Introduction to Game Theory Lecture 13

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

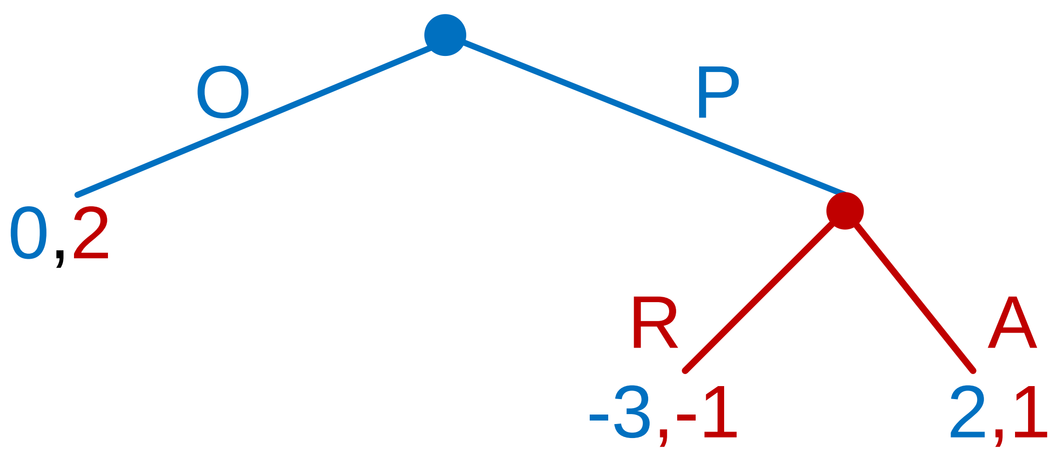
Review

- static games
  - perfect information: NE
  - imperfect information: (Bayesian) NE
- dynamic games
  - perfect information: SPNE
  - imperfect information: weak perfect Bayesian equilibrium

Nash Equilibrium

Dating site:

- P1: Participate (P), Stay Out (O)
- P2: Accept a date (A), Reject (R)



Nash Equilibrium

There are two NE in this game:

- (O,R) -> (0,2)
- (P,A) -> (2,1)

	R	A
O	0,2	0,2
P	-3,-1	2,1

- (O,R) is not a "credible threat" – it is never optimal to Reject after Player 1 Participates

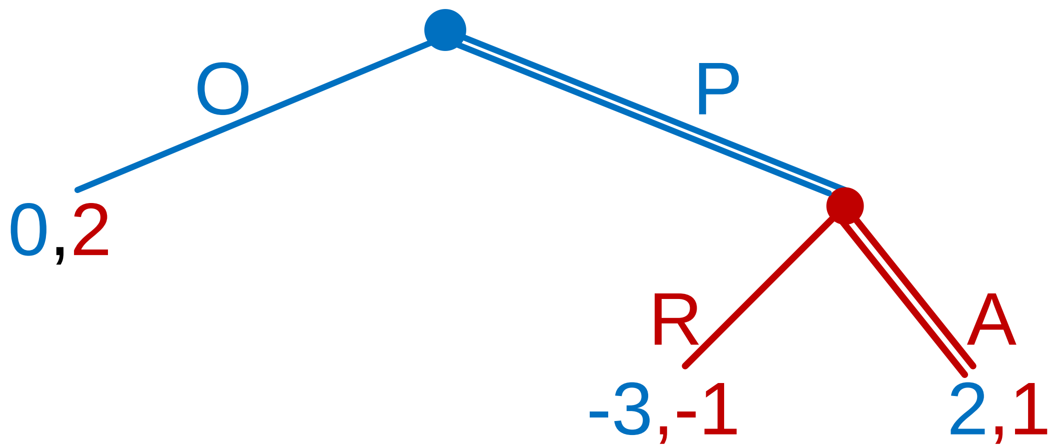
Nash Equilibrium

- What it is: every strategy is the best response to all the other strategies; i.e. nobody can gain anything by changing the strategy unilaterally
- How to find it: construct a payoff table; find best responses; find set of actions where every player is playing best response to other(s)
- Problems: NE concept has a low predictive power, since generally we can have too many NE in a game; furthermore, this concept sometimes gives not sensible predictions (non-credible threats)
- Solution: find Subgame perfect Nash equilibrium

Subgame Perfect Nash Equilibrium

Dating site:

- P1: Participate (P), Stay Out (O)
- P2: Accept a date (A), Reject (R)

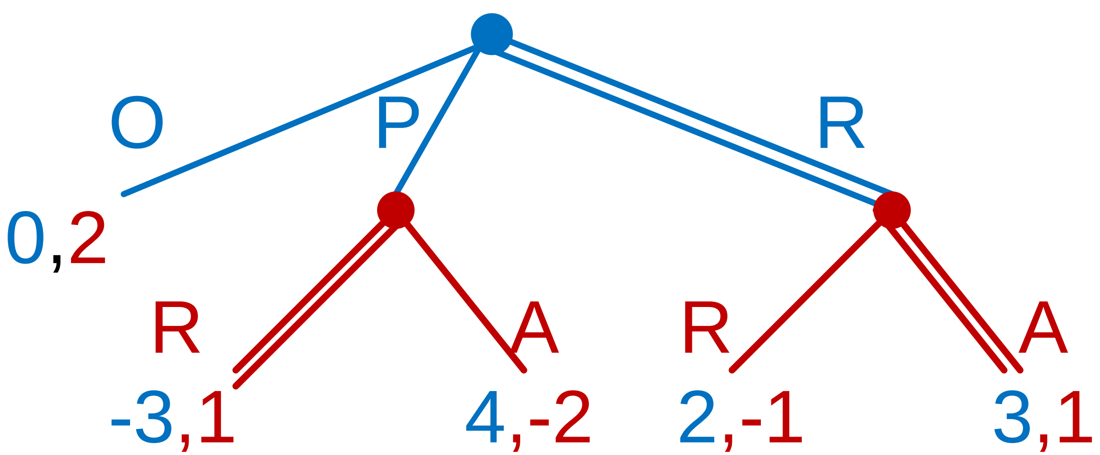


SPNE – only reasonable NE stays – (P,A); non sensible is eliminated

Subgame Perfect Nash Equilibrium

Dating site – more complex:

