



Introduction to Game Theory

Lecture 7

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

Today's Plan

- Dynamic games
 - Subgames
 - Backward Induction
 - Action vs. Strategy
 - Subgame Perfect Nash Equilibrium
- Midterm

Static vs. Dynamic Games

- Static games:
 - players move simultaneously
 - players choose one action only
 - **Nash equilibrium – set of actions**
one action (best response) for each player
- Dynamic games:
 - players move sequentially
 - players choose action in their each decision node
 - **SPNE – set of strategies**
one strategy (set of best responses) for each player

Action vs. Strategy

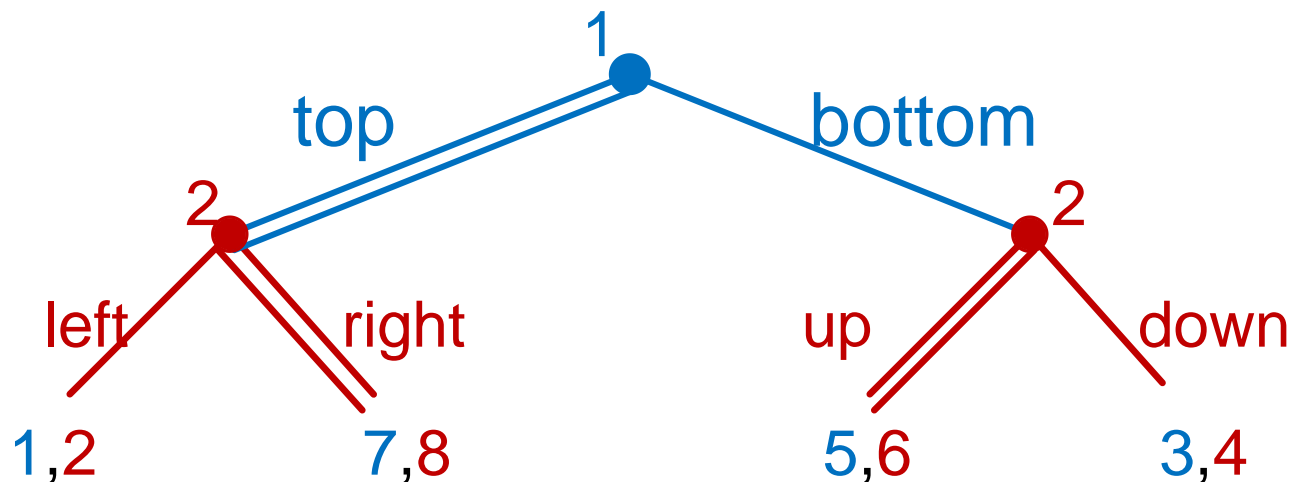
- **action** is a decision in one particular node
(confess, remain silent, head, tail,...)
- **strategy** is a plan of actions for every possible situation that might occur, for every possible node (AF-Accept if Albert goes In, Fight if Albert plays Out)
- **strategy** – it is deciding about the action in each decision node prior to the game
- it is like as if you want your friend to play the game instead of you, you have to tell him in advance what to do in each situation

NE in Dynamic Games

- The concept of SPNE:
 - in static games not needed (only one subgame)
 - in dynamic games – Backward Induction
- The concept of NE:
 - in static games – best responses (circles method)
 - in dynamic games – the concept exists
 - How to find NE in dynamic games?
 - => Same best responses method as in static games but **use strategies instead of actions**

NE in Dynamic Games

- Example:



Player 1: actions = strategies: top, bottom

Player 2: actions: left, right, up, down

strategies: left,up; left,down;

right,up; right,down

SPNE: (top;(right,up))

NE in Dynamic Games

- Example:

Player 1: actions = strategies: top, bottom

Player 2: actions: left, right, up, down

strategies: left,up; left,down;

right,up; right,down

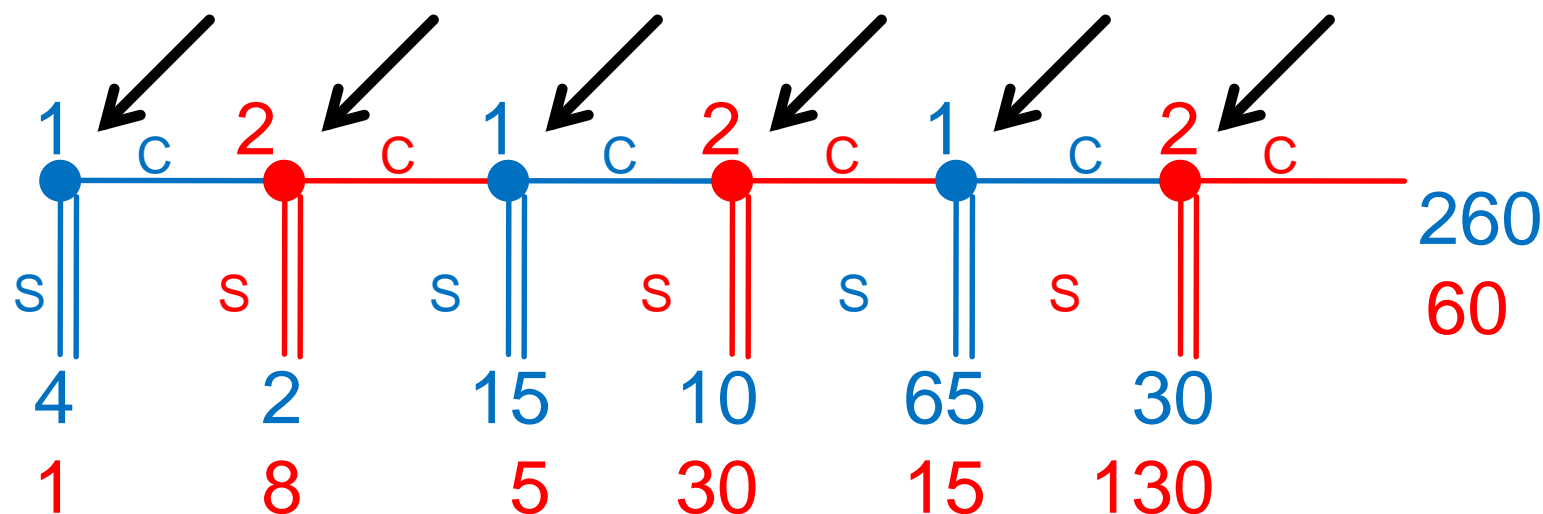
| 1 \ 2 | lu | ld | ru | rd |
|--------|-------|-------|-------|-------|
| top | 1,2 | 1,2 | (7,8) | (7,8) |
| bottom | (5,6) | (3,4) | (5,6) | 3,4 |

Three NE: (bottom,lu), (top,ru), and (top,rd)

Centipede Game

Two-player game, six periods

Each player in each node: Stop, Continue



The only SPNE in this game is to play stop for all players in all decision nodes

Centipede Game

Experimental evidence

- college students in the USA
 - most of the games finished in 4-5 period
 - if experienced – closer to SPNE

