


Introduction to Game Theory
Lecture 12

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

Review

- static games
  - perfect information: NE
  - imperfect information: (Bayesian) NE
- dynamic games
  - perfect information: SPNE
  - imperfect information: weak perfect Bayesian equilibrium

Weak Perfect Bayesian Equilibrium

- weak perfect Bayesian equilibrium consists of equilibrium strategies and beliefs

Definition: A wPBE equilibrium consists of behavioral strategies and beliefs systems satisfying following conditions:

1. Sequential rationality - Each players' strategy is optimal whenever she has to move, given her belief and the other players' strategies
2. Consistency of beliefs with strategies – Each player's belief is consistent with strategy profile (behavioral strategies of all players)

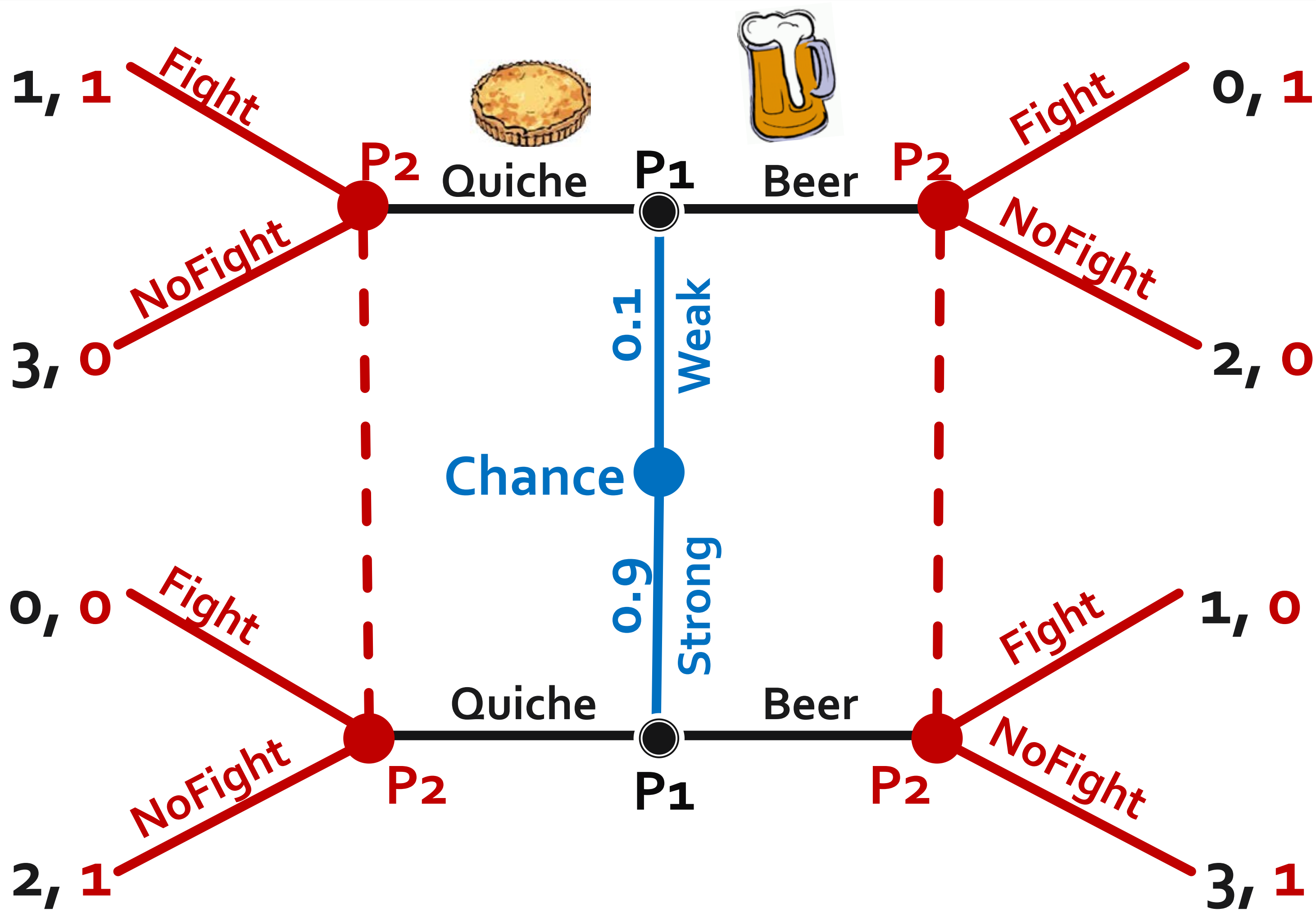
Beer or Quiche Game

- Beer or Quiche is a typical example of a signaling game: P1 sends a signal, P2 reacts
- nature chooses the type of P1: Weak, Strong with certain probabilities
- P1 chooses his breakfast: Beer or Quiche
- P2 does not know the type of P1 but observes what he had for breakfast. He then decides whether to pick up a fight with P1