- P1: Stay Out (O), Real Picture (R), Photoshop (P)
- P2: Accept a date (A), Reject (R)

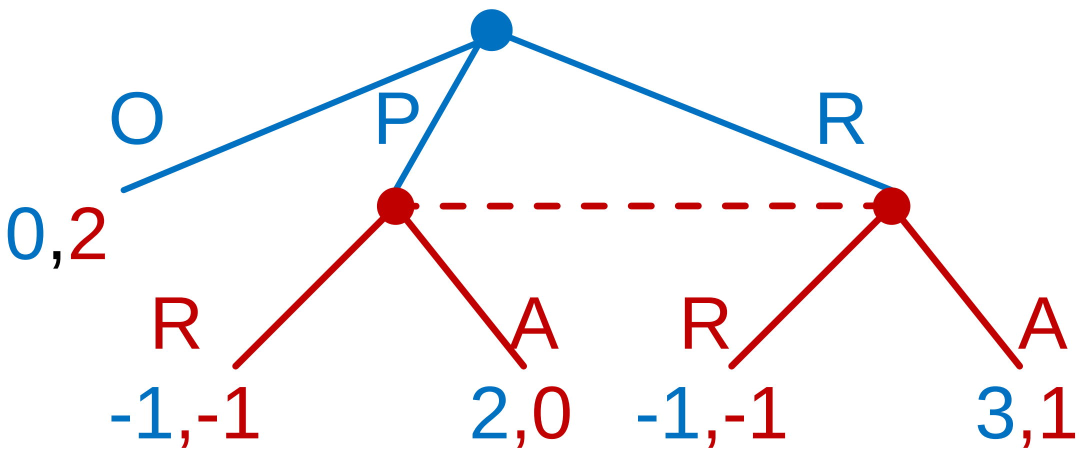


SPNE: (R,RA) – Specify action of each player in each node!!!

Subgame Perfect Nash Equilibrium

Dating site – even more complex:

- P1: Stay Out (O), Real Picture (R), Photoshop (P)
- P2: Accept a date (A), Reject (R)
- P2 can not tell Real picture from Photoshop



All NE are SPNE as well (only one subgame)

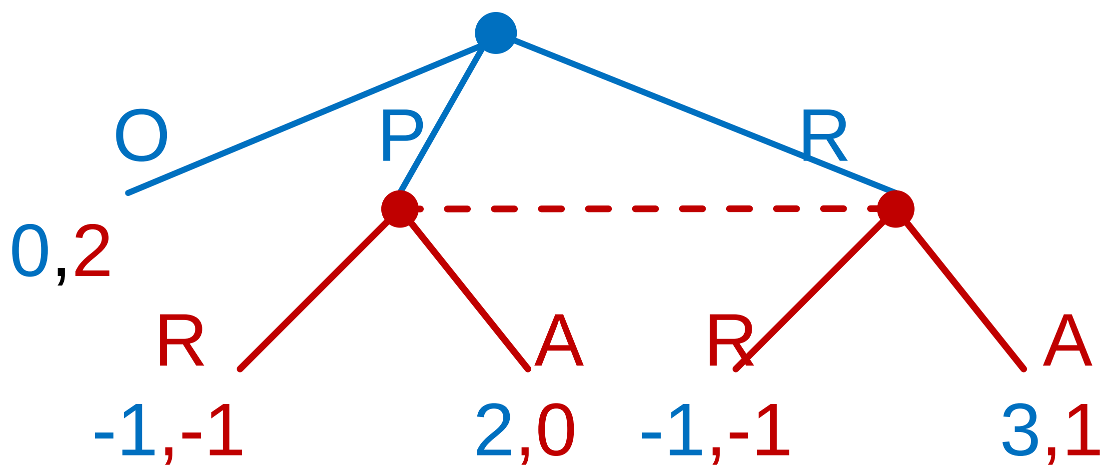
Subgame Perfect Nash Equilibrium

- What it is: optimal actions of all players at every point in the game tree (in every subgame)
- How to find it: backward induction
- Problems: SPNE concept can not be used in games with imperfect information (information sets)
- Solution: weak Perfect Bayesian Equilibrium

Weak Perfect Bayesian Equilibrium

Dating site – even more complex:

- P1: Stay Out (O), Real Picture (R), Photoshop (P)
- P2: Accept a date (A), Reject (R)
- P2 can not tell Real picture from Photoshop



All NE are SPNE as well (only one subgame)

Weak Perfect Bayesian Equilibrium

There are two NE (and SPNE) in this game:

- (O,R) -> (0,2)
- (R,A) -> (3,1)

	R	A
O	0,2	0,2
P	-1,-1	2,0
R	-1,-1	3,1

- (O,R) is not a "credible threat" – it is never optimal to Reject after Player 1 plays P or R (A is dominant)

