Learning by Investing:
Evidence from a Naturally-Occurring Auction

Jan Hanousek and Evžen Kočenda

Abstract

We test for learning among heterogeneous individuals who are engaged in bidding and possess limited information about the auctioned assets. We use an extensive data set taken from an auction through which state-owned enterprises were privatized in the Czech Republic in 1993–1994. To test for learning, we develop new measures of individual performance to accommodate the varying prices of assets available in six successive stages of bidding. We present evidence that learning took place among heterogeneous agents during a large-scale multi-stage auction that is unique in terms of its size, incentives, and variation. The auction's design—with multiple market periods—allowed agents to learn based on accumulated experience. Since individuals had to pay a fee to participate, and since the potential gain was in the magnitude of several months' salary, we conclude that large incentives were driving our results.

Keywords: learning, investing, naturally-occurring auction, stock market, privatization, heterogeneous agents

JEL Classification: C14, C93, D44, D82, D83, G14, P43

Jan Hanousek, Charles University, CERGE-EI, Prague, Politickych veznu 7, 111 21 Prague 1, Czech Republic; Anglo-American University, Prague; CEPR; WDI.
e-mail: jan.hanousek@cerge-ei.cz, tel. (+420) 224 005 119, fax: (+420) 224 227 143

Evžen Kočenda (corresponding author), Charles University, CERGE-EI, Prague, Politickych veznu 7, 111 21 Prague 1, Czech Republic; Anglo-American University, Prague; CEPR; WDI; CESifo.
e-mail: evzen.kocenda@cerge-ei.cz, tel. (+420) 224 005 149, fax: (+420) 224 227 143

For helpful comments we are grateful to Parker Ballinger, Glenn Harrison, John List, Andreas Ortmann, Ondřej Rydval, and an anonymous referee. Isabel Schnabel (Editor) provided us with numerous specific and helpful suggestions. We have also benefited from several presentations. Financial support from the GACR Grant No.402/09/1595 is acknowledged. The usual disclaimer applies.

1. Introduction

This paper considers the second phase of the privatization of state-owned enterprises in the Czech Republic, which took the form of an auction. Citizens used vouchers to purchase stock in privatized companies in a succession of bidding rounds, which created strong incentives to learn. A number of individuals invested on their own, while others invested through Privatization Investment Funds (PIFs) run by professional managers. Our goal is to compare individual portfolios with a market portfolio designed to represent passive investor bidding in an index fund. In particular we measure the number of individuals who successfully placed bids in successive bidding rounds. We distinguish between the number of individuals who improved their performance with respect to the market portfolio over time versus those whose performance worsened. Learning is defined as the observed change in behavior owing to experience and is measured by comparing the number of individuals who improved their performance with respect to the market portfolio with the number of individuals whose performance worsened (based on a random sample of individual bidders). Using this measure, we show that the number of individuals with improved performance is greater between all successive periods than the number of those who did worse; this is verified by a number of robustness tests. Thus, we present evidence that learning took place among agents with limited information who had strong incentives to learn during the course of a naturally-occurring auction whose stakes comprised, on average, several months of pay. By virtue of the auction's design with multiple market periods, which allows agents to learn based on accumulated experience, our results lend insight into the findings of Smith (1962), who shows evidence of inter-period learning and List (2004), who advocates that agents learn on the basis of accumulated experience.

There exists a considerable theoretical literature on learning (see Weibull 1995, Fudenberg and Levine 1998, and Vega-Redondo 2003 among others for overviews), yet empirical evidence is still rudimentary and almost exclusively driven by the experimental approach. Results from learning in laboratory experiments are reviewed by Camerer (2003, p. 265) who argues for the use of experimental data since they "are a good way to test models of learning because control over payoffs and information means we can be sure what subjects know (and know others know, and so on), what they expect to earn from different strategies, what they have experienced in the past, and so forth."

Results from laboratory experiments suffer from various drawbacks. They are nearly always performed with relatively small samples of university students, groups that typically exhibit a relatively high degree of homogeneity, despite variation in certain characteristics such as gender. In addition, monetary rewards for active participation in such experiments are frequently limited, which raises the question to what extent these experiments capture learning. Further, experiments that are small in size in terms of participants and earnings are also usually small in terms of the choice set that participants face. These limitations have been recognized in the past; for example, arguments stressing that incentives must be salient for experimental data to have meaning were already voiced by Smith (1976, 1982).

We avoid these methodological problems by using data from a multi-stage naturally-occurring auction on a quasi-stock market in the Czech Republic. The aim of the auction was to privatize more than 1600 state-owned enterprises. Participants in this auction observed in the field—ordinary citizens—counted in the millions; a participation rate of 87% of eligible citizens minimizes possible sample selection bias. Our study is based on complete individual bidding data for a randomly drawn heterogeneous sample of 5000 citizens from a population of 2.2 million; this sample size allows us to estimate population proportions with a precision of ±1%.[1] To the best of our knowledge, these data constitute one of the largest samples available worldwide that can trace each step of each individual during a naturally-occurring auction. The rewards for participation in these auctions were substantial. The expected returns were several months' average salary, with the maximum returns being several years of average salary.[2] In addition, individuals paid a

---

[1] The data on 5000 individuals corresponds to the maximum size of the representative sample we could obtain given the amount of machine time we received from a government agency to collect the data. Such a sample size allows us to estimate accurately any probability of occurrence within the whole population. For example, Newcombe (1998) presents lower and upper bounds (*LB*, *UB*) for the confidence intervals for a sample proportion using as a normal approximation $[p_{LB}, p_{UB}]$, here $p_{LB} = \hat{p} - z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and $p_{UB} = \hat{p} + z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. As is standard, *z* stands for the quantile of the standard normal distribution and *n* denotes the sample size. The precision of ±1% in estimating population proportions follows directly from the above.

[2] These incentives are higher than those in Kachelmeier and Shehata (1992), where in the highest payoff condition, subjects earned three times their normal monthly revenue in the course of a two-hour experiment that was conducted in another transition economy, the People's Republic of China; or in Slonim and Roth (1998), where in an ultimatum game experiment in the Slovak Republic, financial incentives varied by a factor of twenty-five.

participation fee equal to about one week of salary. Participating individuals clearly had strong incentives to learn.

Our analysis contributes to the debate on learning in three ways. First, we supplement the theoretical literature on learning with evidence that learning took place among heterogeneous individuals during a real-life auction process. Second, we base our evidence on a data set from a naturally-occurring auction that is unique in terms of size, incentives, and variation. Third, in order to detect and quantify learning we develop a new measure of individual bidding performance based on its distance from the performance of two benchmarks (market portfolio and informed investor). These measures account for the varying prices of goods (shares) available in successive stages of bidding.

Using the non-parametric Wilcoxon (1945) sign rank test for paired data we compare individual outcomes over different action stages with respect to benchmarks, account for the changing prices of auctioned shares, and show that significant learning is observed among individuals having limited information about assets on the artificial stock market. We conjecture that the extraordinary incentives that participants faced were the key factor driving our results.

The paper is organized as follows: Section 2 describes the auction observed in the field and the data; Section 3 defines learning formally and reports the empirical results. Section 4 briefly concludes.

2. The Set of Naturally-Occurring Auctions and the Data

The data are from a naturally-occurring auction: the bidding process to acquire shares in privatized firms known as the voucher scheme (see Section 2.1 for details). The subject pool contained all Czech citizens 18 years and older, who were provided with instructions on the bidding process and public information on privatized firms. Thus, bidding was based on publicly-known rules. The individual investors were bidding to acquire shares in privatized firms that could be sold for money after the end of the privatization scheme. The shares represented substantial (future) monetary value, hence the bidding incentives were high. Bidding was performed on an artificial market to proxy for the stock market.