Beer or Quiche

- nature chooses the type of P1: Weak, Strong with probability r, 1-r
- the Weak type likes quiche for his breakfast, the Strong type likes beer; P1 chooses his breakfast
- P2 does not know the type of P1 but observes what he had for breakfast. He then decides whether to pick up a fight with P1
- P1 gets 1 point if he had his favorite meal, and gets additional 2 points if there was no fight
- P2 gets 1 point if he fought the Weak type or if he did not fight the Strong

Beer or Quiche



Beer or Quiche

Separating equilibrium: Each type of sender (P1) chooses a different action, so the receiver (P2) knows the sender's type (e.g.: Strong->Beer; Weak->Quiche)

Pooling equilibrium: All types of sender (P1) choose the same action, so receiver (P2) has no clue about the sender's type (e.g.: Strong->Beer; Weak->Beer)

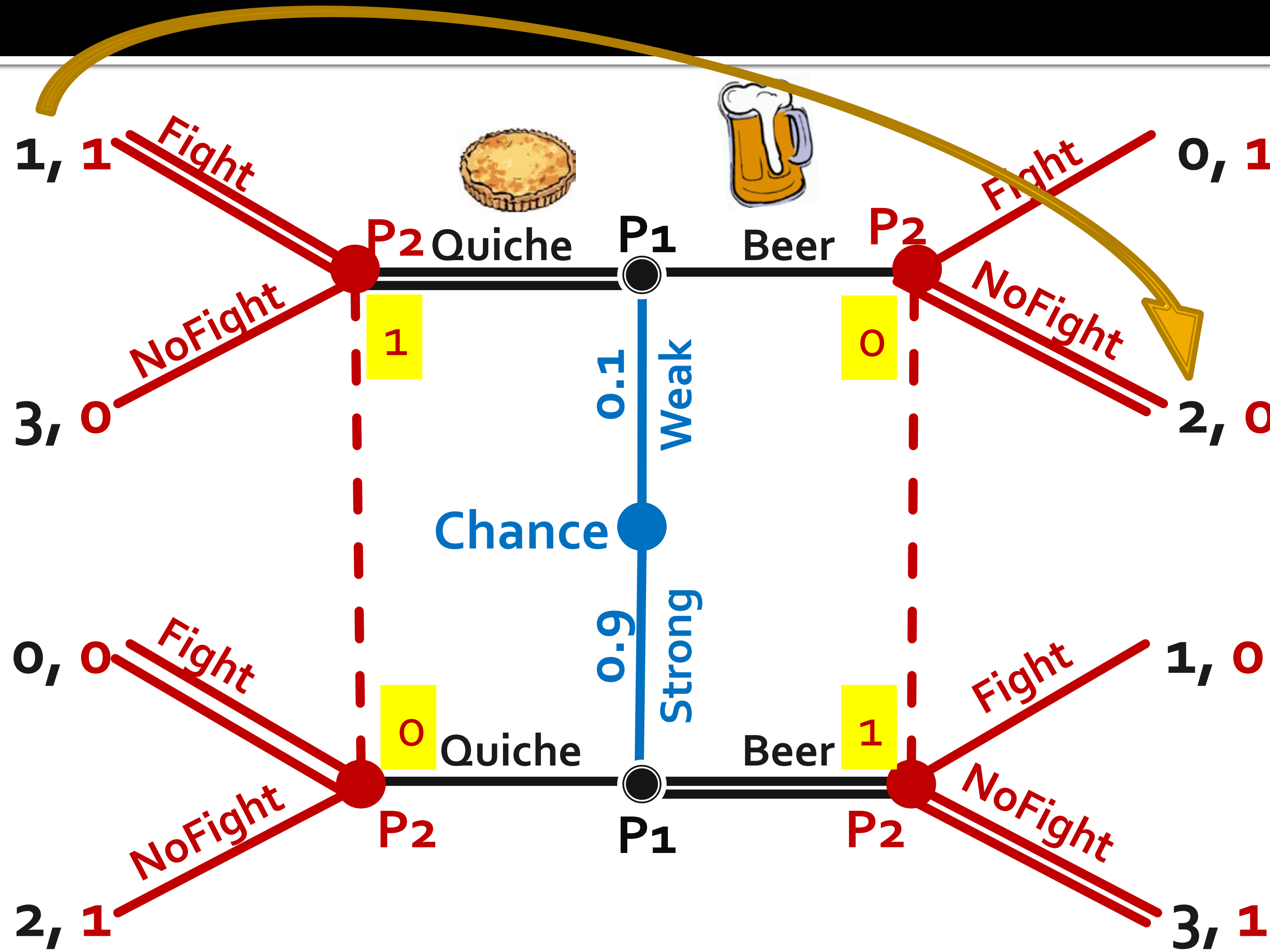
Partially separating/pooling equilibrium: Some types of sender send one message; some types of sender mix the messages (e.g.: Strong->Beer; Weak->mix)