Weak Perfect Bayesian Equilibrium

There are two NE in this game:

- (O,R) -> (0,2)
- (R,A) -> (3,1)

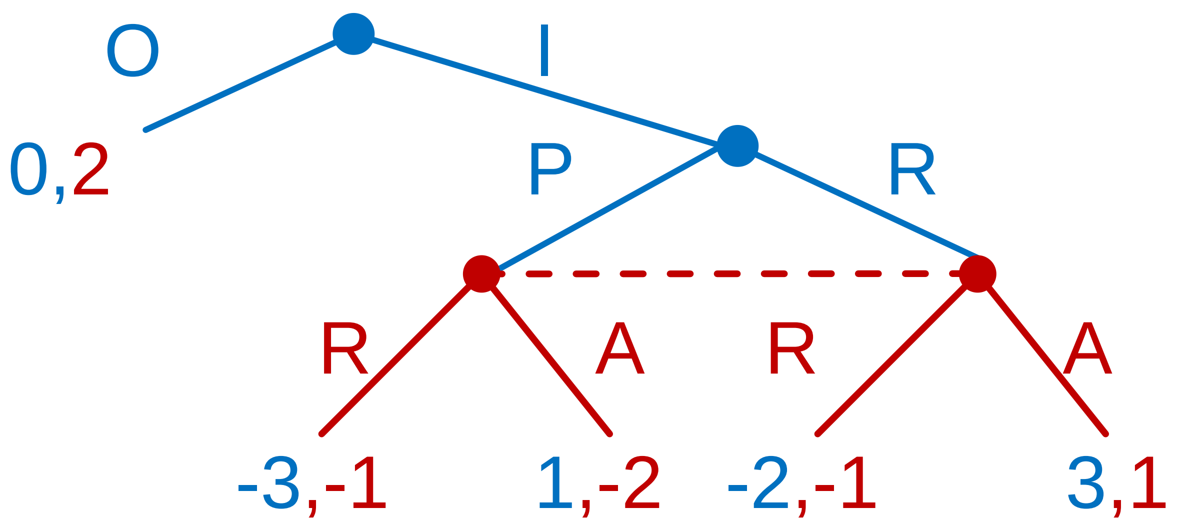
- Rejecting is never optimal, there is no system of beliefs based on which P2 chooses this action => (O,R) is not WPBE

- (R,A) – what is a corresponding system of beliefs?
  - this information set is reached (P1 plays R), so beliefs are determined by Bayes rule -> P2 believes that he is in the right node with Prob=1

Weak Perfect Bayesian Equilibrium

Dating site – even more complex 2:

- P1: Out (O) or In (I), Real Picture (R), Photoshop (P)
- P2: Accept a date (A), Reject (R)
- P2 can not tell Real picture from Photoshop



Weak Perfect Bayesian Equilibrium

There are three NE in this game:

- (OP,R) -> (0,2)
- (OR,R) -> (0,2)
- (IR,A) -> (3,1)

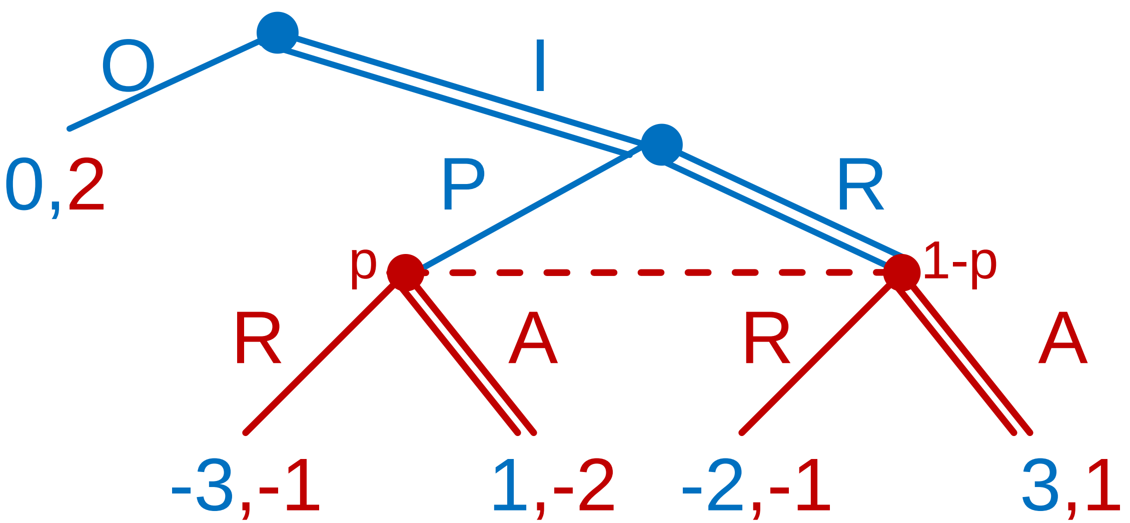
	R	A
OP	0,2	0,2
OR	0,2	0,2
IP	-3,-1	1,-2
IR	-2,-1	3,1

- are all of them WPBE as well? If yes, what is the corresponding system of beliefs?

Weak Perfect Bayesian Equilibrium

NE1: (IR,A) – how about beliefs?