2.1 The voucher scheme as a naturally-occurring auction

The voucher scheme was part of the large-scale privatization process. Two waves of voucher privatization took place, one in 1992–93 and one in 1993–94.[3] Both waves were administered as auctions in the same manner and there were no differences in their setup. During the scheme, a total of 1664 firms were privatized: 988 in the first wave and 676 firms in the second wave, and 185 firms were privatized in both waves in various proportions of their assets. The public could participate in the voucher process. For each wave every eligible citizen was authorized to buy a voucher book that contained 1000 investment "points" for a nominal fee of 1000 crowns (about a week's average wage).

Before the bidding started, individuals had the option of assigning none, some, or all of their points to Privatization Investment Funds (PIFs): these were newly established financial firms vaguely similar in their scope of activities to closed-end mutual funds. These PIFs had to publish basic facts about both the founder and the investment strategy. This part of the scheme is usually called "pre-round" or "zero round".

When entering the voucher scheme, individuals had two basic strategies to choose from. One was to bid for a particular firm to exercise the shareholder's rights of control. However, the limited number of voucher points that were available for each individual during the bidding process effectively prevented individual bidders from exercising control over a privatized firm.[4] Thus, the second strategy was to maximize cash revenues from the future sale of shares, receive dividend payments, or a combination of both.

Before the start of the bidding process, key financial information about each auctioned enterprise was made publicly available. The information was published in the special periodical "Kuponová Privatizace" ("Voucher Privatization") and included figures on employment, wages, capital, sales, costs, profit or loss, liabilities, foreign trade, ownership structure, etc. During the bidding process citizens and PIFs used their voucher points to buy shares of available firms in a series of price-administered bidding rounds. A Pricing Committee in the Ministry of Finance adjusted prices between each round primarily using an excess demand rule. The sequential character of the bidding and closed-economy

---

[3] The privatization process has been extensively described and analyzed. Hence, we refrain from a detailed description but present key facts relevant to our analysis. For details see e.g., Kočenda (1999), Hanousek and Kočenda (2008), and Estrin et al. (2009), among others.

[4] After the voucher scheme ended, this assumption proved to be accurate. The resulting ownership was simply too dispersed to allow individual shareholders to exercise control. For evidence see Kočenda and Valachy (2002) and Hanousek, Kočenda, and Svejnar (2007).

character of the auction prevented incentives for speculation from materializing. The announced goal of this commission was to adjust prices so that by the end of the process citizens had used all their points while distributing as many shares as possible (for details see Filer and Hanousek, 2001). To avoid end-of-game problems, the total number of rounds was not set. The bidding scheme was not a standard auction, since investors' bids were quantities and the prices were in fact administered by the privatization authority.[5]

Each bidding round was divided into four stages. In the first stage participants were told the administered price of the shares of each firm and the number of shares offered. Participants then bid for shares of their preferred firms. The third stage included collecting, matching and analyzing the bids. The last stage coincided with the first stage of the next round; the results of the bidding were announced and the Pricing Committee set the prices for the next round.

The bidding rounds continued until the privatization authority revealed the end of the wave when a negligible proportion of unsold shares along with disposable investment points remained. The final stage of voucher privatization was the real transfer of the purchased shares. For each participant, a share account at the Central Register was created. Those individuals who allocated part or all of the 1000 points to PIFs obtained the shares of the PIFs immediately after the issue. Shares of firms obtained by individuals during the bidding process were traded on the capital market after the end of the privatization scheme.

2.2 Auction design (rules for accepting bids)

The auction design of the Czech privatization scheme was relatively simple and can be described by the following rules:[6]

1. The shares of privatized firms were offered in a sequence of auction stages (rounds). Prices in the first round were equal for all shares of all firms since the number of shares issued was determined by the firm's book value; specifically, the first round

[5] Czech voucher privatization thus resembled, but was not identical to, any classical market mechanism design. It was not a Walrasian tâtonnement since demands were satisfied prior to the determination of the equilibrium price and there was no recontracting. It bears some resemblance to a multi-unit Dutch auction, although there were several key differences including the fact that the initial price was set at a supposed approximation of the true equilibrium price rather than a price higher than the reservation price of any individual bidder. For more details see Hanousek and Kroch (1998) and Filer and Hanousek (2001).

[6] As the formal description of the Czech privatization auction can be found in Aggarwal and Harper (2000), we opt here for a verbal outline.

price of one share of any firm was set at 50 voucher points (artificial currency). The number of points entering the sequence of auctions was known and no new points entered the auction after the first round began.

2. In each successive round, prices were adjusted up or down as a function of the excess demand for or excess supply of shares in the previous round. Thus, if there was a large excess supply (demand) in round r, the price was reduced (increased) in round $r+1$. The price—the number of points per share—was administered by the Pricing Committee, which never publicly revealed its algorithm for adjusting share prices between rounds. It was generally observed that, indeed, prices rose for shares in excess demand and fell for shares in excess supply.
3. If the number of bids for a firm's shares did not exceed its supply of shares, the demand was satisfied and the remaining shares were deferred to the next round.
4. If the demand for a firm's shares exceeded supply by less than 25%, then individual investors had their demand met while bids from investment funds were rationed in proportion to their bids, provided that the funds were allocated to at least 80% of their original bids. In such a case, all shares were sold and shares of the firm were not available for purchase in successive rounds. Once the auction began, the above proportions were made public and kept constant during the auction sequence.
5. In all other cases no shares were sold and all shares were carried over to the following round.
6. Between rounds the public was updated by being provided information on the revised prices and the total demand for shares of each firm by individuals and investment funds in the previous round. Specifically, once the auction began, the authorities after each round publicly revealed the following information: the demand for a particular firm's shares in the previous round (separately by individual investors and PIFs), the amount of shares available for the upcoming round, prices achieved in the past round, and prices set to begin the upcoming round.

Despite the simple design of the auction, to the best of our knowledge, there exists no theoretical model that would solve the problem of the optimal allocation of voucher points or address the strategic behavior of individual bidders as in the Czech voucher scheme.

This is understandable given the extent of the bidding scheme with several hundreds of firms being auctioned to a heterogeneous population of several million individual bidders and several hundred privatization funds.

From the above outline it is evident that the bidding scheme was a way to establish market prices where there was no market, by using—in a sequential process—a market response to adjust administered prices. Although the prices are not market prices, one can consider observed demand to be a market response to the set prices.

2.3 Auction data

As noted earlier, our data come from the series of auction stages that were performed during the voucher scheme. Data from the first wave are not available. The Prague Securities Centre, the state body that maintains the register of dematerialized securities and of their owners archived the data and put an embargo on it due to the "sensitivity of personal information it contains." Therefore, we perform our analysis on the data from the second wave that took place during the period April-December 1994. Acquired shares from the auction were distributed by February 1995 and from March 1995 they started to be traded on the secondary market.

A total of 6.16 million individuals participated in the second wave of the voucher scheme. Out of this total, 3.87 million individuals (62.8%) assigned all of their 1000 points to PIFs (3.46 million individuals put them into only one of a number of PIFs), a negligible number of 0.09 million individuals opted to allocate some points to a PIF and use some for independent bidding, and 2.2 million individuals (35.7%) bid on their own (did not allocate any of their points to PIFs).

