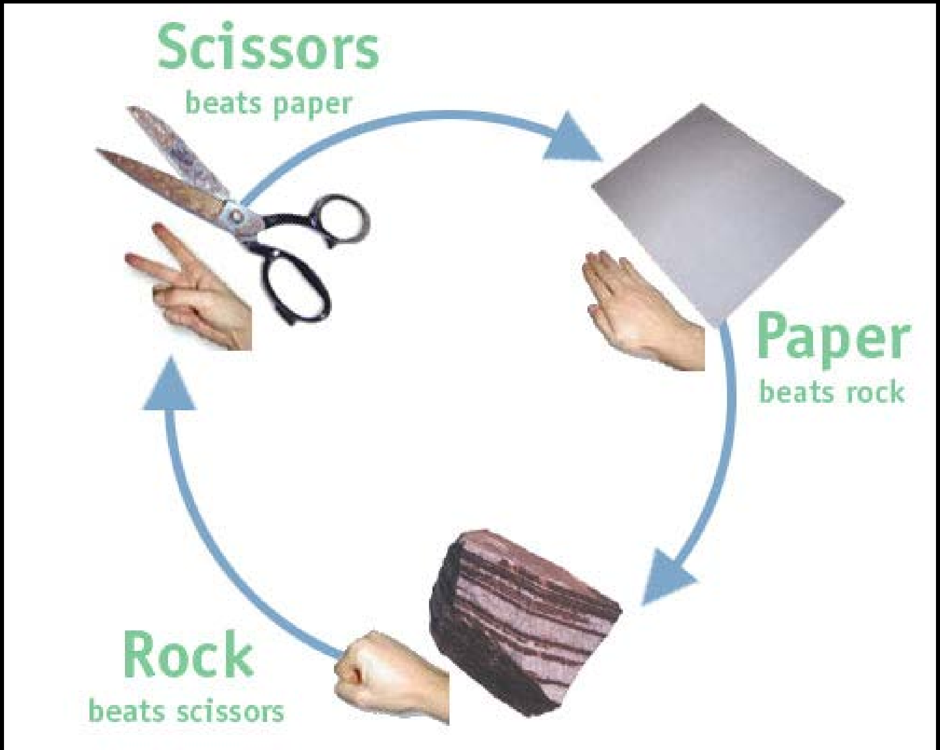

Introduction to Game Theory
Lecture 4

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

Preview

- Review
- Mixed strategy Nash equilibrium
  - review example
  - best response functions – graphs
- Elimination of strategies that are strictly dominated by mixed strategies
  - illustration
  - example

Review

Mixed strategy NE

- need for making oneself unpredictable leads to mixing strategies
- Mixed strategy: player chooses a probability distribution (p_1,p_2,..,p_N) over her set of actions rather than a single action
- If there is no NE without mixing, we will find at least one MSNE (Nash - proof)
- If NE without mixing exists, we may find additional MSNE

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Mixed Strategy NE

- not just mathematical exercise
- examples:
  - matching pennies
  - rock paper scissors
  - penalty kicks
  - baseball pitches
  - tennis service
  - travel agencies pricing policies
- making yourself unpredictable

Mixed Strategies - Example

Matching Pennies:

1 \ 2	Head	Tail
Head	$1,-$1	-$1,$1
Tail	-$1,$1	$1,-$1

- no Nash Equilibria, no pair of actions is compatible with a steady state
- there exists steady state in which each player chooses each action with probability ½