Separating Equilibrium 1

- P1: Strong->Beer; Weak->Quiche
- P2: optimal response to that is:
  if Beer->NoFight; if Quiche->Fight

- not an equilibrium, Weak wants to deviate

Separating Equilibrium 1

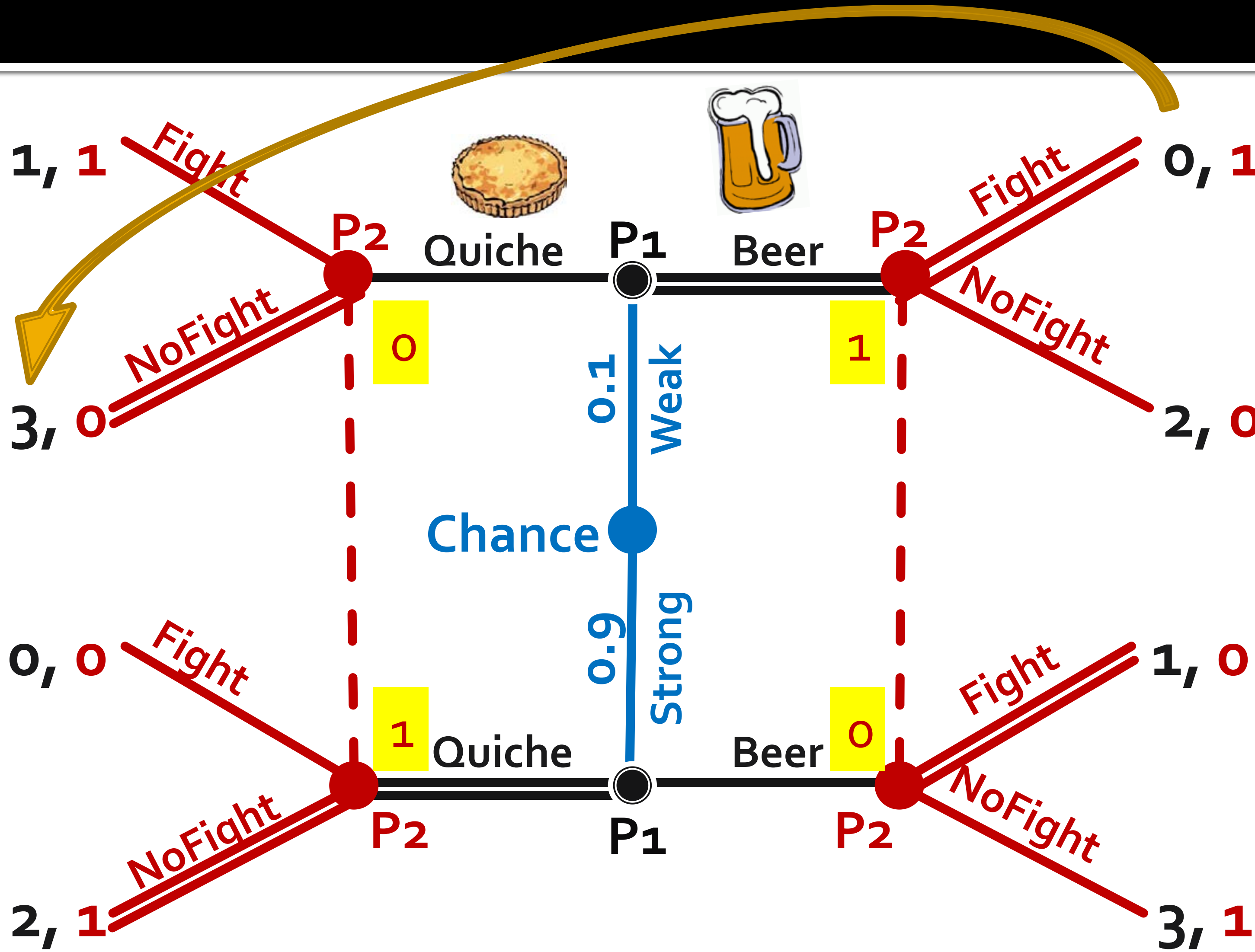


Separating Equilibrium 2

- P1: Strong->Quiche; Weak->Beer
- P2: optimal response to that is:
  if Beer->Fight; if Quiche->NoFight