Thus, the bidding process involved two groups of agents. The first group was formed of ordinary citizens who can be characterized as heterogeneous and less informed agents since they possessed only limited prior knowledge of the process itself and had access only to the publicly available information about firms. PIFs, as the second group, can be considered informed agents: the scope of their business activity was to invest points (artificial currency) *en masse* and to do so they were equipped with teams of analysts. The PIFs had a technical as well as expert advantage in processing public (as well as possibly inside) information over the individual investors.

Our data sample consists of the detailed individual bidding information of the 5000 Czech citizens over the age of 18 who resided in the Czech Republic and were eligible to participate in the voucher scheme. The sample varies across gender, occupation, age and education but none of these characteristics are revealed because 5000 citizens were randomly selected out of the 2.2 million who decided to bid all their points individually and not through any of the PIFs. This way we ensure that every individual in our sample had exactly the same starting conditions in terms of the 1000 points available. We assume that, in general, these individuals were eager to participate actively in the scheme and that they had confidence in their own judgment. As we noted, we analyze a random sample from the sub-population of individuals participating in the voucher scheme. Individuals investing on their own were likely to be more self confident, likely younger, and maybe more educated but we have no information about their characteristics except that inferred by Hingorani, Lehn and Makhija (1997), who show that individuals did not transfer their investment points to funds unless they believed that the funds could make a better investment selection. The bidding data trace in detail every move of each individual during the bidding process. In this respect we sharply differentiate from the rest of the literature in terms of data quality. So far all studies on the voucher privatization scheme have used *aggregated* data, i.e., the bids of all individuals were treated as the outcome of one agent. Such specifications do not take into account that the population of bidding individuals could vary significantly between rounds. In order to analyze if individuals were able to learn and adjust their bidding behavior between rounds, it is imperative to deal with individual bidding data.[7]

3. Learning

3.1 Definition and measures

---

[7] In an earlier article Hanousek and Kroch (1998) analyzed the dynamic effects of information flows between two aggregated agents (investment funds and individuals) using the Arellano-Bond technique in a GMM setup. Their paper lacks any information on individual behavior. Since the paper uses aggregated bidding data on individuals, it implicitly makes an incorrect assumption that the same population of individuals was bidding in each round. The learning model studied in Hanousek and Kroch (1998) is simply a model of dynamic behavior in round-by-round bidding based on updated information. Since their analysis is based on aggregated data it cannot be extended to describe individual behavior. Hence, it cannot claim that individuals really "learned." By using detailed individual data and a non-parametric testing procedure our approach differs substantially from that used by Hanousek and Kroch (1998) as we are able to show the presence of truly individual learning.

In general terms learning can be defined as an "observed change in behavior owing to experience" (Camerer 2003, p. 265). Our analysis concerns the learning of uninformed individuals. If the individuals in our auction were learning, then, owing to experience, they should change their bidding strategy over the course of successive rounds to improve their expected performance. In order to detect (or refute) the presence of learning we have to define appropriate measures of individual performance. In doing so we have to account for the fact that an individual could start the bidding in any round and continue until the end of the scheme or until an individual had no points left, and that prices and the selection of available shares were changing across rounds. To summarize, each auction stage differs in terms of the number of offered firms and their shares to be auctioned, the number of participating individuals and the amount of artificial currency (points) they have available to bid, as well as share prices. The large heterogeneity is accounted for by comparing individual performance pairwise between rounds, and scaling the auctioned prices with respect to an appropriate auction-specific benchmark. This will be explained in detail below.

The voucher scheme used a specific "currency"—voucher points—with which participants could buy shares of privatized companies. After the scheme, unspent vouchers had zero value. Unless all individuals bid only for the same set of shares, we cannot measure and compare their performance between rounds using voucher points. Hingorani, Lehn, and Makhija (1997) faced a similar problem when examining the aggregate bidding behavior and investment performance of individual investors and mutual funds in the same voucher scheme. To assess *ex post* bidding performance they used secondary market prices. Since the book value of all shares was equal by design, the cross-sectional variation in stock market prices provides a market-based measure of the relative valuation of shares (Hingorani, Lehn, and Makhija, 1997). This approach is in the spirit of Erev and Roth (1998), who study both the *ex post* ("best fit") descriptive power of learning models, and their *ex ante* predictive power. Following this approach, we can compare the values of acquired shares using their *ex post* prices on the secondary market after the bidding scheme ended. There is another economic rationale for doing so: when bidding, individuals were in fact motivated by the vision of substantial rewards that they could collect in the secondary market established by the government. Their bidding strategy was aimed at collecting valuable shares of privatized firms (and to sell them later on the stock market). Thus, under the conditions of our auction, learning should factor into the course of actions through which individual investors improved over time in terms of the future value of their bids on the stock market. It is noted that our results may overstate learning if additional information becomes publicly available during the bidding process.

Given the arguments above, we assess individuals' performance in terms of the acquired shares valued by the prices these shares carried on the secondary market after the bidding process ended. In order to have representative and relatively smooth data even for less frequently traded stocks, we use for each share the average price over a three-month period (April-June 1995). This means that in our valuation we employ share prices close to the period after the bulk of the shares from the second wave of the voucher scheme entered the market. Also, during this period the majority of trades were initiated by individual sellers, liquidity was relatively high, and the prices should correspond to the expected payoff for the majority of participants. Further, by choosing this period we do not consider the medium and long run effect on stock prices caused by changes in ownership structure and corporate governance after the end of voucher privatization. As in the standard approach in the financial literature, we allow for a one-month "settle-down period".[8]

3.2 Performance benchmarks

Before evaluating the performance of individual investors, we have to take into account quantitative differences between successive auction rounds. First, the composition of each round differs in terms of the set of shares that are available as well as in terms of their quantities. Some shares (of particular firms) were sold out and were no longer available in the next rounds, while some were sold partially and thus had a reduced quantity available. Second, individual investors placed their bids in various quantities across rounds, e.g. the value of their bids differs with each individual. For both of the above reasons, we have to standardize the value of their bids by considering the value of shares obtained per one

[8] Thus, we do not consider prices from March 1995, when the trading of shares began. The second reason for doing so is the fact that shares were put on the secondary market gradually during this month; from April all of them were available for trading. We also checked against the inclusion of March 1995 and did not find any significant difference.

voucher point. Further, to homogenize the value of shares offered in each round, we link each round with a particular performance benchmark (defined below).

In order to define the value per point (*bid* or *spent*) we consider the following three kinds of investors: (i) an active investor (T), i.e. an individual investor in our sample; (ii) a passive investor bidding according to market capitalization measured in voucher points (also known as the index portfolio or passive index fund), i.e. a market portfolio (M); and (iii) an informed investor (I), i.e. a privatization fund (PIF). We will consider the market portfolio (M) and informed investor/privatization fund (I) as our two benchmarks.

Our first benchmark is the market portfolio (M), which is the value-weighted portfolio, as in the Capital Asset Pricing Model of Sharpe (1964), Lintner (1965), and Mossin (1966). Following Sharpe (1970, p. 82), in each round t of our auction each asset in the market portfolio (M_t) is proportionally represented by its asset market capitalization, which is measured in voucher points. These proportions, or weights, are in our case formally defined as

$$w_{i,t}^{M} = \frac{P_{i,t}Q_{i,t}}{\sum_{k} P_{k,t}Q_{k,t}} \quad \text{for } i = 1, 2, \ldots, N, \tag{1}$$

where $P_{i,t}$ represents the price of a share of privatized firm i expressed in voucher points for a particular round t, where such a price is the result of the bidding mechanism, and $Q_{i,t}$ represents the number of shares offered in round t. The firms' representation is proportional to the market capitalization, which is measured in voucher points.

