Introduction to Game Theory
Lecture 5

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

Today's Plan

- Review

- Dynamic games, Subgames, Subgame Perfect NE (SPNE)

- Actions vs. strategies

- Game

Review

- Games of complete and perfect information
  - Static Games (Nash Equilibrium)
  - Dynamic Games (Backward Induction)

- Games of complete but imperfect information
  - Dynamic Games (Subgame perfect NE)

- Games of incomplete information
  - Static Games (Auctions)
  - Dynamic Games (Signaling)

Review

- Static game:

  - players move simultaneously

  - players are not informed about previous actions

  - there is no order in which players move

  - is represented by normal form (table)

Review

Static games with complete and perfect info:

- Game: players, actions, preferences over outcomes

- Dominance – eliminate systematically worse actions

- Nash Equilibrium – no one can unilaterally improve their situation; everybody plays best response given other players' choices

- Mixed Strategy Nash Equilibrium

Preview

- Games of complete and perfect information
  - Static Games (Nash Equilibrium)
  - Dynamic Games (Backward Induction)

- Games of complete but imperfect information
  - Dynamic Games (Subgame perfect NE)

- Games of incomplete information
 - Static Games (Auctions)
  - Dynamic Games (Signaling)

Preview

- Dynamic game:

  - players move sequentially

  - players are fully informed about previous actions

  - order in which players move is given

  - is represented by extensive form (decision tree)

Dynamic Games

Sequential version of Prisoner's dilemma game
- one prisoner chooses to confess or remain silent as the first player
- second prisoner observes the choice of the first one and decides accordingly
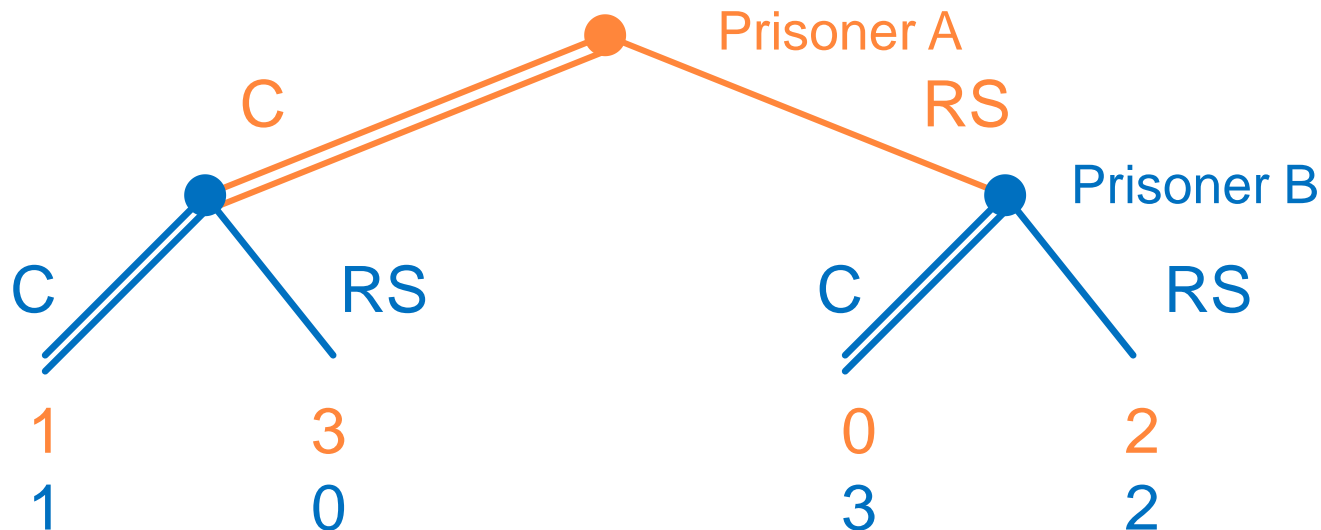
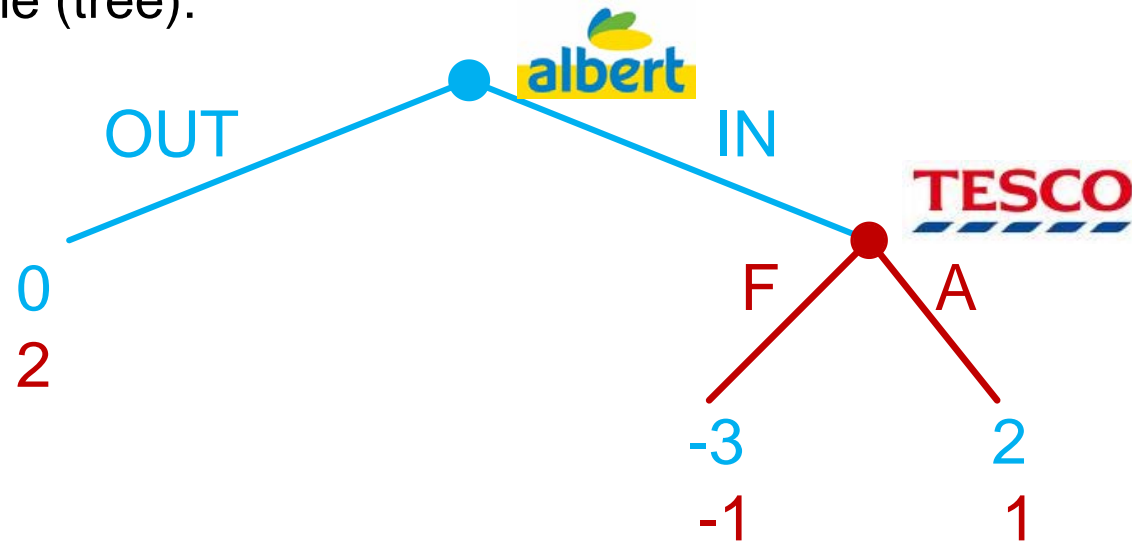We found this as NE in static game. Why bother with this?