Possible explanations for not playing SPNE

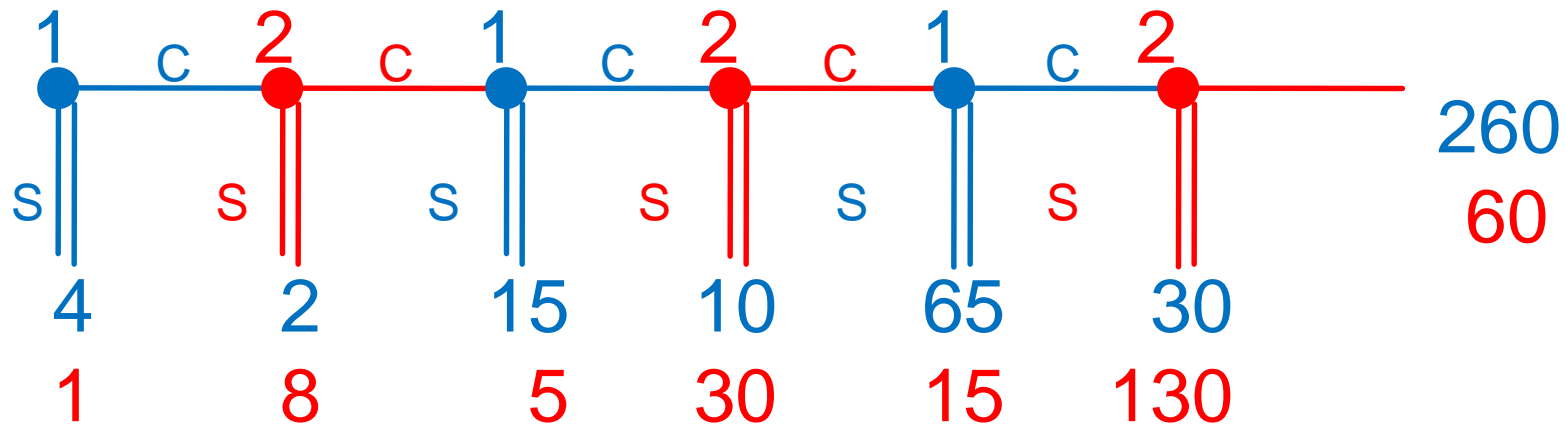
- altruistic behavior
- error
- large incentives – closer to SPNE

Chess players

- better player = lower probability to continue
- grandmasters – stop at their first chance
(more familiar with backward induction)

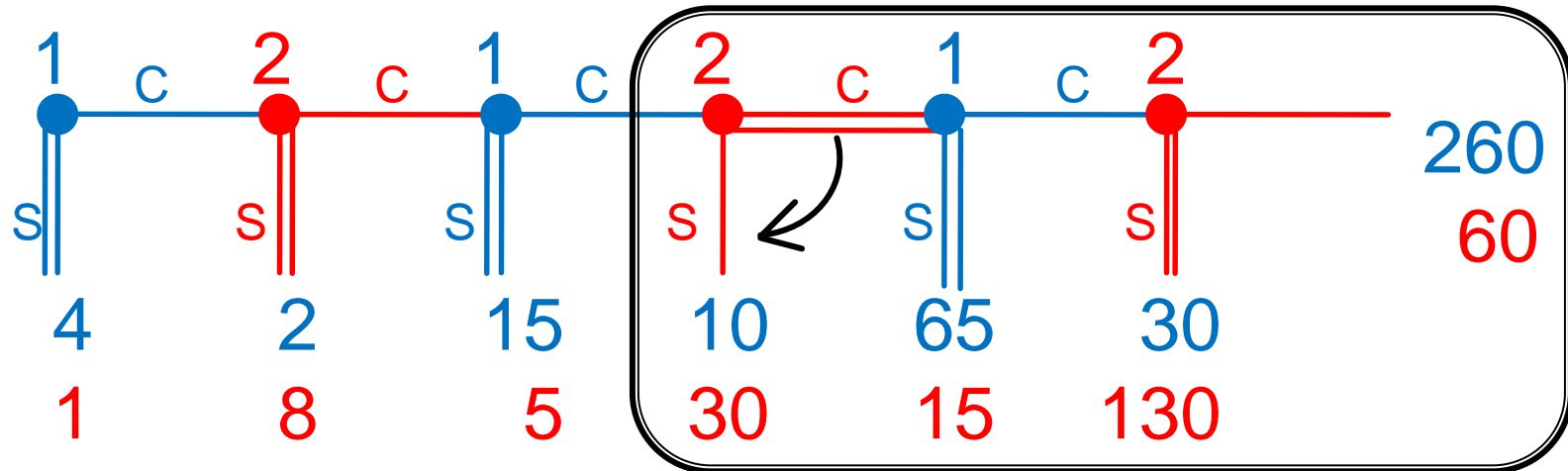
Centipede Game

Actions vs. Strategies



- Actions: Stop, Continue (for each player)
- Strategies:
 - Player 1: e.g. SSS, CSC, CCS, etc. (8 strategies)
 - Player 2: the same

