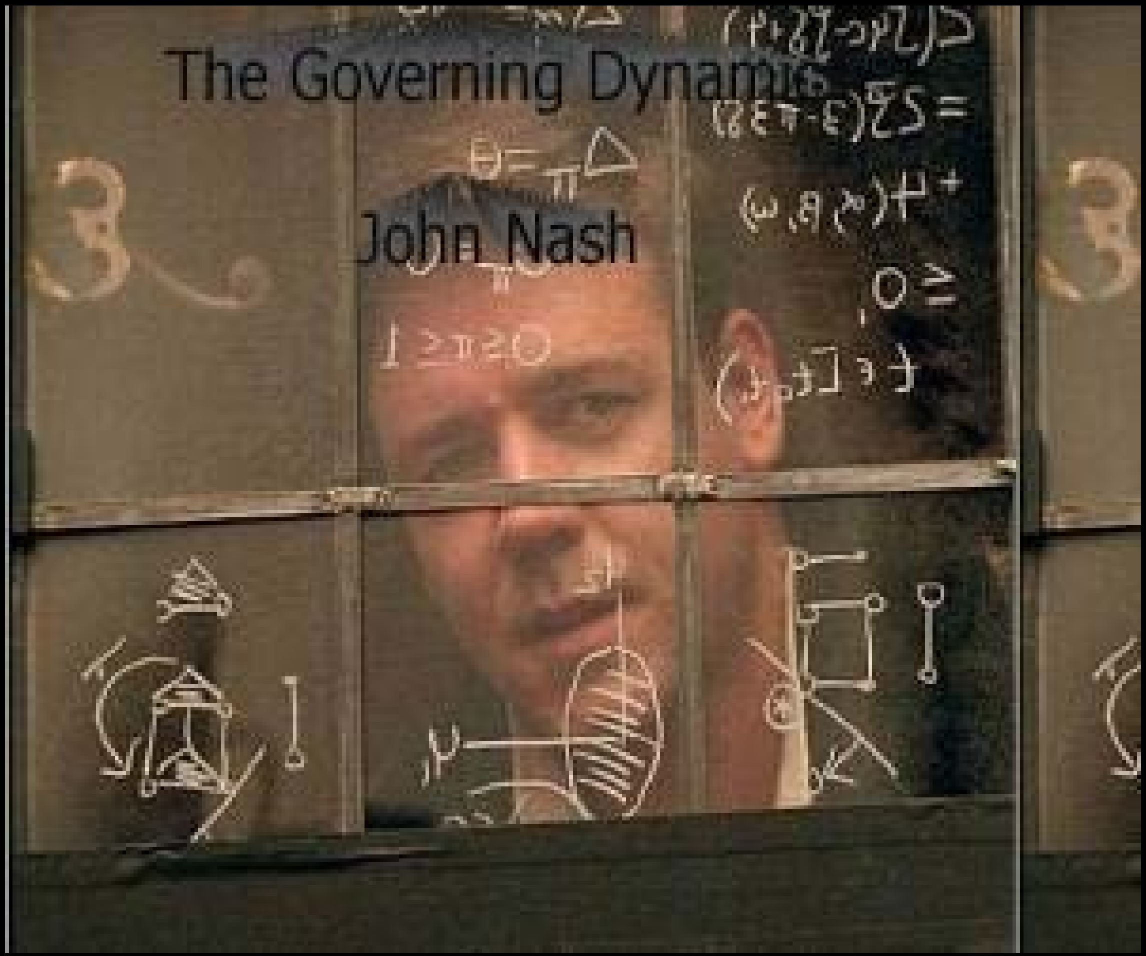


Introduction to Game Theory
Lecture 2

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

Syllabus

- Contact: kalk00@vse.cz
  home.cerge-ei.cz/kalovcova/teaching.html
- Office hours: Wed 7.30pm – 8.00pm, NB339
  or by email appointment
- Osborne, M. J. – An Introduction to Game Theory
  Gibbons, R. – A Primer in Game Theory
  Suggested articles
- Important information on webpage
- Grading: Midterm 30%, Final 60%,
  Homework 10%, Experiments up to 5%

Economic Models & Games

- Game theory is about economic models
- Economic models help us understand behavior of agents, they do not tell us what their optimal action is
- Each game represents some economic situation (Prisoner’s dilemma = Duopoly)
- By solving the game (finding equilibrium) we find plausible outcome of a given situation

Elements of Games

- Strategic game consists of
  - set of players
  - for each player set of actions
  - for each player set of preferences over the set of action profiles
  - preferences represented by payoff function
  - static games: players simultaneously chose actions

    normal form game representation (table)

Games - Classification

Course topics:

- Games of complete and perfect information
  - Static Games (Nash Equilibrium)
  - Dynamic Games (Backward Induction)
- Games of complete but imperfect information
  - Dynamic Games (Subgame perfect NE)
- Games of incomplete information
  - Static Games (Auctions)
  - Dynamic Games (Signaling)

Plan for Today

…previously: “what are models?”…

…today: “how to solve them?”

