
- Assumptions should capture the essence of the situation, not irrelevant details

Economic modeling

- Models are neither “right” nor “wrong”
  - model of car’s braking distance based on its speed
    - Valid according to the situation where it is used
    - Ok for dry road but probably not for snowy road and large difference in speeds
  - Whether a model is useful or not depends, in part, on the purpose for which we use it
    - probability of skid of the car based on its speed
  - We have to think whether it gives us some non-trivial insights into the observed situation and to which extent are the assumptions valid

Economic modeling

- The models of game theory are precise expressions of ideas that can be quite often presented also verbally
- However, the models may help us to analyze more complex situations, precisely describe our way or analysis and facilitate the critique or adoption of our analysis to other people

Game Theory

- GAME THEORY aims to help us understand situations in which decision-makers interact
- Game here is not a game in the everyday sense but stands as a model of various situations:
    - firms competing for business, political candidates competing for votes, bidders competing in an auction, firm and union negotiating next year’s wage contract, legislators negotiating the date of next elections or change of constitution

Theory of rational choice

- Key part of most models in Game Theory
- decision-maker chooses the best action according to her preferences, among all the actions available to her
  - Set A of all possible actions
    - all the bundles that the person can possibly consume
  - Preferences and payoff function
  - For every pair of actions → person knows which of the pair she prefers or if she is indifferent between them
- Consistent preferences: prefers a to b and b to c means that se prefers a to c

Theory of rational choice

- Payoff function
  - represents preferences
  - each action associated with number
  - Higher numbers -> more preferred
- *The action chosen by a decision-maker is at least as good, according to her preferences, as every other available action*
  - *again consistent: if agent chooses a when faced with {a, b} means that she prefers a to b and therefore she must choose either a or c, never b, when facing {a, b, c}*
- No general theory currently challenges the supremacy of rational choice theory

Static game of complete inf.

- Set of players
  - firms, political candidates, bidders, etc.
- For each player set of actions
  - each action may affect also other players
  - $a_1,\ldots, a_N$ – different choices of behavior for each player
- For each player set of preferences over the set of action profiles
  - action profile – set of particular chosen actions of every player
  - 3 players, if player 1 chose a_1, player 2 chose a_{19}, player 3 chose a_5 → action profile $\{a_1, a_{19}, a_5\}$

Static game of complete inf.

- set of preferences
  - for each player – player 1 prefers action profile
    - $\{a_1, a_{19}, a_5\}$ over $\{a_{10}, a_3, a_2\}$
    - $\{a_{10}, a_3, a_2\}$ over $\{a_2, a_4, a_1\}$
    - $\{a_2, a_4, a_1\}$ over $\{a_1, a_{19}, a_2\}$ etc.
- in static game *with ordinal preferences and complete information – every player is aware of all others players' preferences and the actions are chosen simultaneously*

Example 1: Prisoner's Dilemma

Two suspects in a major crime are held in separate cells. There is enough evidence to convict each of them of a minor offense, but not enough evidence to convict either of them of the major crime unless one of them acts as an informer against the other (finks). If they both stay quiet, each will be convicted of the minor offense and spend one year in prison. If one and only one of them finks, she will be freed and used as a witness against the other, who will spend four years in prison. If they both fink, each will spend three years in prison.

Example 1: Prisoner's Dilemma

- Set of players
  - two suspects
- For each player set of actions
  - Stay Quiet or Fink (act as an informer) for both players
- For each player set of preferences over the set of action profiles
  - *Suspect 1's ordering of the action profiles from best to worst:*

    (Fink, Quiet), (Quiet, Quiet), (Fink, Fink), (Quiet, Fink)
  - *Suspect 2's ordering :*

    (Quiet, Fink)=3, (Quiet, Quiet)=2, (Fink, Fink)=1, (Fink, Quiet)=0

Example 1: Prisoner's Dilemma

Suspect 2

Suspect 1

	Quiet	Fink
Quiet	**2, 2**	**0, 3**
Fink	**3, 0**	**1, 1**

Example 1: Prisoner's Dilemma

- Similar situations – that may be modeled as prisoner's dilemma:
  - duopoly

    two firms produce the same good, for which each firm charges either a low price or a high price. Each firm wants to achieve the highest possible profit