Centipede Game



SPNE in Centipede game: $\{SSS, SSS\}$

- where first 3 letters are strategy of the first player and 2nd 3-letter combination is strategy of 2nd player

e.g. $\{SSS, SCS\}$ is not SPNE, because it is not optimal in one of the subgames (15 is less than 30)

Simple Ultimatum Game

Simple ultimatum game:

- split \$10, only two options are possible: offer small amount or offer large amount

1st player: single decision node

actions=strategies: small, large

2nd player: 2 decision nodes based on P1 choice

actions: accept, reject

strategies: ar, aa, etc.

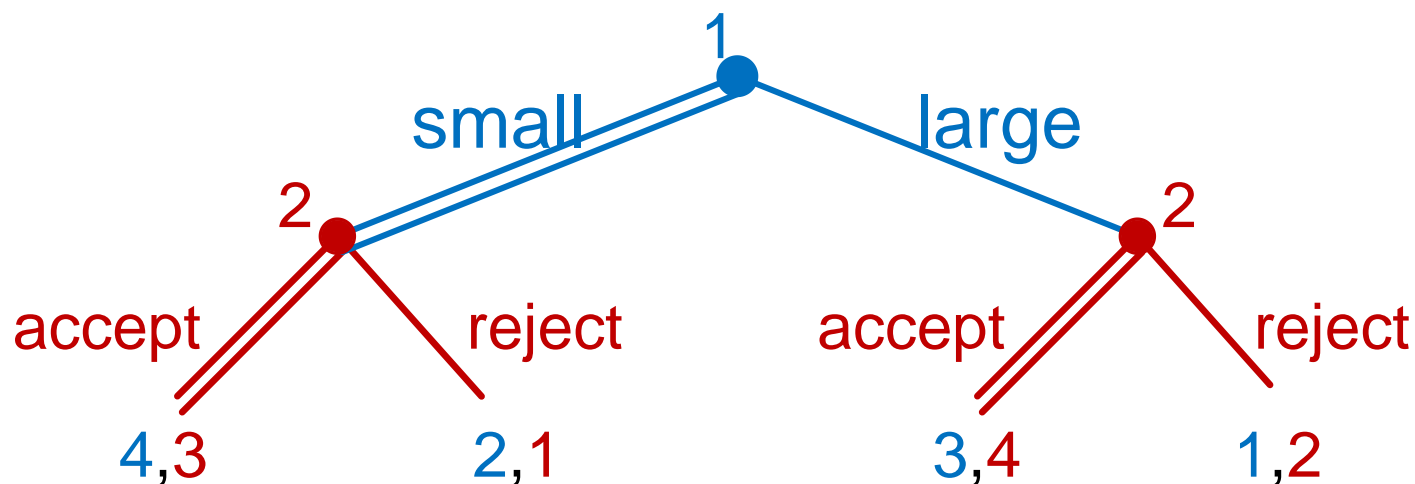
(there are 4 strategies)

SPNE: one number (action, strategy) for player 1

one combination of 2 letters for player 2

Simple Ultimatum Game

- Simple ultimatum game:



Player 1: actions = strategies: small, large

Player 2: actions: accept, reject

strategies: accept,accept; accept,reject,
reject,accept; reject,reject

SPNE: (small,aa)

NE in Dynamic Games

- Simple ultimatum game:

Player 1: actions = strategies: small, large

Player 2: actions: accept, reject

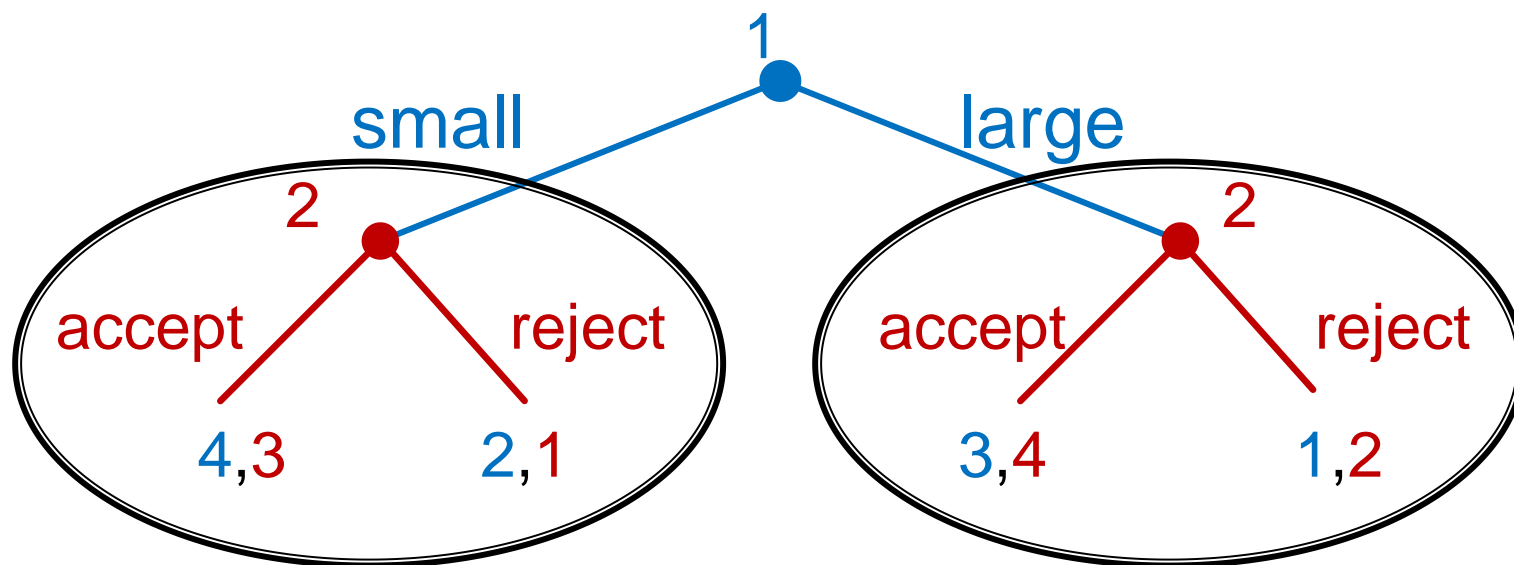
strategies: accept,accept; accept,reject,
reject,accept; reject,reject

| 1 \ 2 | aa | ar | ra | rr |
|-------|-------|-------|-------|-------|
| small | (4,3) | (4,3) | 2,1 | (2,1) |
| large | 3,(4) | 1,2 | (3,4) | 1,2 |

Three NE: (small,aa), (small,ar), and (large,ra)

NE in Dynamic Games

- Three NE: (small,aa), (small,ar), and (large,ra)
 - only the first is SPNE as well



Two subgames

- (small,ar) is not optimal in the right subgame
- (large,ra) is not optimal in the left subgame
- **only (small,aa) is optimal everywhere => SPNE**

Ultimatum Game

Ultimatum game:

- split \$10, only integers between 0 and 10 including are possible

1st player: single decision node

actions=strategies: 0,1,2,...,9,10

2nd player: 11 decision nodes based on P1 choice

actions: accept, reject

strategies: ararararara, aaaaarrrrrr, etc.

(there are 2^{11} strategies)