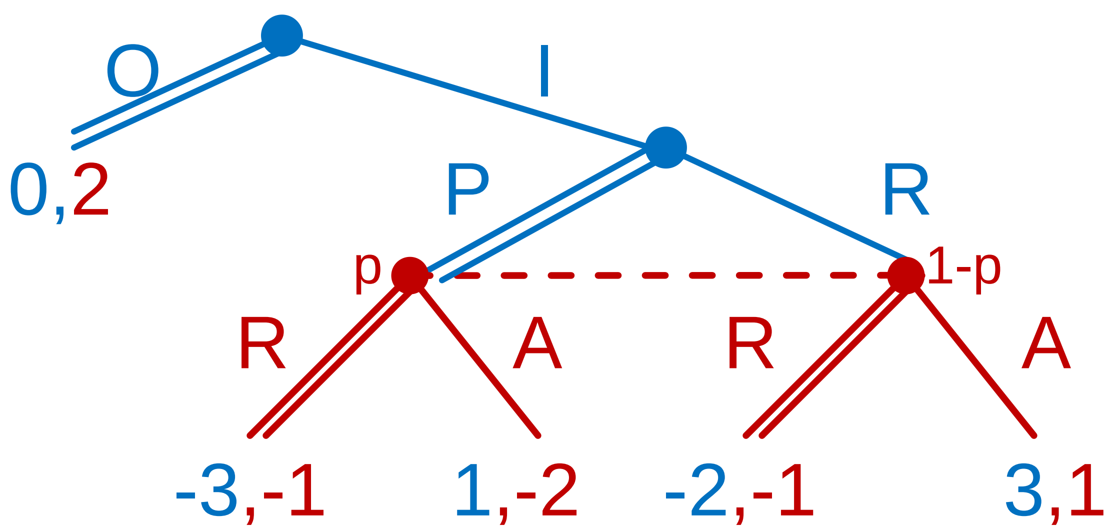
- information set is reached => beliefs are given by actions of P1 and determined by Bayes rule:
- p=0, 1-p=1



Weak Perfect Bayesian Equilibrium

NE2: (OP,R) – how about beliefs?

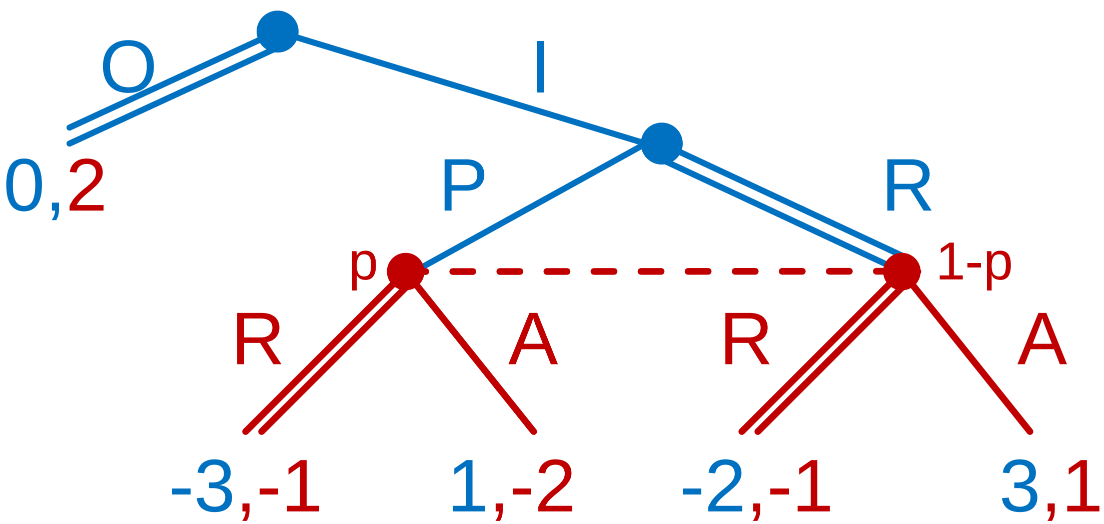
- information set is not reached => beliefs are arbitrary, sequential rationality must be satisfied (given beliefs of P2, his actions are optimal)
- for example: p=1,1-p=0 (any p>2/3) – R is optimal