	High	Low
High	**100**, **100**	**-20**, **120**
Low	**120**, **-20**	**60**, **60**

Example 1: Prisoner’s Dilemma

- Similar situations – that may be modeled as prisoner’s dilemma:
  - the arm race
    Under some assumptions about the countries’ preferences, an arms race can be modeled
  - nice article about prisoner’s dilemma and nuclear conflict: Field (2008): Schelling, Irrationality, and the Event that Didn't Occur
    http://ssrn.com/abstract=1095946

	Refrain	Arm
Refrain	2, 2	0, 3
Arm	3, 0	1, 1

Example 1: Prisoner's Dilemma

- Similar situations – that may be modeled as prisoner's dilemma:
  - Working on a joint project

If your friend works hard then you prefer to goof off (the outcome of the project would be better if you worked hard too, but the increment in its value to you is not worth the extra effort). You prefer the outcome of your both working hard to the outcome of your both goofing off (in which case nothing gets accomplished)

	Hard	Shirk
Hard	2, 2	0, 3
Shirk	3, 0	1, 1

Example 1: Prisoner's Dilemma

- Similar situations – that may be modeled as prisoner's dilemma:
  - Common property

Two farmers are deciding how much to allow their sheep to graze on the village common. Each farmer prefers that her sheep graze a lot than a little, regardless of the other farmer's action, but prefers that both farmers' sheep graze a little than both farmers' sheep graze a lot (in which case the common is ruined for future use)

	A little	A lot
A little	2, 2	0, 3
A lot	3, 0	1, 1

Example 2: Bach or Stravinsky?

Two people wish to go out together. Two concerts are available: one of music by Bach, and one of music by Stravinsky. One person prefers Bach and the other prefers Stravinsky. If they go to different concerts, each of them is equally unhappy listening to the music of either composer.

Example 2: Bach or Stravinsky?

- Set of players
  - two friends
- For each player set of actions
  - Bach or Stravinsky for both players
- For each player set of preferences over the set of action profiles
  - *Friend 1's ordering of the action profiles from best to worst: (Bach, Bach)=2, (Stravinsky, Stravinsky)=1, (Bach, Stravinsky) = (Stravinsky, Bach) =0*
  - *Suspect 2's ordering :*

    (Stravinsky, Stravinsky)=2, (Bach, Bach)=1,

    (Bach, Stravinsky)=(Stravinsky, Bach)=0

Example 2: Bach or Stravinsky?

Friend 2

Friend 1

	Bach	Stravinsky
Bach	**2, 1**	**0, 0**
Stravinsky	**0, 0**	**1, 2**

Example 2: Bach or Stravinsky?

- Similar situations – that may be modeled as BoS:
  - Negotiating of political parties

    two officials of a political party deciding the stand to take on an issue they disagree about the best stand, but are both better off if they take the same stand than if they take different stands
  - two merging firms using different computer technologies

    they will both be better off if they both use the same technology; each firm prefers that the common technology be the one it used in the past.

Example 3: Matching Pennies

Two people choose, simultaneously, whether to show the Head or the Tail of a coin. If they show the same side, person 2 pays person 1 a dollar; if they show different sides, person 1 pays person 2 a dollar. Each person cares only about the amount of money she receives, and (naturally!) prefers to receive more than less.

Example 3: Matching Pennies

- Set of players
  - two persons
- For each player set of actions
  - Head or Tail for both players
- For each player set of preferences over the set of action profiles
  - *Person 1's ordering of the action profiles from best to worst:*

    (Head, Head)=(Tail, Tail)=1, (Head, Tail)=(Tail, Head)=-1
  - *Person 2's ordering :*

    (Head, Tail)=(Tail, Head)=1, (Head, Head)=(Tail, Tail)=-1

Example 3: Matching Pennies

Person 2

Person 1

	Head	Tail
Head	**1, -1**	**-1, 1**
Tail	**-1, 1**	**1, -1**

Example 3: Matching Pennies

- Similar situations – that may be modeled as Matching Pennies:
  - Choices of appearances for new products by an established producer and a new firm in a market of fixed size
  - Relationship between two people in which one person wants to be like the other, whereas the other wants to be different

Example 4: Stag Hunt

Group of hunters who wish to catch a stag (deer). They will succeed if they all remain sufficiently attentive, but each is tempted to desert her post and catch a hare (similar to rabbit).