- Elimination of strictly dominated strategies
- Nash Equilibrium

Iterative Elimination of Strictly Dominated Strategies

How to solve games?

- Consider the following game:
  - Two players
  - Each player chooses between two actions: A and B
  - Payoff for all outcomes is in the table below:

1 \ 2	A	B
A	50,50	100,0
B	0,100	70,70

Dominated Strategies - What?

- Play A – because it is always better, no matter what the other player chooses.

- *Note: playing B could be reasonable if player knows the opponent and it is a repetitive game - it might be advantageous in the long term.*
- *But in this course, we only deal with one shot, non cooperative games.*
- *Repeated games, cheap talk, cooperation, etc – are not part of this introductory course*

Dominated Strategies - What?

- player i's action a strictly dominates her action b if $u_i(a,a_{-i}) > u_i(b,a_{-i})$ for every list a_{-i} of other players' actions
  - u_i is a payoff function that represents player i's preferences
  - $a_{-i} = \{a_1, \ldots , a_{i-1}, a_{i+1}, \ldots, a_N\}$ - actions of others players
- if any action strictly dominates the action b, we say that b is strictly dominated

Dominated Strategies - Why?

- how to “solve” the game (model)?
  what is a plausible outcome for a given game?

1 \ 2	A	B
A	50,50	100,0
B	0,100	70,70

- Iterative elimination of dominated strategies provides insight to what is a plausible outcome of a game

Elimination of Strategies

- Iterative elimination of strictly dominated strategies:
  - rational players do not play strictly dominated actions, hence we can eliminate them
- common knowledge that all players are rational is required:
  - all the players know that all the players are rational, and that all the players know that all the players know that all the players are rational etc.
- the order of elimination does not affect the strategy or strategies we end up with

Prisoner's Dilemma

- Let's get back to Prisoner's Dilemma game:

1 \ 2	Confess	Silent
Confess	1,1	3,0
Silent	0,3	2,2

- Iterative elimination of dominated strategies shows that (Confess, Confess) is likely outcome (consistent with evidence)

Elimination of Strategies

1 \ 2	Left	Center	Right
Up	1,0	1,2	0,1
Down	0,3	0,1	2,0

1. Right is dominated by Center
2. Down is dominated by Up
3. Left is dominated by Center
4. Plausible outcome is {Up,Center}

Party Game

- Consider the following party game where the payoff of two friends depends on whether they come to party early or late:

1 \ 2	Early	Late
Early	10,10	0,3
Late	3,0	5,5

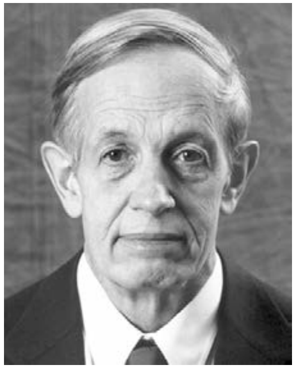
- No strategy is dominant => no elimination
- (Early,Early) is likely outcome of the game

Elimination of Strategies

- Pros:
  - simple – just compare all pairs of strategies and you find if some are dominated
  - if there are many strategies, elimination makes game simpler
- Cons:
  - is weak – take for example Party game – no strategy can be eliminated => no insight about plausible outcome of the game => we need something stronger…