Dynamic Games

Albert is considering opening a new supermarket across Tesco on Narodni trida. If it does, Tesco can either do nothing (accept - A) or start a price war to force Albert to close (fight - F)
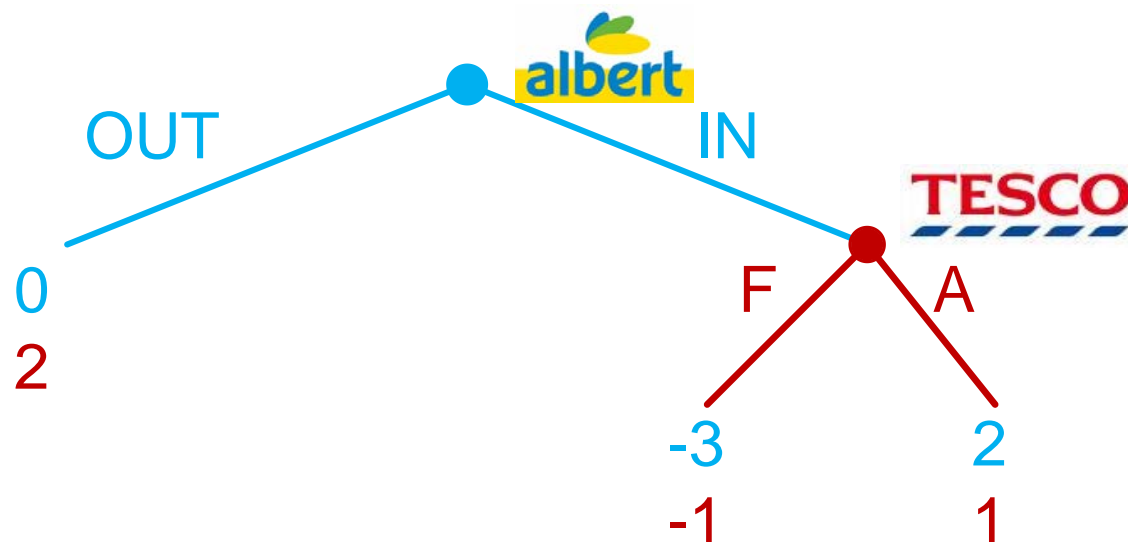
Dynamic Games

Dynamic Game (tree):

Static game (table):

| albert / TESCO	F	A
IN	-3,-1	2,1
OUT	0,2	0,2

NE:
(IN,A)
(OUT,F) - ?