Each of a group of hunters has two options: she may remain attentive to the pursuit of a stag, or catch a hare. If all hunters pursue the stag, they catch it and share it equally; if any hunter devotes her energy to catching a hare, the stag escapes, and the hare belongs to the defecting hunter alone. Each hunter prefers a share of the stag to a hare.

Example 4: Stag Hunt

- Set of players
  - N hunters
- For each player set of actions
  - Stag or Hare for each hunter
- For each player set of preferences over the set of action profiles
  - *each hunter's ordering of the action profiles from best to worst:*

    (Stag, Stag, Stag, Stag , Stag) = 2

    (Hare, … , … , ... , …) = 1

    (Stag, … , Hare, …., …..) = 0

Example 4: Stag Hunt 2 hunters

Hunter 2

Hunter 1

	Stag	Hare
Stag	**2**, **2**	**0**, **1**
Hare	**1**, **0**	**1**, **1**

Example 4: Stag Hunt

- Similar situations – security dilemma
  - Variant of two player Stag Hunt
  - Alternative to Prisoner's Dilemma when cost of arming outweighs the benefit if the other country does not arm itself

Country 2

Country 1

	Refrain	Arm
Refrain	**3**, **3**	**0**, **2**
Arm	**2**, **0**	**1**, **1**

Dominated strategies

- *Strict domination in a static game with ordinal preferences* - action if it is superior no matter what the other players do
- Definition: player *i's action* ***a strictly dominates her action b if*** $u_i(a, a_{-i}) > u_i(b, a_{-i})$ *for every list* a_{-i} *of the other players' actions,* where u_i *is a payoff function that represents player i's preferences*
  - *For every combination of others players' actions payoff when playing a is strictly higher than when playing b*
    - $a_{-I} = \{a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N\}$ *- actions of others players*
- Definition: If any action strictly dominates the action b, *we say that b is* ***strictly dominated***

Example 1: Prisoner's Dilemma

Suspect 2

Suspect 1

	~~Quiet~~	Fink
~~Quiet~~	~~**2**~~, ~~**2**~~	~~**0**~~, **3**
Fink	**3**, ~~**0**~~	**1**, **1**

Iterative elimination

- *Iterative elimination of strictly dominated strategies*
  - Idea: rational players do not play strictly dominated actions
  - Assumption of common knowledge that all players are rational: all the players know that all the players are rational, and that all the players know that all the players know that all the players are rational etc.
  - often produces a very imprecise prediction about the play of the game in real life or in experiments
  - the order of elimination does not affect the strategy or strategies we end up with

Iterative elimination: Example 1

Player 2

Player 1

	Left	Middle	Right
Up	**1, 0**	**1, 2**	**0, 1**
Down	**0, 3**	**0, 1**	**2, 0**

Iterative elimination: Example 1

Player 2

Player 1

	Left	Middle	~~Right~~
Up	**1, 0**	**1, 2**	**0, ~~1~~**
Down	**0, 3**	**0, 1**	**2, ~~0~~**

Iterative elimination: Example 1

Player 2

Player 1

	Left	Middle
Up	**1, 0**	**1, 2**
Down	**0, 3**	**0, 1**

Iterative elimination: Example 1

Player 2

Player 1

	Left	Middle
Up	**1, 0**	**1, 2**
~~Down~~	**$\cancel{0}$, 3**	**$\cancel{0}$, 1**

Iterative elimination: Example 1

Player 2

	Left	Middle
Up	**1, 0**	**1, 2**

Player 1

Iterative elimination: Example 1

Player 2

Player 1

	~~Left~~	Middle
Up	**1**, **~~0~~**	**1**, **2**

Iterative elimination: Example 2

Player 2

Player 1

	Left	Middle	Right
Left	**0**, **4**	**4**, **0**	**5**, **3**
Middle	**4**, **0**	**0**, **4**	**5**, **3**
Right	**3**, **5**	**3**, **5**	**6**, **6**