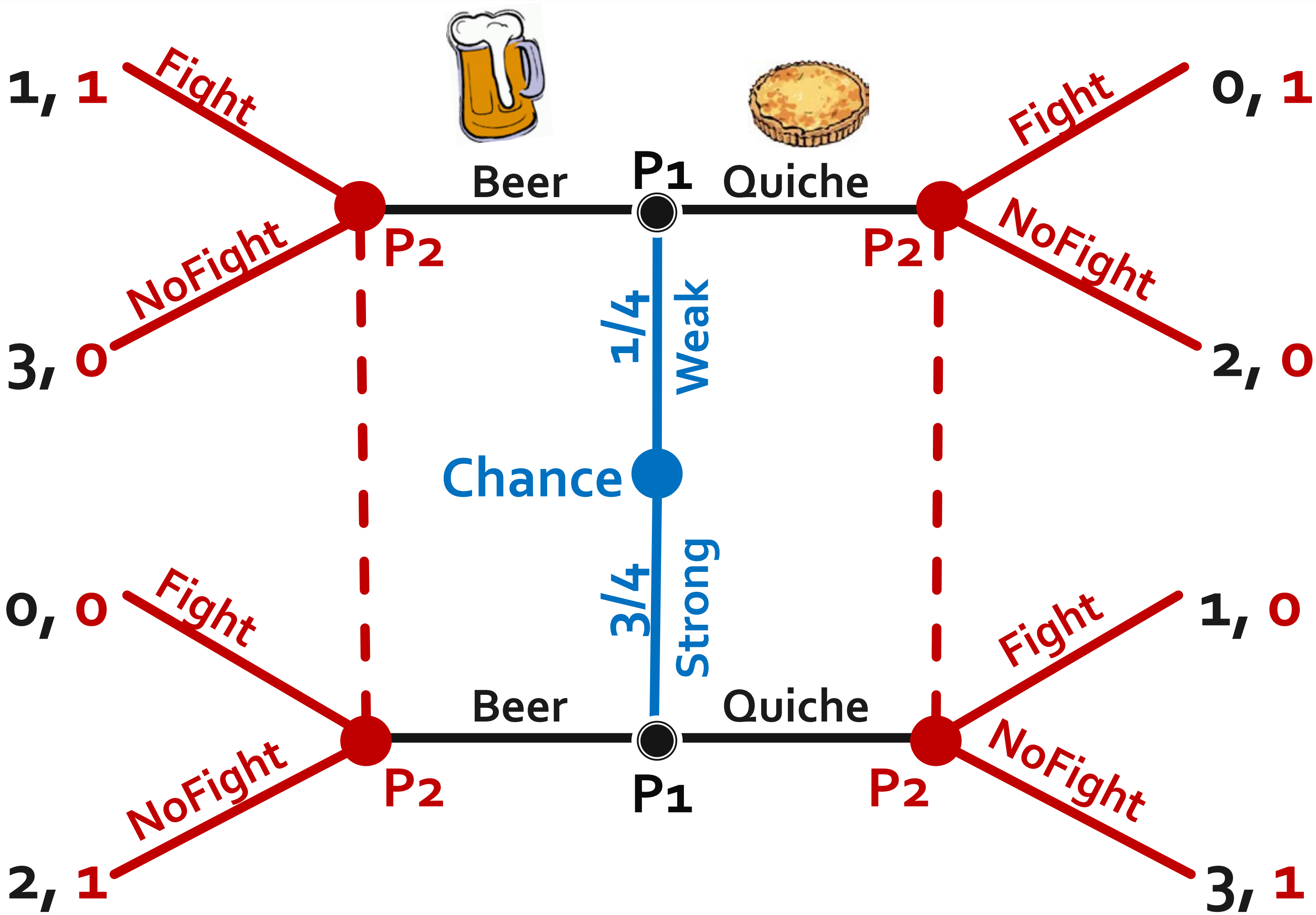
Weak Perfect Bayesian Equilibrium

NE3: (OR,R) – how about beliefs?

- information set is not reached => beliefs are arbitrary, sequential rationality must be satisfied (given beliefs of P2, his actions are optimal)
- for example: p=1,1-p=0 (any p>2/3) – R is optimal



Beer or Quiche



Beer or Quiche

- We analyze all for possible equilibria:
  - Separating equilibrium 1: Weak-Quiche, Strong-Beer
  - Separating equilibrium 2: Weak-Beer, Strong-Quiche
  - Pooling equilibrium 1: Weak-Beer, Strong-Beer
  - Pooling equilibrium 2: Weak-Quiche, Strong-Quiche

In each case:

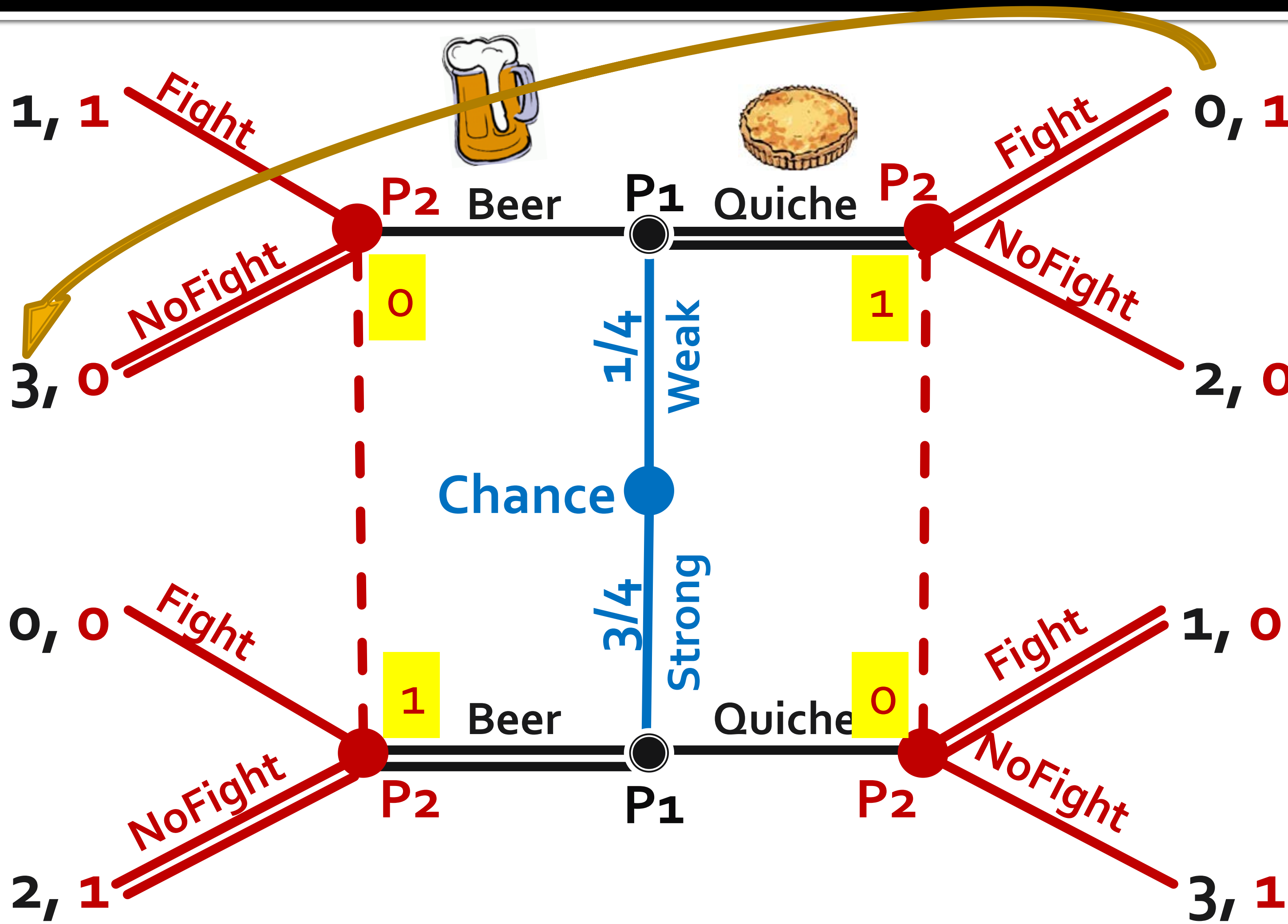
1. Start with Player 1 actions
2. Determine Player 2’s beliefs
3. Find Player 2’s optimal response
4. Check if Player 1’s action is optimal