The market portfolio, also known as the market index fund, is a naïve or passive portfolio since it does not use other information than the market capitalization of the firm at the time of the investment. Because the market portfolio represents an average return it is a natural benchmark for evaluating various investment strategies and its application is used for individuals and funds, and this approach has been also adopted in the behavioral finance literature (Odean, 1999; Barber and Odean, 2000; Barber and Odean, 2001). The performance comparison with the market portfolio benchmark is not limited by the problem of whether an individual could invest in this portfolio or not. The comparison simply evaluates how good or bad a specific investment strategy was with respect to the average return. For our purpose the market portfolio works also as an adjustment factor that allows us to compare bidding in different rounds because firms auctioned in these rounds differ in terms of prices and number of outstanding shares. Hence, a market portfolio based on market capitalization (i.e. the product of the price and number of outstanding shares) becomes the natural benchmark to use for comparing the investment decisions of individuals who want to maximize their earnings.

Our second benchmark is an informed investor (*I*) or privatization fund (PIF). It is the aggregate performance of privatization funds participating in the auction. Formally this benchmark is defined as a portfolio with weights defined as:

$$w_{i,t}^{I} = \frac{P_{i,t} D_{i,t}}{\sum_k P_{k,t} D_{k,t}} \quad \text{for } i = 1, 2, \ldots, N, \qquad (2)$$

where $P_{i,t}$ represents the price of a share of privatized firm i expressed in voucher points for a particular round t, and $D_{i,t}$ represents the number of shares acquired by an informed investor (PIF) in round t.

Informed investor (PIF) performance can be a good benchmark in earlier rounds of the auction (1–2), but in later rounds (3–6) the funds faced legal restrictions on the percentage of shares they were allowed to hold in any single firm and re-balanced their portfolios accordingly. More importantly, several hundreds of heterogeneous PIFs took part in the auctions. They differed chiefly in size (acquired points, assets) and strategy. Some funds behaved as profit maximizers and built balanced portfolios. Some bid aggressively to acquire control over specific firms but had to do so with respect to legal restrictions on maximum holdings. Since there were groups of funds established by single founders, these groups of funds were able to acquire controlling stock as a group and thus effectively circumvent the legal restrictions. Moreover, the PIFs might have been learning as well but such an effect would be impossible to disentangle given the legal restrictions imposed in later rounds and the behavior masked by being a member of a group of funds. All these facts make the informed investor (PIF) a less convenient benchmark than the market portfolio. Nevertheless, in Section 4 we present the results from using both benchmarks.

3.3 Measures of learning and hypotheses

To detect and quantify the hypothesized learning from our data, we define measures of learning based on the value of shares achieved per point spent in the bidding. From this perspective, we detect learning if the number of individuals improving their performance during the auction stages with respect to the benchmark is larger than the number of individuals with worsening performance. In terms of our set of auctions, *learning is taking place when an individual investor decreases a distance from the benchmark or, for an underperforming individual, the distance from the benchmark is getting smaller over time*.

In order to make such an assessment, let us denote the value per point (spent or bid) for the active individual *j*-th investor in round *t* as:

$$V_T(j,t) = \frac{\sum_i q_{i,t} P_i^S}{\sum_i q_{i,t} P_{i,t}}, \tag{3}$$

where $q_{i,t}$ is the number of shares in firm *i* acquired by the active individual investor in round *t*, evaluated at secondary market prices P_i^S. Hence, at each round the value per point for the active (individual) investor is defined as a ratio of the value of shares priced at the secondary market to the value of shares measured in voucher points (artificial currency used during auction). The value per point for the market portfolio in round *t* is defined as:

$$V_M(t) = \frac{\sum_i w_{i,t}^M P_i^S}{\sum_i w_{i,t}^M P_{i,t}}. \tag{4}$$

Hence, the value per point of the market portfolio is defined as the ratio of the portfolio value priced at the secondary market to the portfolio value measured in voucher points.

Similarly, the value per point for the informed investor (PIF) in round *t* is defined as:

$$V_I(t) = \frac{\sum_i w_{i,t}^I P_i^S}{\sum_i w_{i,t}^I P_{i,t}}. \tag{5}$$

An observed change due to experience (e.g. learning) means that an individual investor performs better over time in the bidding process, in the sense that individual performance either exceeds or is getting closer to the selected benchmark. Thus, we could observe how the value $V_T(j,t)$ developed over time; in a similar fashion we could observe the development of the value of the ratios $V_T(j,t)/V_M(t)$ and $V_T(j,t)/V_I(t)$. The ratios eliminate the effect of a particular auction stage but alone they would not allow us to detect (or refute) learning. Table 1 contains the performance values of the market portfolio, informed investors (PIFs) and individuals in each round. All performance values are measured in CZK per 1 point to facilitate comparison. Since all participants have 1000 points available, the mean payoffs of individuals (and their counterparts) are obtained by multiplying by 1000. High maximum payoffs can be observed in rounds 2–4. Since in round 1 the prices were set uniformly in an artificial pattern, we conjecture that during later rounds the potential payoff increases dramatically and corresponds to the high motivation of individuals. Due to the fact that the price setting mechanism is based on an excess demand rule, the prices in later rounds move closer to their equilibrium, and the market portfolio is close to the optimal bidding strategy. The decrease in prices over the succession of auction stages is also due to increasing market efficiency. The above pattern, in which an excessive rent is vanishing over time and prices converge to equilibrium, conforms to the classic argument of Smith (1962).

For the above reasons it is imperative to observe the evolution of the distance between the value of the performance of the individual investor and the performance of both benchmarks. Changes in this distance allow for accurate assessment since they are free from changes in the values of shares in successive rounds. Following the above line of reasoning, we define the measure of performance of an individual in the bidding process in the form of performance distance from the two benchmarks.

The distance of individual relative performance from market portfolio performance is defined as

$$D_M(j,t) = \max\left(1 - \frac{V_T(j,t)}{V_M(t)}; 0\right). \tag{6}$$

In a similar fashion, the distance of individual relative performance from informed investor (PIF) performance is defined as

$$D_I(j,t) = \max\left(1 - \frac{V_T(j,t)}{V_I(t)}; 0\right). \tag{7}$$

In the context of our performance indicators, learning occurs when the performance indicator calculated for each individual investor *j* improves over time. The performance indicator is bounded by zero because as the auction stages progress the auctioned prices move closer to equilibrium and it is more difficult for individual investors to outperform a benchmark. This means that a decrease in the extent of outperforming the benchmark cannot be regarded as a worsening of performance. Therefore, the zero bound represents the fact that we consider all cases when a benchmark is outperformed as a common outcome. This approach is more restrictive for those individuals performing below the selected benchmark and less restrictive for those who outperform the given benchmark.

Due to individual investors' heterogeneity, we pairwise compare the performance of each individual in the current round with that in previous rounds. Thus, the pairwise comparison is also embedded in how we formulate our null and alternative hypotheses and in that respect we fully eliminate individual heterogeneity between rounds. The null hypothesis of "no learning" indicates the case when individual performance does not improve or remains the same over time. On the other hand, in case of learning the distance between individual performance and the benchmark decreases over time; this can be conveniently translated into our alternative hypothesis. Specifically, we formulate our null hypothesis and its alternative as:

$$H_0 : D_b\left(j,t\right) \leq D_b\left(j,t+1\right), \quad H_A : D_b\left(j,t\right) > D_b\left(j,t+1\right), \quad b = I, M \ . \tag{8}$$

Thus, learning is detected when we reject the null hypothesis.