1 \ 2	Early	Late
Early	10,10	0,3
Late	3,0	5,5

Nash Equilibrium



Nash Equilibrium

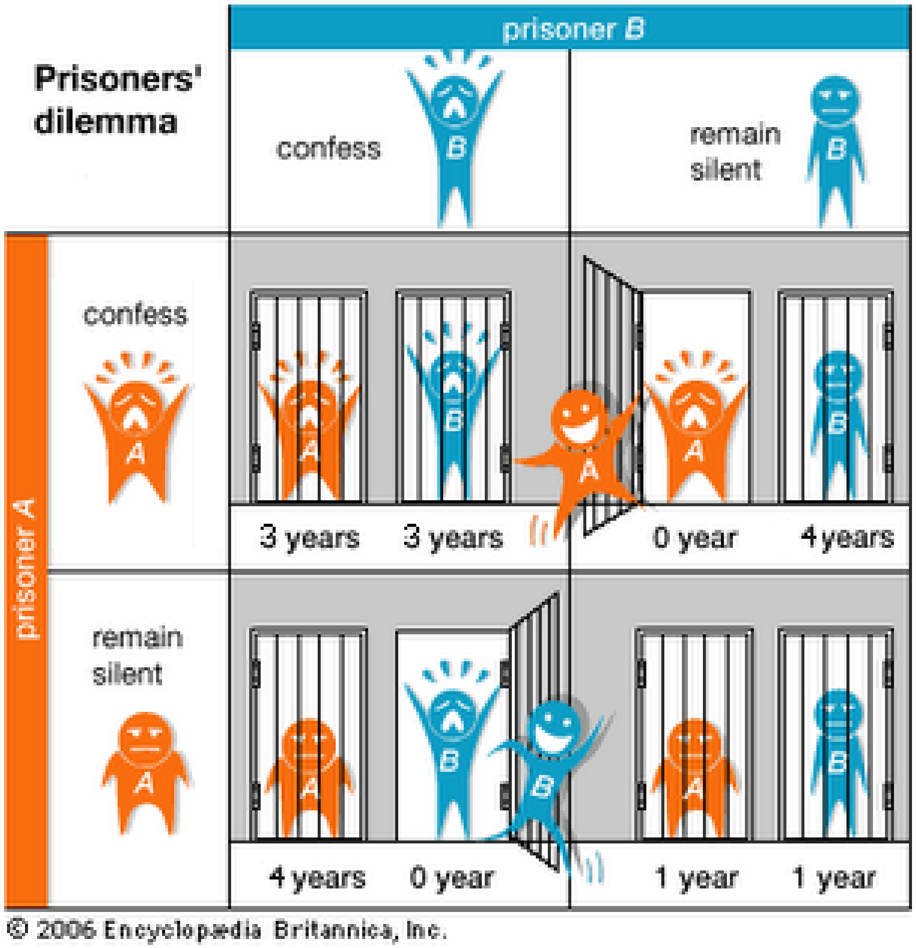
- The action profile (list of action of each player) $\mathbf{a}^*$ is a Nash equilibrium if, for every player **i** and every action $\mathbf{b_i}$ of player i, a^* is at least as good according to player i's preferences as the action profile $(\mathbf{b_i}, \mathbf{a}^*_{-i})$ in which player **i** chooses $\mathbf{b_i}$ while every other player chooses $\mathbf{a}^*_{-i}$
- Equivalently, for every player **i**, $u_i(a^*) \geq u_i(b_i, a^*_{-i})$ for every action $\mathbf{b_i}$ of player i, where u_i is a payoff function that represents player i's preferences

Nash Equilibrium

- Equivalently, the action profile a^* is a Nash equilibrium if and only if every player's action is a best response to the other players' actions
- *Translation: In Nash equilibrium, nobody can unilaterally improve their payoff, everybody is playing the best they can*

Nash Equilibrium

What actions will be chosen by players in a strategic game?