Mixed Strategy NE – How to Find

1 \ 2	H (q)	T (1-q)
H (p)	\$1,-\$1	-\$1,\$1
T (1-p)	-\$1,\$1	\$1,-\$1

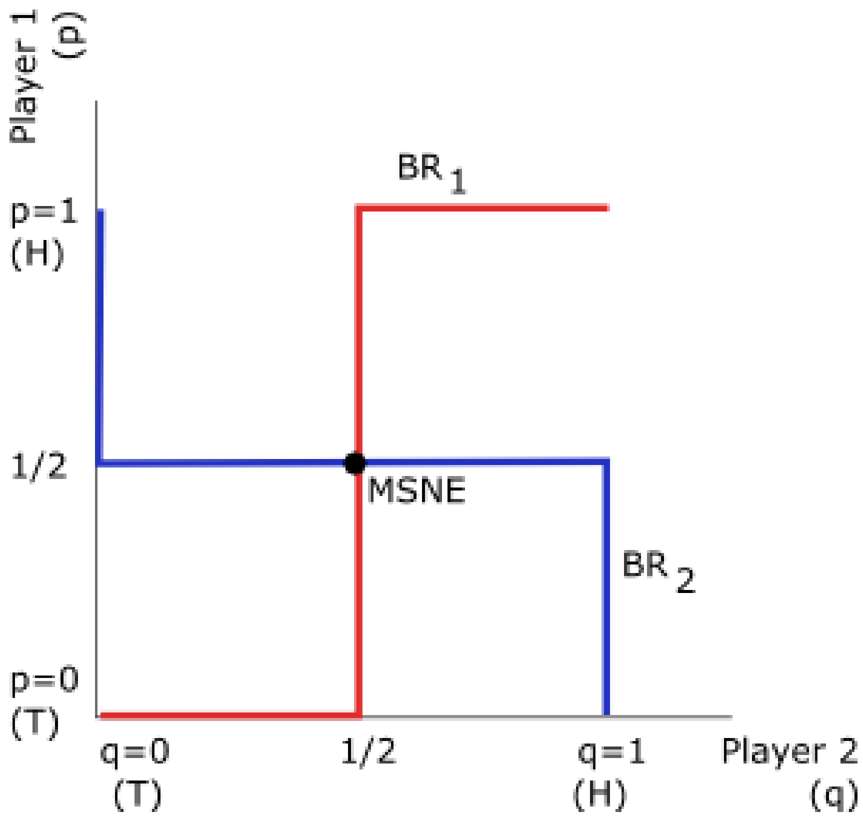
If q<½: T is better than H
If q>½: H is better than T
If q=½: H is as good as T

$$B_1(q) = \begin{cases} \{0\} & \text{if } q < \tfrac{1}{2} \\ \{p: 0 \le p \le 1\} & \text{if } q = \tfrac{1}{2} \\ \{1\} & \text{if } q > \tfrac{1}{2} \end{cases}$$

$$B_2(p) = \begin{cases} \{1\} & \text{if } p < \tfrac{1}{2} \\ \{q: 0 \le q \le 1\} & \text{if } p = \tfrac{1}{2} \\ \{0\} & \text{if } p > \tfrac{1}{2} \end{cases}$$

Mixed Strategies - Example

- player 1 (2) chooses H with probability p (q) and T with probability 1-p (1-q)



Mixed Strategy NE

1 \ 2	B (q)	S (1-q)
B (p)	2,1	0,0
S (1-p)	0,0	1,2

two NE: (B,B) and (S,S)
any MSNE?

- P1 must be indifferent between B and S (otherwise not mixing, playing pure strategy):

$$\underbrace{q*2+(1-q)*0}_{B} = \underbrace{q*0+(1-q)*1}_{S} \Rightarrow q=1/3$$

- P2 must be indifferent between B and S:
  $p*1+(1-p)*0 = p*0+(1-p)*2 \Rightarrow p=2/3$

Mixed Strategy NE

1 \ 2	B (q)	S (1-q)
B (p)	2,1	0,0
S (1-p)	0,0	1,2

If q<1/3: S is better than B
If q>1/3: B is better than S
If q=1/3: B is as good as S