4. Statistical Inference and Empirical Evidence

4.1 Non-parametric approach

To verify our hypotheses formulated in Section 3 we essentially have two options. The first is to estimate a model and compare the relevant coefficients. In such a case, we would exploit the dynamic panel structure of our data and inspect time-varying coefficients between bidding stages, which would be equivalent to conventional *t*-tests. However, a formal model that would capture the complexity of our auction could easily be misspecified, which might result in biased coefficients and a less-than-accurate inference. Furthermore, much of the current learning literature deals with distinguishing among different learning models (see Camerer 2003, Chapter 6, for an exhaustive review). This means that the question posed is not whether there is learning, but rather which learning model best organizes the data. In contrast, we aim to answer whether there is learning evidenced by the data without being constrained by a specific model. Therefore, we opt to take a different avenue.

The second option does not involve estimation to detect learning as in essence we are interested in movements of coefficients up and down (an increase or decrease in performance) rather than in estimating the specific values of such coefficients. Therefore, it is better to use a panel data structure to properly match individual performance between rounds and test whether we observe an improvement in performance. Moreover, we do not need any distributional assumptions and employ a non-parametric approach.

For the above reasons we use the second option. We chose to verify the postulated hypotheses by conducting a series of the Wilcoxon (1945) sign rank test for matched data.[9] It tests whether the median is equal to a specific value. In that way it is able to distinguish between groups of data with different medians. Since the procedure does not involve any distributional assumptions, it is a distribution-free or non-parametric testing procedure. This robust non-parametric approach has been used as an alternative to the matched *t*-test for a long time, typically in cases when the normality assumption on data does not hold. Since subsamples of our bidding data for specific rounds are not normally distributed, the Wilcoxon sign rank test is the preferred procedure. Lehmann (1998) provides a general statistical exposition of the test. For our purposes, the test is also appropriate since it accounts for the magnitude in differences of individual performance from a benchmark that varies in different auction stages. Finally, the procedure has been frequently used in the empirical finance literature on stock or exchange rate returns, and we employ it in this sense as well.[10]

The approach outlined above allows us to track the performance of each individual investor over time at each round and compare such performance with that of our benchmark. Thanks to the extensive panel structure of our data we are able to trace the learning of individuals across rounds as well. Rejecting the null hypothesis means that learning is detected.

4.2 Empirical observations of learning

By virtue of bidding rules and limited resources (1000 points per individual only) individual investors could not revoke their accepted bids, nor could PIFs revoke their bids.

[9] In this approach we treat our data as a panel with fixed effects.
[10] For the use of the sign and sign rank tests see Flores (1986), Zivney and Thompson (1989), Corrado and Zivney (1992), Abrevaya (2000), and Fatum and Hutchinson (2003), among others.

For this reason any sign of learning detected by our empirical tests would be a lower-bound measure of potential learning in a less constrained setting. However, that would mean that if learning is detected, it is a strong indication of learning.

We present our results in several tables. The principal results of the tests of learning when the market portfolio is used as a benchmark are presented in Table 2. Row *i* and column *j* in Table 2 correspond to rounds *i* and *j* in which sub-populations of our sample successfully placed their bids. The numbers presented in this (*i, j*) intersection correspond to the results of the sign test of the hypothesis that there is no learning effect found between the corresponding round *i* and *j*. To be specific, for example, the first cell in Table 2 contains the following symbols and numbers: **L 261** **, NL 80, p–value: 0.001. This means there were 341 (=261+80) individuals who successfully placed their bids in rounds 1 and 2, for 261 individuals the distance from the benchmark (the market portfolio) decreased while for 80 individuals the distance from the benchmark increased, the p–value of the underlying sign test was 0.001, the bold face letters highlight that there was a significant increase in relative performance between rounds 1 and 2, and ** marks the level of significance (1%). Other cells on the diagonal should be interpreted in the same manner. The evidence of learning between consequent rounds (diagonal cells) is frequent: in all cells on the diagonal, learning is detected at the highest significance levels. This we can identify as an instantaneous effect of learning. Further, the number of individual investors who bid in each round and actually allocated their bids successfully increases up to the fourth round. We conclude that we detect evidence of learning between two successive periods with the two independent benchmarks used.

As a robustness check of our main results, presented in Table 2, we also present the results using a definition of distance without the max-operator (see equation 6). This robustness check is performed to assess the specific extent of bidding performance of an individual who might significantly outperform the benchmark during the beginning rounds but be less successful in later stages. With the max-operator included in our definition (6) the performance indicator is bounded by zero. When we exclude the max-operator we are more restrictive to those individuals who outperform the benchmark in successive rounds. In reality this means that we consider learning only in those cases when an individual outperforms the benchmark by the same relative performance. Our results that are positioned on the diagonal of Table 3 exhibit a similar pattern as those in Table 2: we reject the hypothesis of no learning between successive rounds with the single exception of the last stage. Further, evidence of learning in Table 3 now has a slightly different meaning than in Table 2. Due to the fact that the max-operator is removed, evidence of learning is stronger. This is because to qualify for learning, an individual outperforming the benchmark by, say, 20% in one round must outperform the benchmark by at least 20% in other rounds as well.

In Table 4 we show results when the performance of an informed investor (PIF) is taken as a benchmark. Again, row *i* and column *j* correspond to the rounds *i* and *j* in which sub-populations of our sample placed their executed bids. The numbers presented in this (*i, j*) intersection correspond to the sign test of the hypothesis that there is no learning effect found between corresponding rounds *i* and *j*. Specifically, as the first cell in Table 4 contains the following symbols and numbers: **L 187** *, NL 154, p–value: 0.042, we know that there were 341 (=187+154) individuals who successfully placed their bids in rounds 1 and 2, for 187 individuals the distance from the benchmark (the market portfolio) decreased while for 154 individuals the distance from the benchmark increased, the p–value of the underlying sign test was 0.042, the bold face letters highlight that there was a significant increase in relative performance between rounds 1 and 2, and * marks the level of significance (5%). Results on the diagonal provide evidence of learning between consecutive bidding rounds: in three cases learning was detected, in one case it was not, and one case offers a statistically insignificant result. Hence, learning (measured by the distance from the individual relative performance to the informed investor (PIF) performance) was evidenced at a moderate level. However, this evidence has to be understood in light of the different objectives of the privatization funds (mostly control of the companies), institutional barriers (a PIF could control only up to 20% of a company) and different behavior at the end of the game. For these reasons we concentrate on results provided by tests performed with the market portfolio as the benchmark. Finally, it has to be noted that it is not possible that everybody outperforms the benchmark as our evidence shows. Therefore, in terms of our comparison of uninformed and informed bidders, the advantage of the informed investors (PIFs) is depleted by the learning of the uninformed individuals.

In addition, we also consider a stronger version of learning by analyzing individuals' behavior between two non-consecutive rounds. The relevant hypothesis for both benchmarks would be formulated in the following form:

$$H_0 : D_b\left(j,t\right) \leq D_b\left(j,t+s\right), \quad H_A : D_b\left(j,t\right) > D_b\left(j,t+s\right), \qquad s > 1, b = I, M. \quad (9)$$

In fact, in hypothesis (9) we ask to what extent learning has a long-term effect. As we can see from Table 4 (PIF as the benchmark), we find evidence of long-term learning in four cases while the rest of the results lack statistical significance. Even stronger results can be found in Table 2 (market portfolio as a benchmark): in eight out of ten possible tests between non-consecutive rounds we reject the null hypothesis of no improvement in performance among individual investors. Recall that these are the results of the nonparametric test: we can analyze whether the test rejects the hypothesis on a given significance level and we can do this over time. But it would be rather misleading to analyze patterns of statistics forming the series of such tests since the statistics are formed on populations of bidders that differ across all rounds. For this reason, we concentrate on the evidence or lack of evidence of learning rather than on the degree of learning, since the degree of learning cannot be properly assessed. In any event, based on our results we can confirm with a high level of significance the presence of long-term learning effects in the investors' behavior.