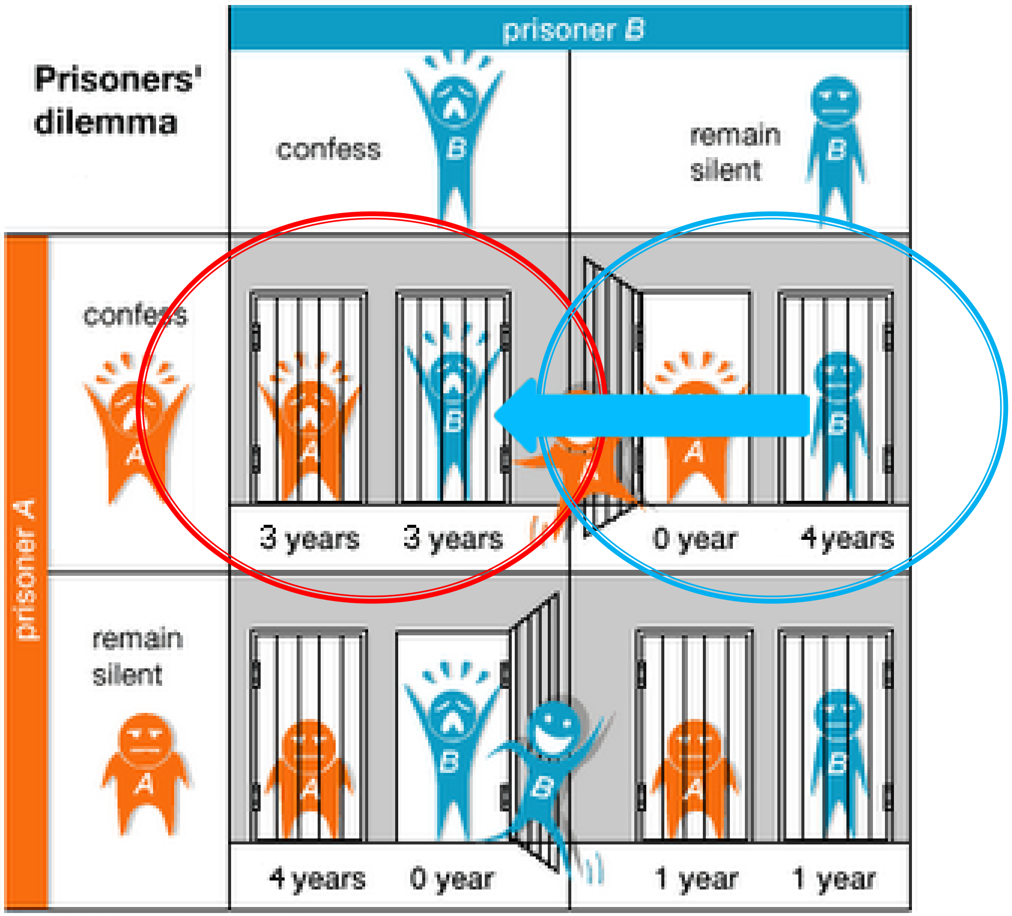


Nash Equilibrium

- A Nash equilibrium (NE) is such combination of actions of all players that no player can do better by choosing a different action given that every other player sticks to NE action
- {Confess,Confess} is NE, because no prisoner can do better by switching to “Remain Silent” while their opponent plays “Confess”
- {Confess,Remain Silent} is not NE, because Prisoner B could do better by switching to “Confess” while his opponent plays “Confess”

Nash Equilibrium

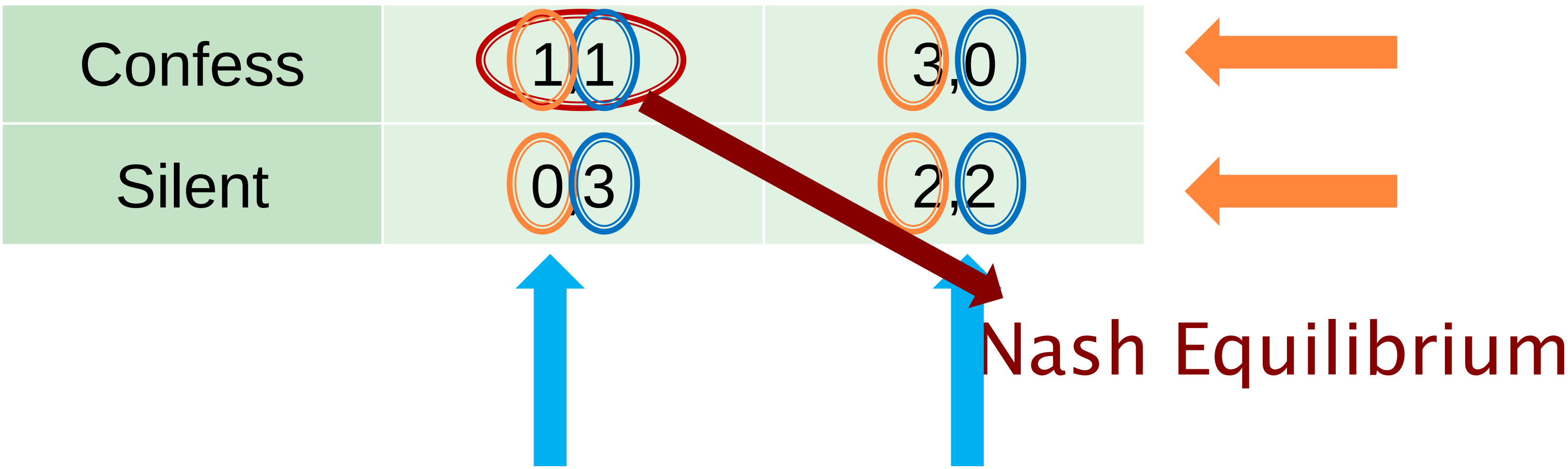
What actions will be chosen by players in a strategic game?