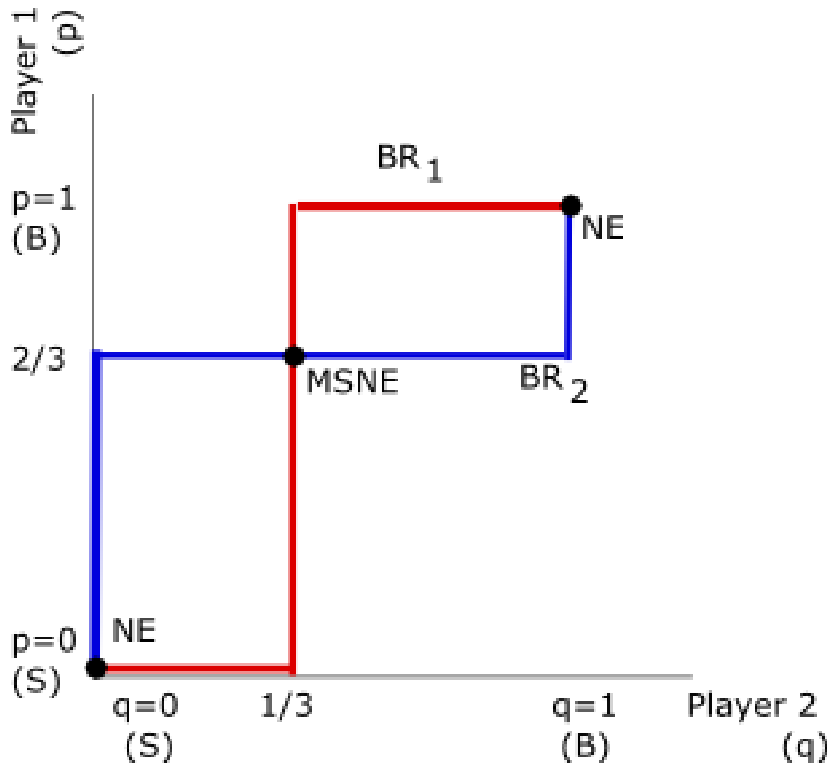
$$B_1(q) = \begin{cases} \{0\} & \text{if } q < 1/3 \\ \{p: 0 \le p \le 1\} & \text{if } q = 1/3 \\ \{1\} & \text{if } q > 1/3 \end{cases}$$

$$B_2(p) = \begin{cases} \{0\} & \text{if } p < 2/3 \\ \{q: 0 \le q \le 1\} & \text{if } p = 2/3 \\ \{1\} & \text{if } p > 2/3 \end{cases}$$

MSNE:
$\{({}^{2}/_{3}, {}^{1}/_{3}); ({}^{1}/_{3}, {}^{2}/_{3})\}$

Mixed Strategy NE

- player 1 (2) chooses B with probability p (q) and S with probability 1-p (1-q)



Mixed Strategy NE

1 \ 2	L (q)	R (1-q)
T (p)	0,1	0,2
B (1-p)	2,2	0,1

two NE: (T,R) and (B,L)
any MSNE?

- P1 must be indifferent between T and B (otherwise not mixing, playing pure strategy):

$$\underbrace{q*0+(1-q)*0}_{T} = \underbrace{q*2+(1-q)*0}_{B} \Rightarrow q=0$$

- P2 must be indifferent between L and R:
  $p*1+(1-p)*2 = p*2+(1-p)*1 \Rightarrow p=1/2$

Mixed Strategy NE

1 \ 2	L (q)	R (1-q)
T (p)	0,1	0,2
B (1-p)	2,2	0,1

If q>0: B is better than T
If q=0: B is as good as S

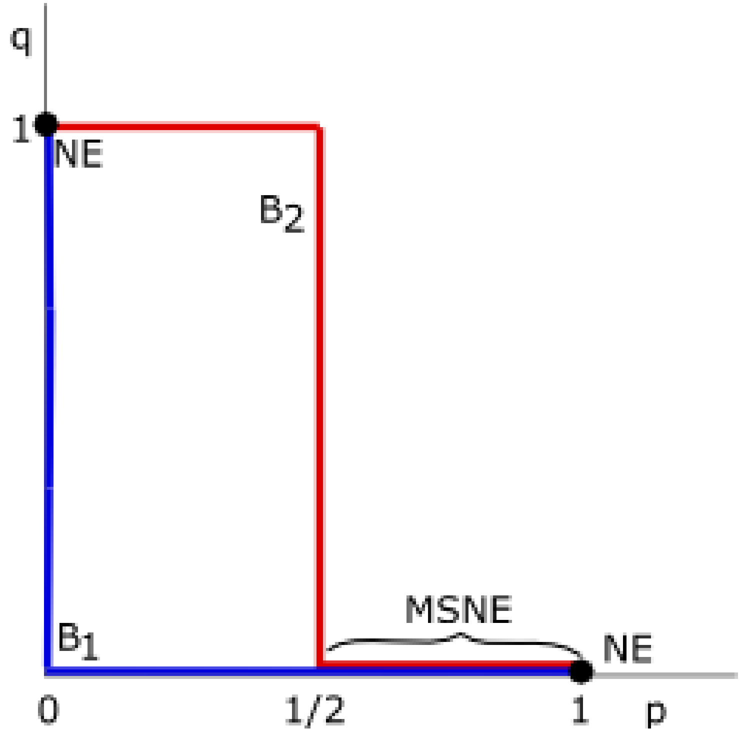
$$B_1(q) = \begin{cases} \{0\} & \text{if } q > 0 \\ \{p: 0 \le p \le 1\} & \text{if } q = 0 \end{cases}$$