The strict budget constraints (all bidders had vouchers of 1000 points) may evoke the possibility of an alternative explanation for the narrowing of portfolio performance over time by speculating that informed bidders bid more aggressively (and at a higher price) in earlier rounds, thus reducing their numbers of points, their flexibility of portfolio choice, and their advantage over less informed bidders. Such an alternative explanation is, however, faulty. Informed investors (PIFs) managed a total of 3.96 billion points (63.5% of all points available). Out of these available points the funds spent 19.7% and 42.2% of the points during the first and second rounds, respectively. Thus, about 40% of their points were still left for bidding in the remaining rounds.

Nevertheless, we explore the behavior of bidders with limited information who faced a realistic budget constraint from round to round as their allotment of points decreased and the relative value was affected by changes in the prices of different shares. The results in Tables 5–6 account for the "budget constraint" of 200 and 300 points being available as a minimum for the two bidding rounds compared. We impose such a budget constraint since we hypothesize that it could affect or interfere with learning.

The 200-point constraint (Table 5) does not significantly change the results from the no-constraint case. The distance between the benchmark (market portfolio) increases for a somewhat smaller number of individual investors when compared with the no-constraint case (Table 4), but the evidence of learning between consequent rounds (diagonal cells) is again substantial. In all cells on the diagonal, learning is detected at the highest significance levels, which we identify as an instantaneous effect of learning even when the 200-point budget constraint is considered. Further, as in the no-constraint case, the number of individual investors who bid in each round and who successfully allocated their bids increases up to the fourth round.

The same pattern emerges when we augment the budget constraint of available points between successive rounds to 300 points (Table 6). The significant increases in relative performance are less frequent than in the no-constraint and 200-point-constraint cases, but the evidence of learning is again solid. The exception to this pattern is the lack of significance related to bidding in the sixth round. Such an outcome should be expected, though. Given the announcement of the privatization authorities, individuals knew that the sixth round was the last round and that unspent points would be worthless afterwards. Thus, with the imposed budget constraint, individual investors tend to bid for any shares still available regardless of their price (up to their available points) and learning should not be expected in a situation like this.

The Wilcoxon sign rank test takes into account the magnitude of the differences, e.g. the magnitude of the movements of individual bidders towards the benchmark. Thus, the test effectively eliminates cases when, for example, some of the individuals could have an extremely suboptimal bidding strategy in, say, round 1 and alter it insignificantly but in the right direction in round 2. Further, learning took place in a consistent manner across individual bidders since learning was detected not only between successive rounds (on the diagonal in Tables 2–6) but also between non-consecutive rounds (off the diagonal). If individual bidders followed the above hypothetical bidding strategy on a large scale, such a pattern would not emerge in the first place.

The results presented in Tables 2–6 provide evidence that learning occurred across a large number of bidders: both better and worse performers were able to reduce the distance from the benchmark. The majority of bidders were able to bid close to the benchmark, but the proportion decreased with successive bidding stages. This pattern is quite understandable: as the auction stages progressed, the market became more efficient and prices reflected the available information. To conclude, the majority of individuals learned during the bidding process but a substantial number of bidders did not learn as their distances from both benchmarks increased. Thus, our results suggest that there was learning on average.

As originally uninformed private bidders became experienced through learning, our results are in line with the findings of List (2004, p. 1133), who concludes that "market experience plays a role in the distribution of rents: experienced market players earn more rents than inexperienced agents." Further, the design of our auction contains multiple market periods that allow agents to learn based on their accumulated experience.[11] We have shown that learning occurred across a large number of bidders between successive as well as non-successive auction stages. In this context, our results lend a new perspective to the findings of Smith (1962) who shows evidence of inter-period learning and List (2004) who advocates that agents learn on the basis of accumulated experience.

5. Conclusions

We have tested for learning among heterogeneous individuals engaged in bidding on a proxy stock market who possessed limited information about that market's assets. We used an extensive data set taken from a naturally-occurring auction through which state-owned enterprises were privatized in the Czech Republic (1993–1994). We developed new measures of individual performance to accommodate for the varying prices of goods (shares) available in six successive stages of bidding.

Specifically, we used the equivalent of a market portfolio and an informed investor as benchmarks and we scaled the performance of individual investors to these benchmarks. Based on our specification we observe learning when an individual investor decreases a

[11] Since less-informed bidders observed the behavior of well-informed bidders (by knowing *ex post* what the PIFs invested in) and to an extent factored it into their bidding, less informed bidders show the features of social learning as described by Ballinger, Palumbo and Wilcox (2003).

distance from the benchmark. As a robustness check we also imposed the budget constraint of a minimum number of points available for the two compared (successive) bidding rounds, which we hypothesized would negatively affect learning. We performed all possible pair-matched Wilcoxon sign rank tests and we were able to reject the null hypothesis of no learning. Moreover, learning was found to be significant in nearly all tests performed for both successive and non-successive rounds. An exception are the insignificant results with respect to the sixth bidding stage, especially when a significant budget constraint was imposed. This result is due to the fact that the sixth stage was announced as the last bidding stage and individuals were spending their remaining points no matter what a result might be since the points would be worthless afterwards.

Our analysis supplements the theoretical literature on learning by providing evidence of learning that took place during a naturally-occurring auction. This auction observed in the field generated a large data set that in size, incentives, and variation is superior to any laboratory experiment we know of. Since individuals had to pay a fee to participate, and since the potential gain was in the magnitude of several months' salary, we conclude that large incentives were driving our results.

References

Abrevaya, J. (2000). 'Testing for a treatment effect in a heterogeneous population: a modified sign–test statistic and a leapfrog statistic', *Journal of Applied Statistics*, 27(6), pp. 679–687.

Aggarwal, R. and Harper, J. T. (2000). 'Equity valuation in the czech voucher privatization auctions', *Financial Management*, 29(4), pp. 77–100.

Arellano, M. and Bond, S. (1991). 'Some tests of specification for panel data: monte carlo evidence and an application to employment equations', *Review of Economic Studies*, 58(2), pp. 277–297.

Ballinger, T. P., Palumbo, M. G. and Wilcox, N. T. (2003). 'Precautionary saving and social learning across generations: an experiment', *The Economic Journal*, 113(490), pp. 920–947.

Barber, B. M. and Odean, T. (2000). 'Trading is hazardous to your wealth: the common stock investment performance of individual investors', *Journal of Finance*, 55(2), pp. 773–806.

Barber, B. M. and Odean, T. (2001). 'Boys will be boys: Gender, Overconfidence, and Common Stock Investment', *Quarterly Journal of Economics*, 116(1), pp. 261–292.

Camerer, C. F. (2003). *Behavioral Game Theory: Experiments in Strategic Interaction*, Princeton, New Jersey: Princeton University Press.

Corrado, C. J. and Zivney, T. L. (1992). 'The specification and power of the sign test in event study hypothesis tests using daily stock return', *Journal of Financial and Quantitative Analysis*, 27(3), pp. 465–478.

Erev, I. and Roth, A. E. (1998). 'Predicting how people play games: reinforcement learning in experimental games with unique, mixed strategy equilibria', *American Economic Review*, 88(4), pp. 848–881.