How to Find Nash Equilibrium?

What actions will be chosen by players in a strategic game?

1 \ 2	Confess	Silent
Confess	1,1	3,0
Silent	0,3	2,2



Note: our circles are best responses, that is why "circle method" leads to NE

$BR_1(C)=C$; $BR_1(S)=C$; $BR_2(C)=C$; $BR_2(S)=C$;

Nash Equilibrium

Note, that the NE definition implies

- neither that a strategic game necessarily has a Nash equilibrium
- nor that it has at most one

- Possible outcomes:
  - no Nash Equilibrium*
  - one Nash Equilibrium
  - many Nash Equilibria

**In pure strategies*

Nash Equilibrium

Prisoners' Dilemma Game:

1 \ 2	Confess	Silent
Confess	1,1	3,0
Silent	0,3	2,2

One Nash Equilibrium: {Confess,Confess}

How to Find Nash Equilibrium?

Party Game:

1 \ 2	Early	Late
Early	10,10	0,3
Late	3,0	5,5

Two Nash Equilibria: {E,E} and {L,L}
Players agree on which one is better
$BR_1(E)=E$; $BR_1(L)=L$; $BR_2(E)=E$; $BR_2(L)=L$;

How to Find Nash Equilibrium?

Stag Hunt:

1 \ 2	Stag	Hare
Stag	2,2	0,1
Hare	1,0	1,1

Two Nash Equilibria: {S,S} and {H,H}

Players agree on which one is better

$BR_1(S)=S$; $BR_1(H)=H$; $BR_2(S)=S$; $BR_2(H)=H$;

Nash Equilibrium

Battle of Sexes Game:

1 \ 2	Boxing	Shopping
Boxing	2,1	0,0
Shopping	0,0	1,2

Two Nash Equilibria: {B,B} and {S,S}
Players disagree on which one is better
$BR_1(B)=B$; $BR_1(S)=S$; $BR_2(B)=B$; $BR_2(S)=S$;

Nash Equilibrium

Matching Pennies:

1 \ 2	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

No Nash Equilibria

$BR_1(H)=H$; $BR_1(T)=T$; $BR_2(H)=T$; $BR_2(T)=H$;

How to Find Nash Equilibrium?

Yet another game:

1 \ 2	L	M	R
T	1,1	1,0	0,1
B	1,0	0,1	1,0

One Nash Equilibrium: {T,L}

$BR_1(L)=\{T,B\}$; $BR_1(M)=T$; $BR_1(R)=B$;

$BR_2(T)=\{L,R\}$; $BR_2(B)=M$;

Nash Equilibrium - Assumptions

- Each player chooses best available action
  - best action depends on other players' actions
- Each player has belief about other players' actions
  - derived from past experience playing the game
  - experience sufficient to know how opponents will behave
  - does not know action of her particular opponents
- Idealized circumstances:
  - for each player - population of many such players
  - players are selected randomly from each population
  - players gain experience about "typical" opponents, but not any specific set of opponents