Dynamic Games

- NE – imperfections:

  - NE concept has a low predictive power, since generally we can have too many NE in a game

  - This concept sometimes gives not sensible predictions (non-credible threats such as "Stay out or I will fight")

- Not sensible predictions – NE concept does not require reasonable behavior in those parts of the game which players do not expect to play

- To overcome this – we introduce Subgame Perfect NE (SPNE) which solves this problem
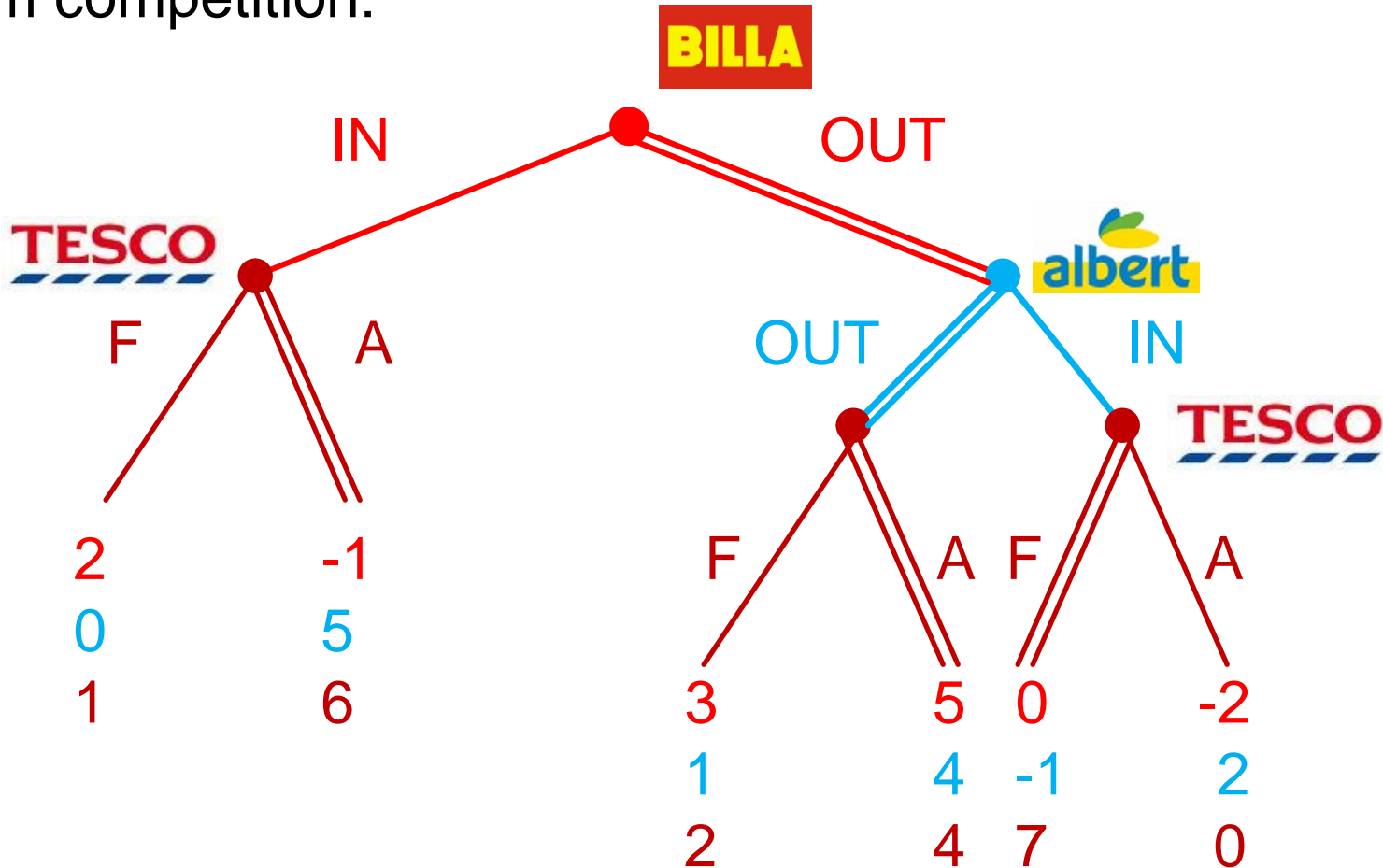
Dynamic Games