Separating Equilibrium 1

- P1: Strong->Beer; Weak->Quiche
- P2: optimal response to that is:
  if Beer->NoFight; if Quiche->Fight

- Weak P1 wants to deviate => no WPBE

Separating Equilibrium 1

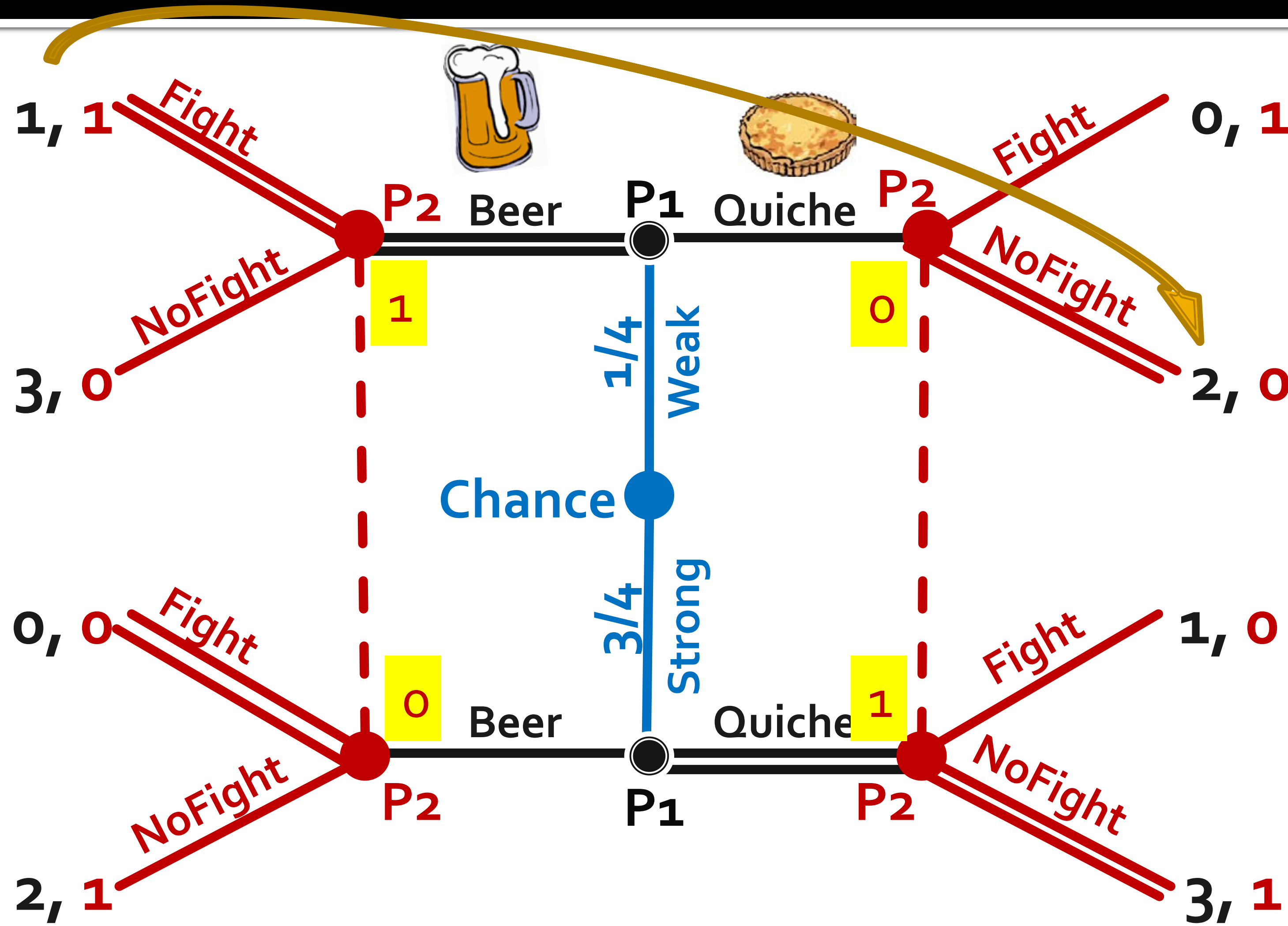


Separating Equilibrium 2

- P1: Strong->Quiche; Weak->Beer
- P2: optimal response to that is:
  if Beer->Fight; if Quiche->NoFight