$$B_2(p) = \begin{cases} \{1\} & \text{if } p < 1/2 \\ \{q: 0 \le q \le 1\} & \text{if } p = 1/2 \\ \{0\} & \text{if } p > 1/2 \end{cases}$$

MSNE:
{(p,1-p); (0,1)}
p≥1/2

Mixed Strategy NE

- player 1 chooses T with probability p and B with probability 1-p
- player 2 chooses L with probability q and R with probability 1-q



Mixed Strategy NE

Holmes vs. Moriarty

- Holmes (a genius) gets on the train London-Canterbury-Dover to get to Dover
- Moriarty (equally smart guy) rents a special and follows Holmes

- Holmes prefers to get off on different station
- Moriarty prefers the same station

H \ M	D (q)	C (1-q)
D (p)	0,8	8,-4
C (1-p)	4,-4	-4,2

Mixed Strategy NE

Holmes vs. Moriarty

- Holmes: Moriarty knows that I want to go to D, so I'd better get off in C
- Holmes: Moriarty is almost as smart as I am he knows this and goes to C, so I'd better go to D
- Holmes: But Moriarty knows that I know…

Mixed Strategy NE

…so whatever my reasoning is, Moriarty will figure it out and get me

Mixed Strategy NE

- Solution to Holmes' dilemma: If Holmes himself does not know which action he will choose, Moriarty cannot take advantage of knowing Holmes's action

=> Ignorance is a bliss

Mixed Strategy NE

H \ M	D (q)	C (1-q)
D (p)	0,8	8,-4
C (1-p)	4,-4	-4,2

- no pure strategy NE ➡ players have to mix:
- for example: {(½,½),(½,½)} – could this work?

H \ M	D (q)	C (1-q)	$^1/_2D+^1/_2C$
D (p)	0,8	8,-4	4,2
C (1-p)	4,-4	-4,2	0,-1
$^1/_2D+^1/_2C$	2,2	2,-1	2,0.5

- still no NE, we need different probabilities for mixing

Mixed Strategy NE

- how about: {(1/3,2/3),(3/4,1/4)} – could this work?

M  / H 	D (q)	C (1-q)	$^3/_4$D+$^1/_4$C
D (p)	0,8	8,-4	2,5
C (1-p)	4,-4	-4,2	2,-2.5
$^1/_3$D+$^2/_3$C	8/3,0	0,0	2,0

- Yes, this leads to one Mixed strategy NE

Mixed Strategy NE

H  \  M	D (q)	C (1-q)
D (p)	0,8	8,-4
C (1-p)	4,-4	-4,2

- Holmes must be indifferent between D and C (otherwise not mixing, playing pure strategy):

$$\underbrace{q*0+(1-q)*8}_{D} = \underbrace{q*4+(1-q)*(-4)}_{C} \Rightarrow q = {}^{3}/_{4}$$