Elimination vs. Circle Method

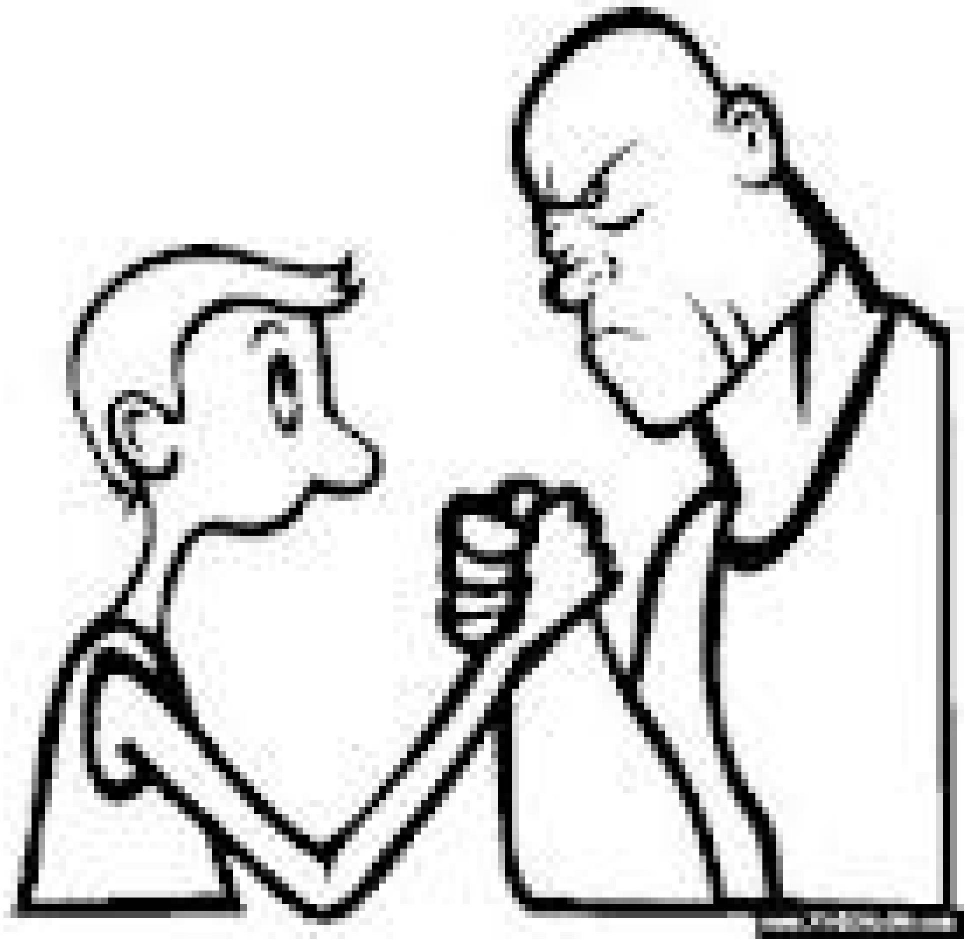
We can find plausible outcome (Nash equilibrium) of the game by:

- Elimination of strictly dominated strategies
- Circle Method

How do these methods relate?

- Elimination requires common knowledge and sometimes is too imprecise (no strictly dominated strategies, no elimination, no prediction)
- We need something stronger – Nash Equilibrium (found by Circle Method)
- IF there is a single NE, Elimination and Circle Method lead to the same outcome

Strict vs. Weak Dominance

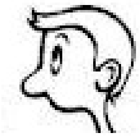


Strict Dominance

- Definition: player i's action $\mathbf{a_i}$ strictly dominates her action $\mathbf{b_i}$ if $\mathbf{u_i(a_i,a_{-i})>u_i(b_i,a_{-i})}$ for every list a_{-i} of the other players' actions, where u_i is a payoff function that represents player i's preferences

- Definition: If any action strictly dominates the action b_i, we say that b_i is strictly dominated

Weak Dominance



- Definition: player i's action $\mathbf{a_i}$ weakly dominates her action $\mathbf{b_i}$ if $\mathbf{u_i(a_i,a_{-i}) \geq u_i(b_i,a_{-i})}$ for every list a_{-i} of the other players' actions, where u_i is a payoff function that represents player i's preferences

- Definition: If any action weakly dominates the action b_i, we say that b_i is weakly dominated

Strict vs. Weak Dominance

1 \ 2	Left	Center	Right
Up	1,2	1,1	0,2
Down	0,1	0,1	2,3

- Right strictly dominates Center
- Right weakly dominates Left
- Left weakly dominates Center

Strct vs. Nonstrict NE

- Strict NE:
  - requires that the equilibrium action is better than any other action (given that all other players stick to NE actions)

- Nonstrict NE:
  - requires that the equilibrium action is not worse than any other action (given that all other players stick to NE actions)

Strct vs. Nonstrict NE

Example:

1 \ 2	L	R
T	2,2	1,1
B	2,2	2,3

Nonstrict NE

Strict NE

If we eliminate T which is weakly dominated by B, and then eliminate L which is dominated by R we lose nonstrict NE {T,L}

Strct vs. Nonstrict NE

Note: Elimination of weakly dominated strategies leads to:

- strict Nash equilibria
- but can eliminate nonstrict Nash equilibria

That is why we only eliminate strictly dominated strategies

Summary

- Nash Equilibrium is a concept of a *steady state* in given situation
- No one can unilaterally improve their payoff, therefore no one has incentive to deviate from equilibrium action
- Equilibrium behavior is based on general knowledge and experience with similar players and situations; not on particular circumstances

Summary

- We can find Nash equilibria by:
  - Elimination of strictly dominated strategies
  - “Circle Method”
- Elimination method is sometimes imprecise, NE (Circle Method) is stronger.