- Weak P1 wants to deviate => no WPBE

Separating Equilibrium 2

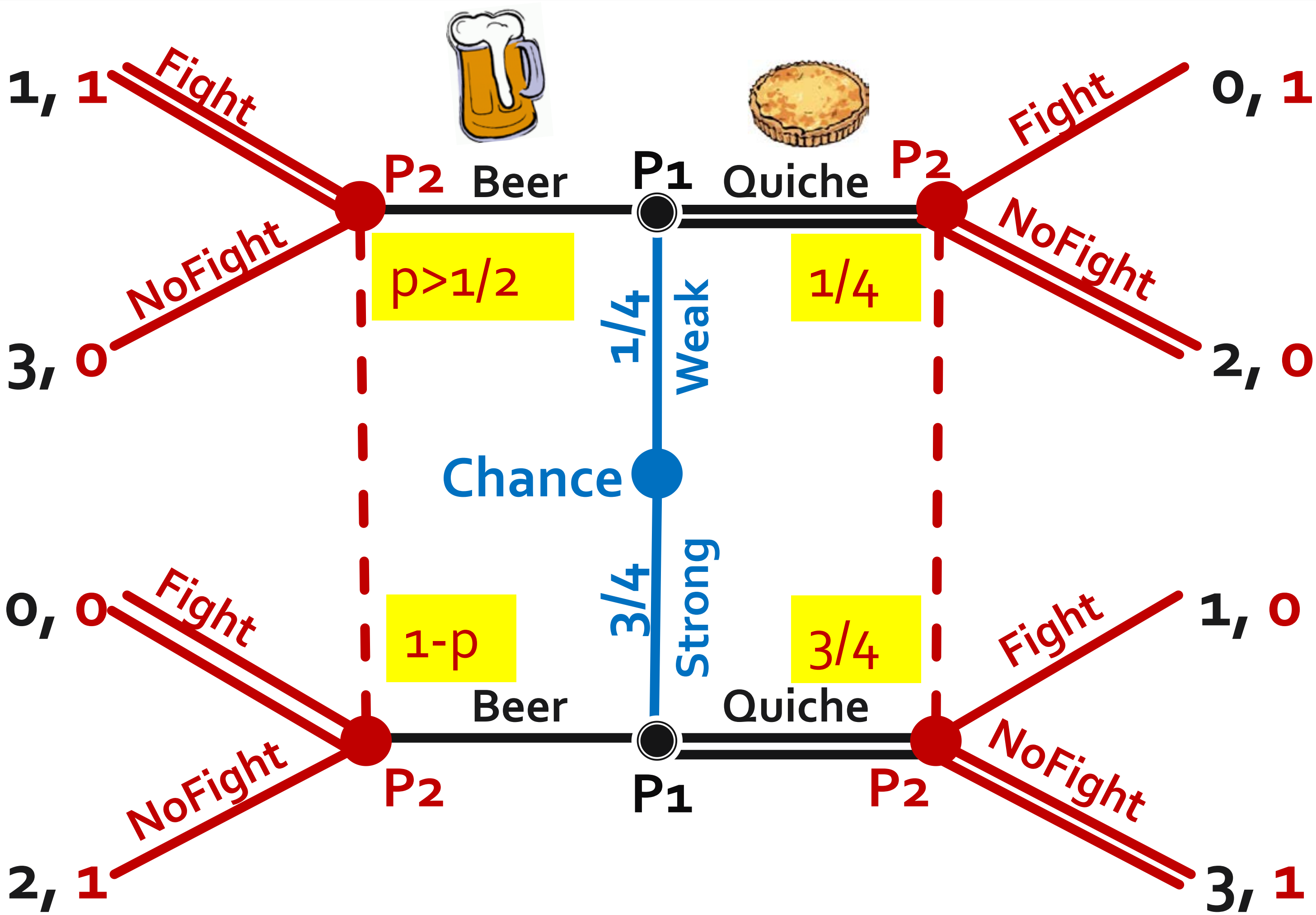


Pooling Equilibrium 1

- P1: Strong->Quiche; Weak->Quiche
- P2: optimal response to that:
  NoFight if Quiche

- Strong Player 1 gets the highest possible payoff here, whatever Player 2 decides to do in the left information set, Strong P1 has no regrets.
- To make Weak P1 have no regrets, the action in the left information set has to be Fight.
- For Fight to be optimal in the left information set, EP(Fight) has to be larger than EP(NoFight): $p>1/2$
- This is WPBE.