SPNE: one number (action, strategy) for player 1

one combination of 11 letters for player 2

Ultimatum Game

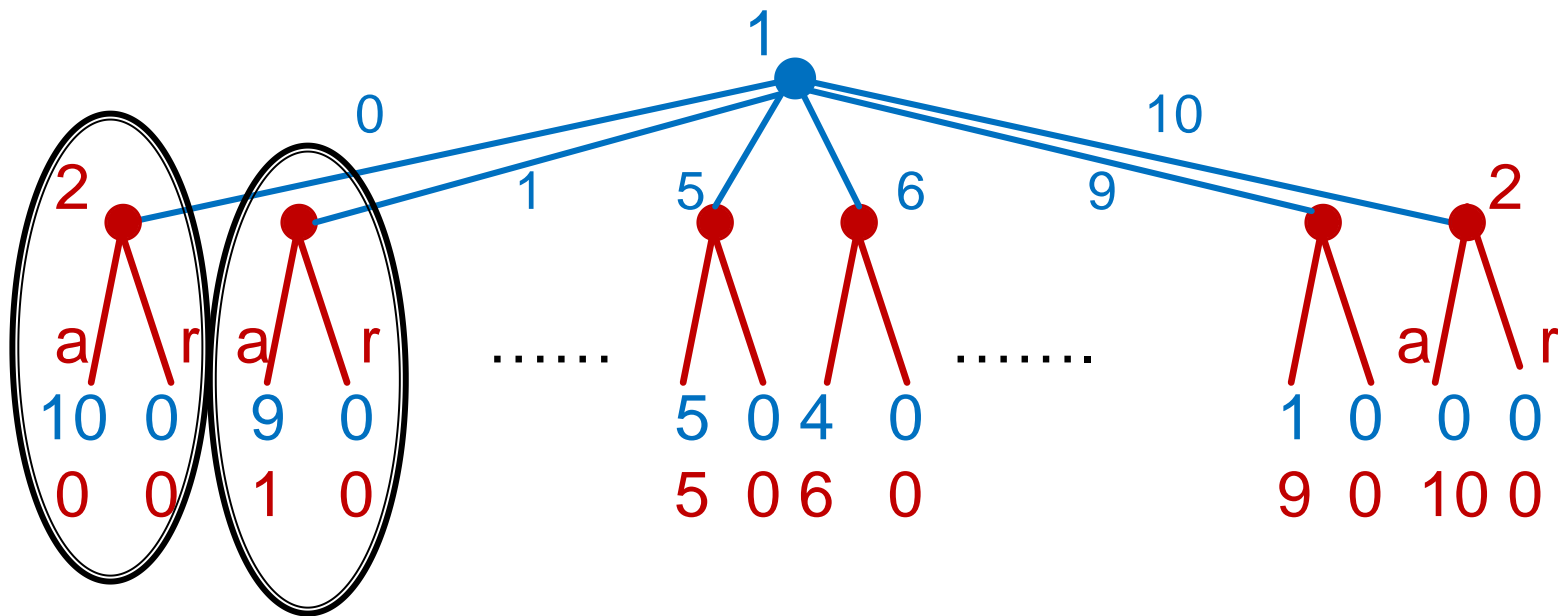
Action vs. Strategy:

- SPNEs are:
 - $\{0, \text{aaaaaaaaaaa}\}$
 - $\{1, \text{raaaaaaaaaaaa}\}$
- Any other offer from the 1st player is not optimal because he could increase his payoff by offering less
- Any other combination of accept/reject choice would not be credible threat
 - e.g. $\{5, \text{rrrrraaaaaaa}\}$ is NE, but 2nd player's threat to reject everything below 5 is not credible => not SPNE

Ultimatum Game

NE vs. SPNE:

- e.g. $\{5, rrrrrraaaaaa\}$ is NE, but 2nd player's threat to reject everything below 5 is not credible \Rightarrow not SPNE (not optimal in the second subgame)



Summary

- SPNE is refinement of NE \Rightarrow $SPNE \subset NE$
- NE in static games – best responses
- NE in dynamic games – strategies instead of actions, best responses
- SPNE in static games – not needed
- SPNE in dynamic games – Backward Induction
- action – decision in one node
- strategy – decisions for each node in the game
- NE – actions; SPNE – strategies

Midterm Exam

!!!! Surname starting A - M – 18:00 !!!!

!!!! Surname starting N - Z – 18:45 !!!!

Static games:

actions, action profiles, iterative elimination of dominated strategies, NE, mixed strategies, dominated strategies in mixed strategies, MSNE

Dynamic games:

Backward induction, strategies, NE in dynamic games, SPNE

Osborne – chapters 1,2,4,5,6