Subgame Perfect NE

- • solves the problem with not sensible predictions
- • is found using Backward Induction
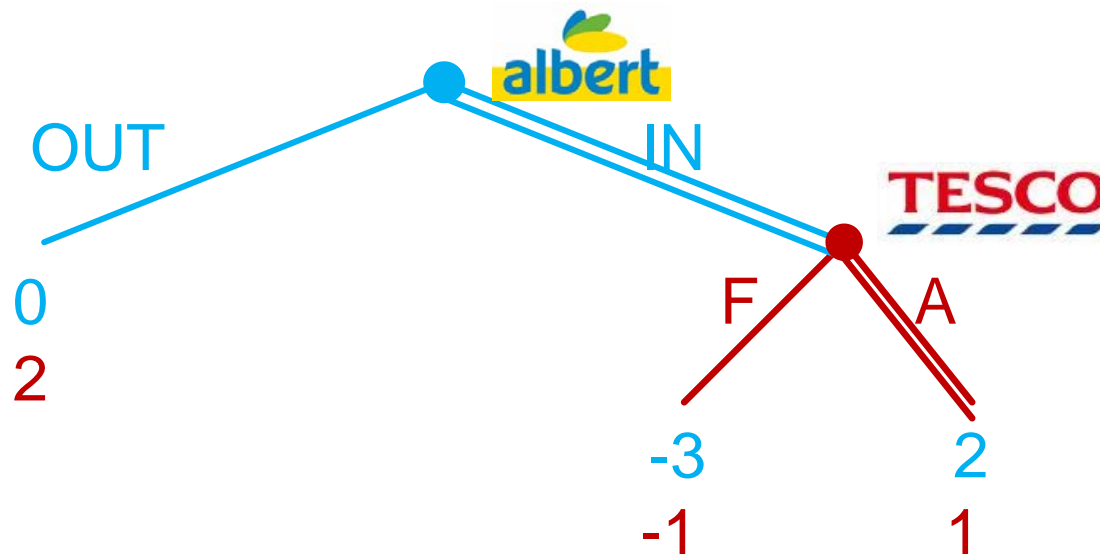
Backward induction - we only keep the "sensible" equilibrium and ignore the "bad" (empty threat) one

# Dynamic Games - Example

3-frim competition:

SPNE: {OUT, OUT, AAF}

Dynamic Games - Example

- There are two NE in this game: (OUT,OUT,AAF) and (IN,IN,AFA)

- Exercise – write down tables for static version of this game to check that (IN,IN,AFA) is NE (note that three players mean that you need more than one table

- Only the first one is based on reasonable behavior in every part of the game.