Estrin, S., Hanousek, J., Kočenda, E. and Svejnar, J. (2009). 'Effects of privatization and ownership in transition economies', *Journal of Economic Literature*, 47(3), pp. 699-728.

Fatum, R. and Hutchison, M. M. (2003). 'Is sterilised foreign exchange intervention effective after all? an event study approach', *The Economic Journal*, 113(487), pp. 390–411.

Filer, R.K. and Hanousek, J. (2001). 'Efficiency of price setting based on a simple excess demand rule: the natural experiment of czech voucher privatization', *European Economic Review*, 45(9), pp. 1619–1646.

Flores, B. E. (1986). 'Use of the sign test to supplement the percentage better statistic', *International Journal of Forecasting*, 2(4), pp. 477–489.

Fudenberg, D. and Levine, D. (1998). *The Theory of Learning in Games*, Cambridge, Massachusetts: MIT Press.

Hanousek, J. and Kočenda, E. (2008). 'Potential of the state to control privatized firms', *Economic Change and Restructuring*, 41(2), pp. 167-186.

Hanousek, J. and Kroch, E. A. (1998). 'The two waves of voucher privatization in the czech republic: a model of learning in sequential bidding', *Applied Economics*, 30, pp. 133–143.

Hanousek, J., Kočenda, E. and Svejnar, J. (2007). 'Origin and concentration: corporate ownership, control and performance in firms after privatization', *Economics of Transition*, 15(1), pp. 1-31.

Hingorani, A., Lehn, K. and Makhija, A. K. (1997). 'Investor behavior in mass privatization: The case of the Czech voucher scheme', *Journal of Financial Economics*, 44(3), pp. 349-396.

Kachelmeier, S. J. and Shehata, M. (1992). 'Examining risk preferences under high monetary incentives: experimental evidence from the People's Republic of China', *American Economic Review*, 82(5), pp. 1120–41.

Kočenda, E. (1999). 'Residual state property in the Czech Republic', *Eastern European Economics*, 37(5), pp. 6–35.

Kočenda, E. and Valachy, J. (2002). 'Firm ownership structures: dynamic development', *Prague Economic Papers*, 11(3), pp. 255-268.

Lehmann, E. L. (1998). *Nonparametrics: Statistical Methods Based on Ranks, Revised*, Engelwood Cliffs, New Jersey: Prentice Hall.

Lintner, J. (1965). 'The valuation of risk assets and the selection of risky investment in stock portfolios and capital budgets', *Review of Economics and Statistics*, 47, pp. 13–37.

List, J. A. (2004). 'Testing neoclassical nompetitive theory in multilateral decentralized markets', *Journal of Political Economy*, 112(5), pp. 1131–1156.

Mosin, J. (1966). 'Equilibrium in a capital asset market', *Econometrica,* 34, pp. 768–783.

Newcombe, R. G. (1998). 'Two–sided confidence intervals for the single proportion: comparison of seven methods', *Statistics in Medicine*, 17(8), pp. 857–872.

Odean, T. (1999). 'Do investors trade too much?' *American Economic Review*, 89(5), pp. 1279–98.

Sharpe, W. (1964). 'Capital asset prices: a theory of capital market equilibrium under condition of risk', *Journal of Finance*, 19, pp. 425–442.

Sharpe, W. (1970). *Portfolio Theory and Capital Markets*, New York: McGraw–Hill.

Slonim, R. and Roth, A. E. (1998). 'Learning in high stakes ultimatum games: an experiment in the slovak republic', *Econometrica*, 66(3), pp. 569–96.

Smith, V. L. (1962). 'An experimental study of competitive market behavior', *Journal of Political Economy*, 70(2), pp. 111–137.

Smith, V. L. (1976). 'Experimental economics: induced value theory', *American Economic Review Proceedings*, 66, pp. 247–279.

Smith, V. L. (1982). 'Microeconomic systems as an experimental science', *American Economic Review*, 72, pp. 923–955.

Vega–Redondo, F. (2003). *Economics and the Theory of Games,* Cambridge: Cambridge University Press.

Weibull, J. (1995). *Evolutionary Game Theory,* Cambridge: MIT Press.

Wilcox, N. (2006). 'Theories of learning in games and heterogeneity bias', *Econometrica,* 74, pp.1271-1292.

Wilcoxon, F. (1945). 'Individual comparisons by ranking methods', *Biometrics*, 1, pp. 80–83.

Zivney, T. L. and Thompson, D. J. (1989). 'The specification and power of the sign test in measuring security price performance: comments and analysis', *Financial Review*, 24(4), pp. 581–88.

Table 1. Descriptive Statistics.
Performance of portfolios in CZK per 1 point.

Round	Market portfolio	Informed investor portfolio	Individuals		
	mean	mean	mean	maximum	95% confidence interval
1	4.285	8.013	7.330	25.149	(7,070 ; 7,590)
2	6.514	12.514	15.043	313.010	(13,535 ; 16,551)
3	6.060	7.895	10.543	227.540	(9,717 ; 11,369)
4	7.384	9.418	7.522	126.400	(7,304 ; 7,739)
5	10.077	11.416	10.307	55.897	(10,047 ; 10,566)
6	14.048	17.312	13.463	34.337	(12,958 ; 13,969)

Sample size: 5000 individuals.

Note: CZK is the abbreviation for the Czech currency unit.

The values of the portfolios presented in this table are valued at prices on the secondary market after the voucher scheme (auction) ended. To facilitate comparison, the values of all portfolios are measured in CZK per 1 point. Since all participants have 1000 points available, the mean payoffs of individuals (and their counterparts) are obtained by multiplying by 1000.

Table 2. Summary of Tests of Learning in Future Rounds.
Performance with respect to the market portfolio benchmark in each round.

	Round 2	Round 3	Round 4	Round 5	Round 6
Round 1	L **261** *** NL 80 p–value: 0.001	L **267** *** NL 94 p–value: 0.001	L **284** *** NL 206 p–value: 0.001	L **135** *** NL 78 p–value: 0.001	L 56 NL 44 p–value: 0.136
Round 2		L **354** *** NL 127 p–value: 0.001	L **330** *** NL 237 p–value: 0.001	L **170** *** NL 96 p–value: 0.001	L 65 NL 70 p–value: 0.697
Round 3			L **503** *** NL 390 p–value: 0.001	L **248** *** NL 157 p–value: 0.001	L **104** ** NL 76 p–value: 0.022
Round 4				L **398** *** NL 206 p–value: 0.001	L **163** *** NL 111 p–value: 0.001
Round 5					L **180** *** NL 127 p–value: 0.002

L denotes learning, NL denotes no learning observed. On the bottom we list the p-value of the sign test.
** and *** denote the 5% and 1% significance levels, respectively.

Total sample size: 5000 individuals.

Note: Row *i* and column *j* in this table correspond to rounds *i* and *j* in which sub-populations of our sample placed their bids. The numbers presented in this (*i*, *j*) intersection correspond to the sign test of the hypothesis that there is no learning effect found between the corresponding rounds *i* and *j*. For example, the first cell contains the following symbols and numbers: L **261** ***, NL 80, p–value: 0.001. This means that there were 341 (=261+80) individuals who successfully placed their bids in rounds 1 and 2, for 261 individuals the distance from the benchmark (the market portfolio) decreased while for 80 individuals the distance from the benchmark increased, the p-value of the underlying sign test was lower than 0.001, and bold face letters highlight that there was a significant increase in relative performance between rounds.