Iterative elimination: Example 3

P 2

P 1

P3:A	A	B	C
A	0, 4, 1	3, 0, 3	1, 8, 8
B	4, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 4, 7	1, 3, 5

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 2	7, 3, 5
C	2, 5, 3	6, 5, 6	1, 6, 1

P 2

P 1

P3:C	A	B	C
A	1, 4, 7	3, 6, 7	3, 1, 7
B	2, 1, 3	4, 0, 3	4, 3, 4
C	1, 5, 4	3, 4, 4	1, 4, 3

Iterative elimination: Example 3

P 2

P3:A	A	B	C
~~A~~	~~0, 4, 1~~	~~3, 0, 3~~	~~1, 8, 8~~
B	4, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 4, 7	1, 3, 5

P 1

P 2

P3:B	A	B	C
~~A~~	~~1, 7, 8~~	~~5, 0, 1~~	~~2, 2, 5~~
B	4, 0, 3	6, 4, 2	7, 3, 5
C	2, 5, 3	6, 5, 6	1, 6, 1

P 2

P3:C	A	B	C
~~A~~	~~1, 4, 7~~	~~3, 6, 7~~	~~3, 1, 7~~
B	2, 1, 3	4, 0, 3	4, 3, 4
C	1, 5, 4	3, 4, 4	1, 4, 3

P 1

Iterative elimination: Example 3

P 2

P3:A	A	B	C
B	4, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 4, 7	1, 3, 5

P 1

P 2

P3:B	A	B	C
B	4, 0, 3	6, 4, 2	7, 3, 5
C	2, 5, 3	6, 5, 6	1, 6, 1

P 2

P3:C	A	B	C
B	2, 1, 3	4, 0, 3	4, 3, 4
C	1, 5, 4	3, 4, 4	1, 4, 3

P 1

Iterative elimination: Example 3

P 2

P3:A	A	B	C
B	4, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 4, 7	1, 3, 5

P 1

P 2

P3:B	A	B	C
B	4, 0, 3	6, 4, 2	7, 3, 5
C	2, 5, 3	6, 5, 6	1, 6, 1

P 2

P3:C	A	B	C
B	2, 1, 3	4, 0, 3	4, 3, 4
C	1, 5, 4	3, 4, 4	1, 4, 3

P 1

Iterative elimination: Example 3

P 2

P3:A	A	B	C
B	4, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 4, 7	1, 3, 5

P 1

P 2

P3:C	A	B	C
B	2, 1, 3	4, 0, 3	4, 3, 4
C	1, 5, 4	3, 4, 4	1, 4, 3

P 1

Iterative elimination: Example 3

P 2

P 1

P3:A	A	~~B~~	C
B	4, 5, 4	~~4, 2, 3~~	2, 3, 7
~~C~~	~~2, 5, 4~~	~~1, 4, 7~~	~~1, 3, 5~~

P 2

P 1

P3:C	A	~~B~~	C
B	2, 1, 3	~~4, 0, 3~~	4, 3, 4
~~C~~	~~1, 5, 4~~	~~3, 4, 4~~	~~1, 4, 3~~

Iterative elimination: Example 3

P 2

P3:A	A	C
B	4, 5, 4	2, 3, 7

P 1

P 2

P3:C	A	C
B	2, 1, 3	4, 3, 4

P 1

Iterative elimination: Example 3

P 2

P3:A	A	~~C~~
B	**4, 5, 4**	~~**2, 3, 7**~~

P 1

P 2

P3:C	A	C
B	**2, 1, 3**	**4, 3, 4**

P 1

Player 1: B
Player 2: A
Player 3: A

Summary

- Static game with complete information
- Dominant and dominated strategies
- Iterative elimination of strictly dominated strategies
- Gibbons 1.1.A-B; Osborne 1-2.5

NEXT WEEK:

Nash equilibrium, best response function

Experiment