- (IN,IN,AFA) is NE, but it is an empty threat from Tesco

- Note, that SPNE is a refinement (subset) of NE

Dynamic Games - Example

(IN,IN,AFA) is NE because no firm wants to deviate:
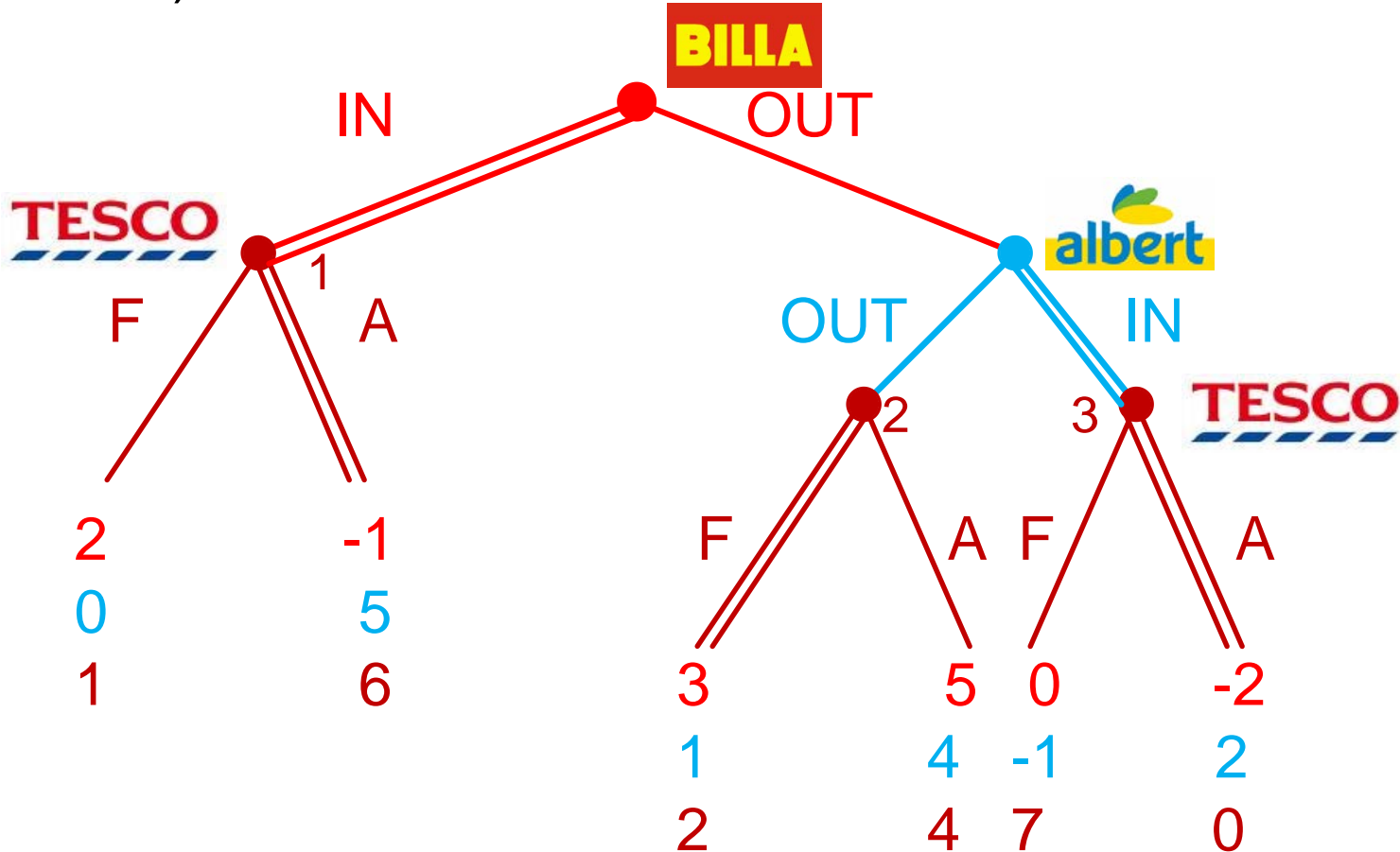
Not SPNE: Tesco's strategy is not optimal in nodes 2,3

Subgame Perfect NE

- SPNE overcomes empty threats, because it requires sequential rationality – rational decision in every decision node

- starting at a given decision node – actions are optimal for the rest of the game – for a given subgame (even if player do not expect to get to that subgame)

- What is subgame?

Subgame Perfect NE

- Subgame
  - part of the game that follows after some actions were already taken