Table 3. Summary of Tests of Learning in Future Rounds.
Performance with respect to the market portfolio benchmark in each round.
Definition of the distance without the max-operator.

	Round 2	Round 3	Round 4	Round 5	Round 6
Round 1	L **202** *** NL 139 p–value: 0.001	L **208** *** NL 157 p–value: 0.008	L **284** *** NL 206 p–value: 0.001	L **118** *** NL 95 p–value: 0.066	L 45 NL 55 p–value: 0.184
Round 2		L **255*** NL 226 p–value: 0.098	L **308** ** NL 259 p–value: 0.022	L 136 NL 130 p–value: 0.379	L 59 NL 76 p–value: 0.939
Round 3			L **478** ** NL 415 p–value: 0.019	L **233** *** NL 172 p–value: 0.001	L 95 NL 85 p–value: 0.251
Round 4				L **361** *** NL 243 p–value: 0.001	L **152** ** NL 122 p–value: 0.040
Round 5					L 159 NL 148 p–value: 0.284

L denotes learning, NL denotes no learning observed. On the bottom we list the p-value of the sign test.
* , **, and *** denote the 10%, 5%, and 1% significance levels, respectively.

Total sample size: 5000 individuals.

Note: Row *i* and column *j* in this table correspond to rounds *i* and *j* in which sub-populations of our sample placed their bids. The numbers presented in this (*i, j*) intersection correspond to the sign test of the hypothesis that there is no learning effect found between the corresponding rounds *i* and *j*. For example, the first cell contains the following symbols and numbers: L **202** ***, NL 139, p–value: 0.001. This means that there were 341 (=202+139) individuals who successfully placed their bids in rounds 1 and 2, for 202 individuals the distance from the benchmark (the market portfolio) decreased while for 139 individuals the distance from the benchmark increased, the p-value of the underlying sign test was lower than 0.001, and bold face letters highlight that there was a significant increase in relative performance between rounds.

Table 4. Summary of Tests of Learning in Future Rounds.
Performance with respect to the informed investor (PIF) benchmark in each round.

	Round 2	Round 3	Round 4	Round 5	Round 6
Round 1	L **187** ** NL 154 p-value: 0.042	L **254** *** NL 107 p-value: 0.001	L 231 NL 259 p-value: 0.905	L **132** *** NL 81 p-value: 0.001	L 56 NL 44 p-value: 0.136
Round 2		L **344** *** NL 137 p-value: 0.001	L 289 NL 278 p-value: 0.337	L **178** *** NL 88 p-value: 0.001	L 76 NL 59 p-value: 0.084
Round 3			L 328 NL **565** *** p-value: 0.001	L 200 NL 205 p-value: 0.617	L 91 NL 89 p-value: 0.470
Round 4				L **391** *** NL 213 p-value: 0.001	L **151** ** NL 123 p-value: 0.050
Round 5					L 151 NL 156 p-value: 0.634

L denotes learning, NL denotes no learning observed. On the bottom we list the p-value of the sign test. ** and *** denote the 5% and 1% significance levels, respectively.

Total sample size: 5000 individuals.

Note: Row *i* and column *j* in this table correspond to rounds *i* and *j* in which sub-populations of our sample placed their bids. The numbers presented in this (*i*, *j*) intersection correspond to the sign test of the hypothesis that there is no learning effect found between the corresponding rounds *i* and *j*. For example, the first cell contains the following symbols and numbers: L **187** **, NL 154, p–value: 0.042. This means that there were 341 (=187+154) individuals who successfully placed their bids in rounds 1 and 2, for 187 individuals the distance from the benchmark (the market portfolio) decreased while for 154 individuals the distance from the benchmark increased, the p-value of the underlying sign test was lower than 0.05, and bold face letters highlight that there was a significant increase in relative performance between rounds.

Table 5. Summary of Tests of Learning in Future Rounds.
Performance with respect to the market portfolio benchmark in each round.
Budget constraint restriction: at least 200 points available in each compared round.

	Round 2	Round 3	Round 4	Round 5	Round 6
Round 1	L **260** *** NL 80 p–value: 0.001	L **261** *** NL 93 p–value: 0.001	L **276** *** NL 199 p–value: 0.001	L **110** *** NL 71 p–value: 0.001	L 34 NL 25 p–value: 0.149
Round 2		L **353** *** NL 124 p–value: 0.001	L **310** *** NL 224 p–value: 0.001	L **147** *** NL 82 p–value: 0.001	L 49 NL 52 p–value: 0.697
Round 3			L **481** *** NL 378 p–value: 0.001	L **216** *** NL 137 p–value: 0.001	L **71** ** NL 51 p–value: 0.043
Round 4				L **350** *** NL 186 p–value: 0.001	L **163** *** NL 111 p–value: 0.001
Round 5					L **141** *** NL 88 p–value: 0.001

L denotes learning, NL denotes no learning observed. On the bottom we list the p-value of the sign test.
** and *** denote the 5% and 1% significance levels, respectively.

Total sample size: 5000 individuals.

Note: Row *i* and column *j* in this table correspond to rounds *i* and *j* in which sub-populations of our sample placed their bids. The numbers presented in this (*i*, *j*) intersection correspond to the sign test of the hypothesis that there is no learning effect found between the corresponding rounds *i* and *j*. For example, the first cell contains the following symbols and numbers: L **260** ***, NL 80, p–value: 0.001. This means that there were 340 (=260+80) individuals who successfully placed their bids in rounds 1 and 2, for 260 individuals the distance from the benchmark (the market portfolio) decreased while for 80 individuals the distance from the benchmark increased, the p-value of the underlying sign test was lower than 0.001, and bold face letters highlight that there was a significant increase in relative performance between rounds.

Table 6. Summary of Tests of Learning in Future Rounds.
Performance with respect to the market portfolio benchmark in each round.
Budget constraint restriction: at least 300 points available in each compared round.

	Round 2	Round 3	Round 4	Round 5	Round 6
Round 1	L **257** *** NL 80 p–value: 0.001	L **250** *** NL 82 p–value: 0.001	L **244** *** NL 175 p–value: 0.001	L **69** *** NL 41 p–value: 0.005	L 14 NL 12 p–value: 0.422
Round 2		L **320** *** NL 108 p–value: 0.001	L **272** *** NL 199 p–value: 0.001	L **107** *** NL 52 p–value: 0.001	L 23 NL 26 p–value: 0.716
Round 3			L **442** *** NL 338 p–value: 0.001	L **141** *** NL 88 p–value: 0.001	L 30 NL 27 p–value: 0.396
Round 4				L **239** *** NL 114 p–value: 0.001	L 38 NL 34 p–value: 0.362
Round 5					L 60 NL 51 p–value: 0.224

L denotes learning, NL denotes no learning observed. On the bottom we list the p-value of the sign test.
*** denotes the 1% significance level.

Total sample size: 5000 individuals.

Note: Row *i* and column *j* in this table correspond to rounds *i* and *j* in which sub-populations of our sample placed their bids. The numbers presented in this (*i*, *j*) intersection correspond to the sign test of the hypothesis that there is no learning effect found between the corresponding rounds *i* and *j*. For example, the first cell contains the following symbols and numbers: L **257** ***, NL 80, p–value: 0.001. This means that there were 337 (=257+80) individuals who successfully placed their bids in rounds 1 and 2, for 257 individuals the distance from the benchmark (the market portfolio) decreased while for 80 individuals the distance from the benchmark increased, the p-value of the underlying sign test was lower than 0.001, and bold face letters highlight that there was a significant increase in relative performance between rounds.