Pooling Equilibrium 1

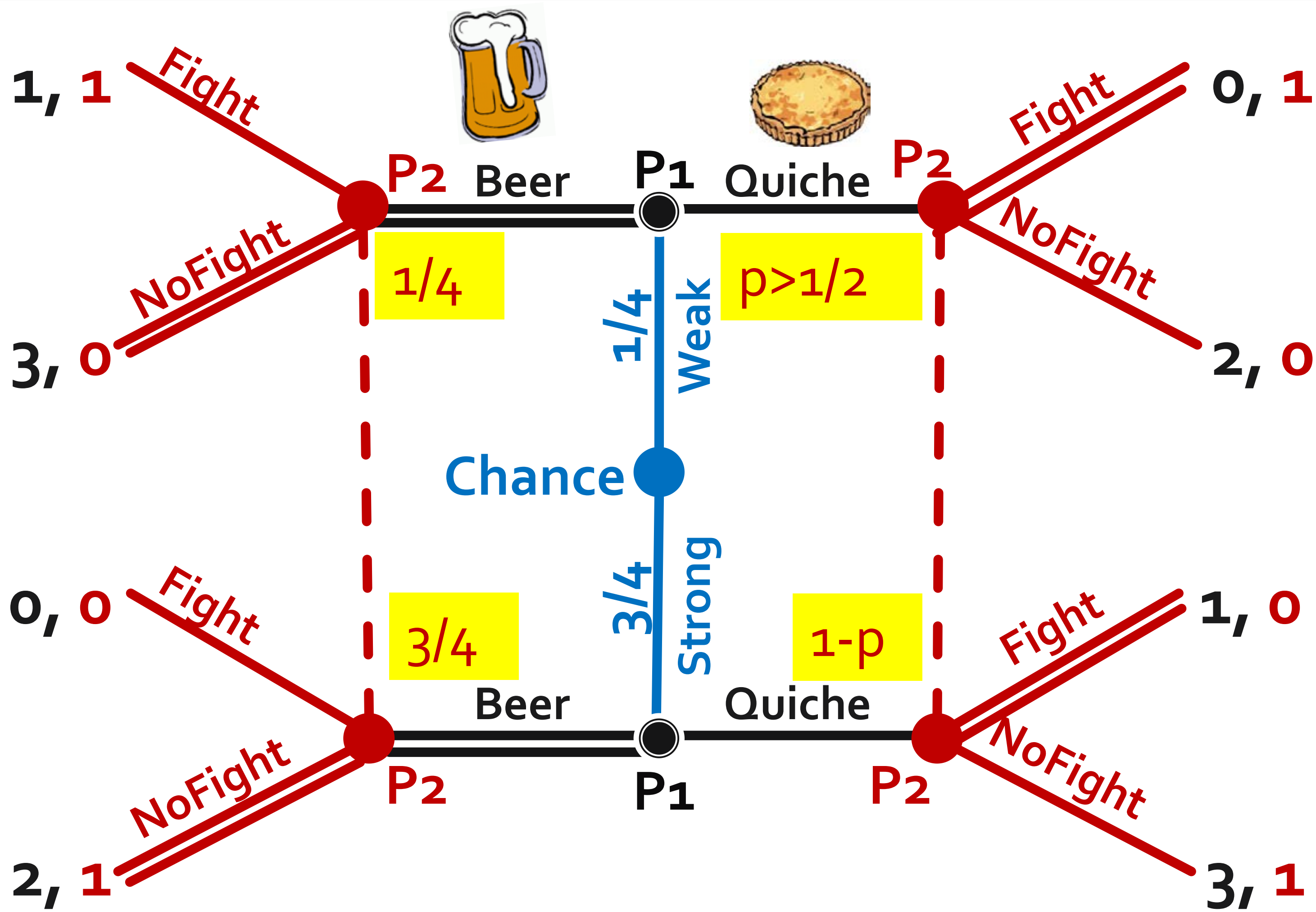


Pooling Equilibrium 2

- P1: Strong->Beer; Weak->Beer
- P2: optimal response to that:
  NoFight if Beer

- Weak Player 1 gets the highest possible payoff here, whatever Player 2 decides to do in the left information set, Weak P1 has no regrets.
- To make Strong P1 have no regrets, the action in the right information set has to be Fight.
- For Fight to be optimal in the right information set, EP(Fight) has to be larger than EP(NoFight): $p>1/2$
- This is WPBE.

Pooling Equilibrium 2



Weak Perfect Bayesian Equilibrium

- What it is: optimal actions of all players at every point in the game tree and system of consistent beliefs such the actions are sequentially rational
- How to find it: find NE first, then look for beliefs: information set is reached - Bayes; information set is not reach – anything, consistent with own actions; alternatively, analyze all possible pooling and separating equilibria one by one
- Problems: WPBE puts no restrictions on beliefs in information sets which are not reached – sometimes leads to “unreasonable” beliefs
- Solution: Sequential Equilibrium (not covered in our course)

Bayesian Nash Equilibrium

- What it is: Nash equilibrium in games with uncertainty about type of opponent or state
- every strategy is the best response to all the other strategies of all the other types of players or in all possible states
- How to find it: construct a new big payoff table with expected payoffs (rather than certain payoffs); find Nash equilibriua in a standard way

Game

- Theory vs. Real Life
- Usually the actions of players are far from the theoretical prediction

  (Dictator Game, Ultimatum Game, Beauty Contest Game, Public Good Game, Pirate Game, Centipede Game, etc.)

- Reasons:
  - People aren't always rational
  - People are overconfident
  - People are reluctant to change their minds
  - People care about fairness as demonstrated by the ultimatum game
  - People are inconsistent over time

Game

- Theory vs. Real Life
- Game Theory does not tell us what people do
- It provides a new way of thinking about strategic interactions
- It provides a framework for analyzing these situations

What's Next?

- Cooperative Games
  - Coalitions
  - Cheap Talk
- Repeated Games
  - Folk Theorem
  - Reputation
  - Grim Trigger Strategy
- Nash Bargaining
  - Rubinstein, Osborne
- Auctions

Final Exam

Materials:

- Lecture notes, homeworks
- Osborne – chapters: 1,2,4,5,9,10
- Gibbons – chapters: all four, relevant parts

- Office hours – by email appointment