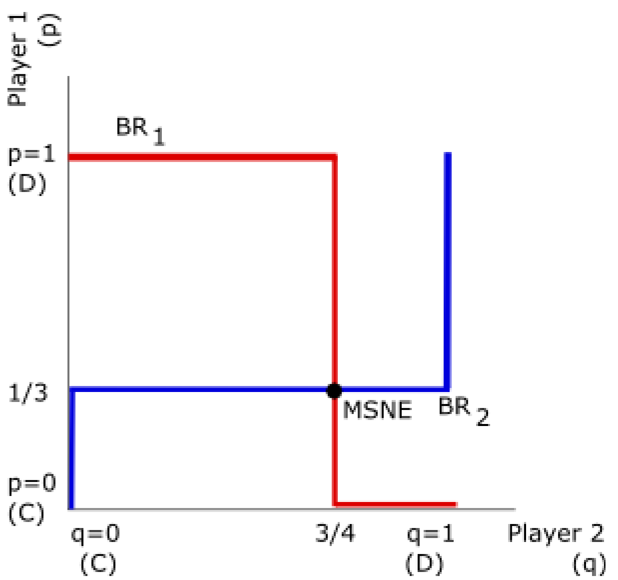
- Moriarty must be indifferent between D and C: $p*8+(1-p)*(-4) = p*(-4)+(1-p)*2 \Rightarrow p = {}^{1}/_{3}$

Mixed Strategy NE

$$B_1(q) = \begin{cases} \{1\} & \text{if } q < 3/4 \\ \{p: 0 \le p \le 1\} & \text{if } q = 3/4 \\ \{0\} & \text{if } q > 3/4 \end{cases}$$

$$B_2(p) = \begin{cases} \{0\} & \text{if } p < 1/3 \\ \{q: 0 \le q \le 1\} & \text{if } p = 1/3 \\ \{1\} & \text{if } p > 1/3 \end{cases}$$

MSNE:
$\{({}^1/_3, {}^2/_3); ({}^3/_4, {}^1/_4)\}$



Preview

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Elimination by Mixed Strategies

- no pure strategy is dominated by another pure strategy

1 \ 2	A	B	C
D	5,4	3,5	2,7
E	2,7	8,2	3,5
F	3,4	4,5	2,4
½D+½E	3.5,5.5	5.5,3.5	2.5,6

- however, ½D + ½E strictly dominates F
  (3.5,5.5,2.5) > (3,4,2)

Elimination by Mixed Strategies

- Example:

1 \ 2	L	C	R
T	3,4	4,5	1,7
M	1,7	8,2	3,5
B	4,4	3,5	2,4

- only strategy that is never best response to opponent's actions is T
- there exists p and (1-p) such that:
  pM + (1-p)B > T

Elimination by Mixed Strategies

1 \ 2	L	C	R
T	3,4	4,5	1,7
M	1,7	8,2	3,5
B	4,4	3,5	2,4

- pM + (1-p)B > T
  - p*1+(1-p)*4>3 => p<1/3
  - p*8+(1-p)*3>4 => p>1/5
  - p*3+(1-p)*2>1 => always true
- we can choose for example p=1/4

Elimination by Mixed Strategies

1 \ 2	L	C	R
~~T~~	~~3,4~~	~~4,5~~	~~1,7~~
M	1,7	8,2	3,5
B	4,4	3,5	2,4

- pM + (1-p)B > T
- $^{1}/_{4}$M+ $^{3}/_{4}$B = ($^{13}/_{4}$,$^{17}/_{4}$,$^{9}/_{4}$) > (3,4,1) = T

Elimination by Mixed Strategies

1 \ 2	L	C	R
M	1,7	8,2	3,5
B	4,4	3,5	2,4

- only strategy that is never best response to opponent's actions is R
- there exists p and (1-p) such that: pL + (1-p)C > R

Elimination by Mixed Strategies

1 \ 2	L	C	R
M	1,7	8,2	3,5
B	4,4	3,5	2,4

- pL + (1-p)C > R
  - p*7+(1-p)*2>5 => p>3/5
  - p*4+(1-p)*5>4 => p<1
- we can choose for example p=4/5
- $^4/_5$L+ $^1/_5$C = (6,4.2) > (5,4) = R

Elimination by Mixed Strategies

- after iterative elimination of dominated strategies we get:

1 \ 2	L	C
M	1,7	8,2
B	4,4	3,5

- no further elimination is possible because every action is best response to some of opponent's actions

Summary

- Mixed strategies Nash equilibrium
  - making your actions unpredictable
  - duopoly, sport
- Iterative elimination of strictly dominated strategies
  - strategy can be dominated by pure strategy
  - strategy can be dominated by mixed strategy
- Homework deadline next Wednesday