not subgame
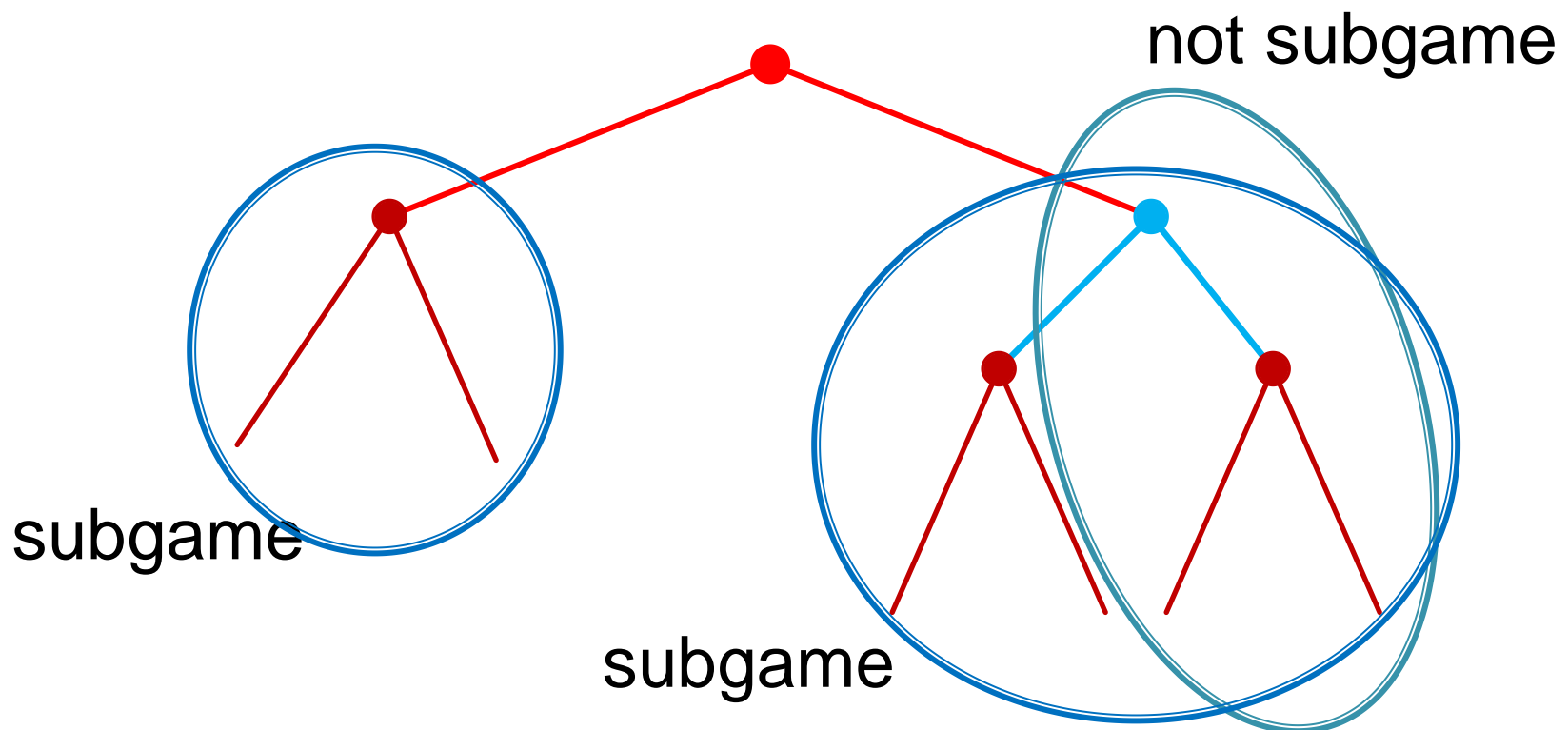
subgame

subgame

Subgame Perfect NE

A subgame perfect equilibrium is a strategy profile s* with the property that in no subgame can any player *i* do better by choosing a strategy different from s_i^*, given that every other player *j* plays s_j^*

- NE – set of optimal actions
- SPBE – set of optimal strategies

- What is strategy? Strategy is a plan of action that specifies move (action) in every single decision mode of a given player

Action vs. Strategy

In our 3-firm example, Tesco has:
- three decision nodes: 1,2,3
- two actions in each node: A, F
- eight strategies: AAA, AAF, AFA, FAA, AFF,…, FFF

- strategy is a plan of actions for every possible situation that might occur (for every possible node)

- strategy – it is deciding about the action in each decision node prior to the game

- it is like as if you want your friend to play the game instead of you, you have to tell him in advance what to do in each situation

Action vs. Strategy

More examples:

- 1-round matching pennies (each player can choose H or T)
    - actions: H, T (two actions)
    - strategies: H, T (two strategies)

- 2-round matching pennies (each player can choose H or T in each round)
    - actions: H, T (two actions)
    - strategies: HHHHH, HHHHT, HHHTH,… (2^5 strategies)
        - o first letter determines action in the first round
        - o remaining four letters determine action in the second round conditional on the first round outcome (there are four possible outcomes of the first round)

Summary

- Concept of NE in dynamic games permits players to make empty threat because actions at decision nodes that are not reached do not have any effect on player's payoff

- What player says she will do at unreached node can insure the other player to play such that this node really is unreached

- SPNE – refinement of NE – requires that actions are optimal in every subgame (sequential rationality)

- SPNE is refinement of NE – some of NE are also SPNE and there is no SPNE such that it is not also NE