- not an equilibrium, Weak wants to deviate

Separating Equilibrium 2



Pooling Equilibrium 1

- P1: Strong->Beer; Weak->Beer
- P2: optimal response to that:

  NoFight if Beer

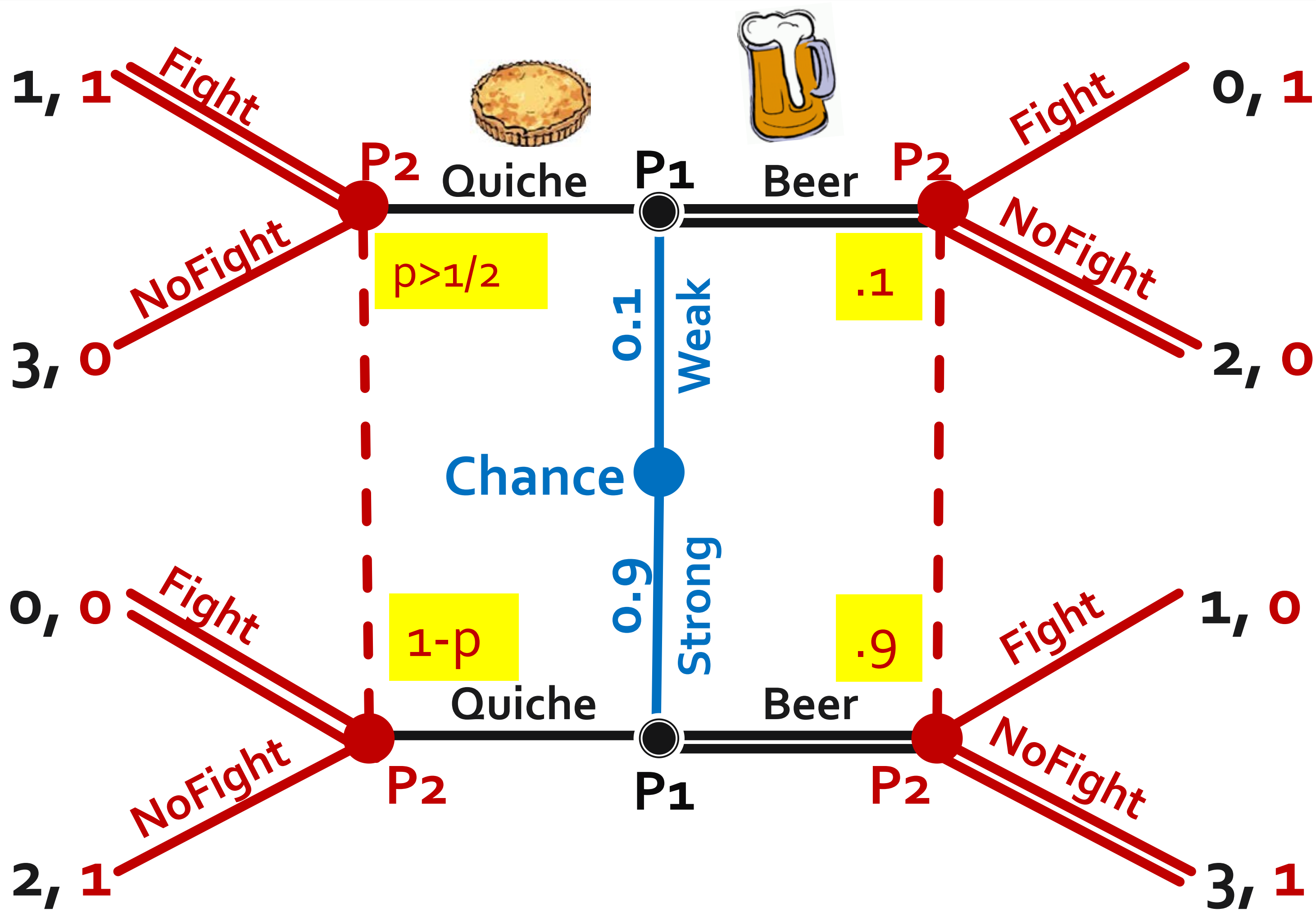
  Fight if Quiche (so P1 - Weak does not deviate)

- WPBE:

  if Beer, Pr(Weak)=0.1; Pr(Strong)=0.9

  if Quiche, Pr(Weak)=p; Pr(Strong)=1-p; where p>1/2

Pooling Equilibrium 1



Pooling Equilibrium 2

- P1: Strong->Quiche; Weak->Quiche
- P2: optimal response to that:

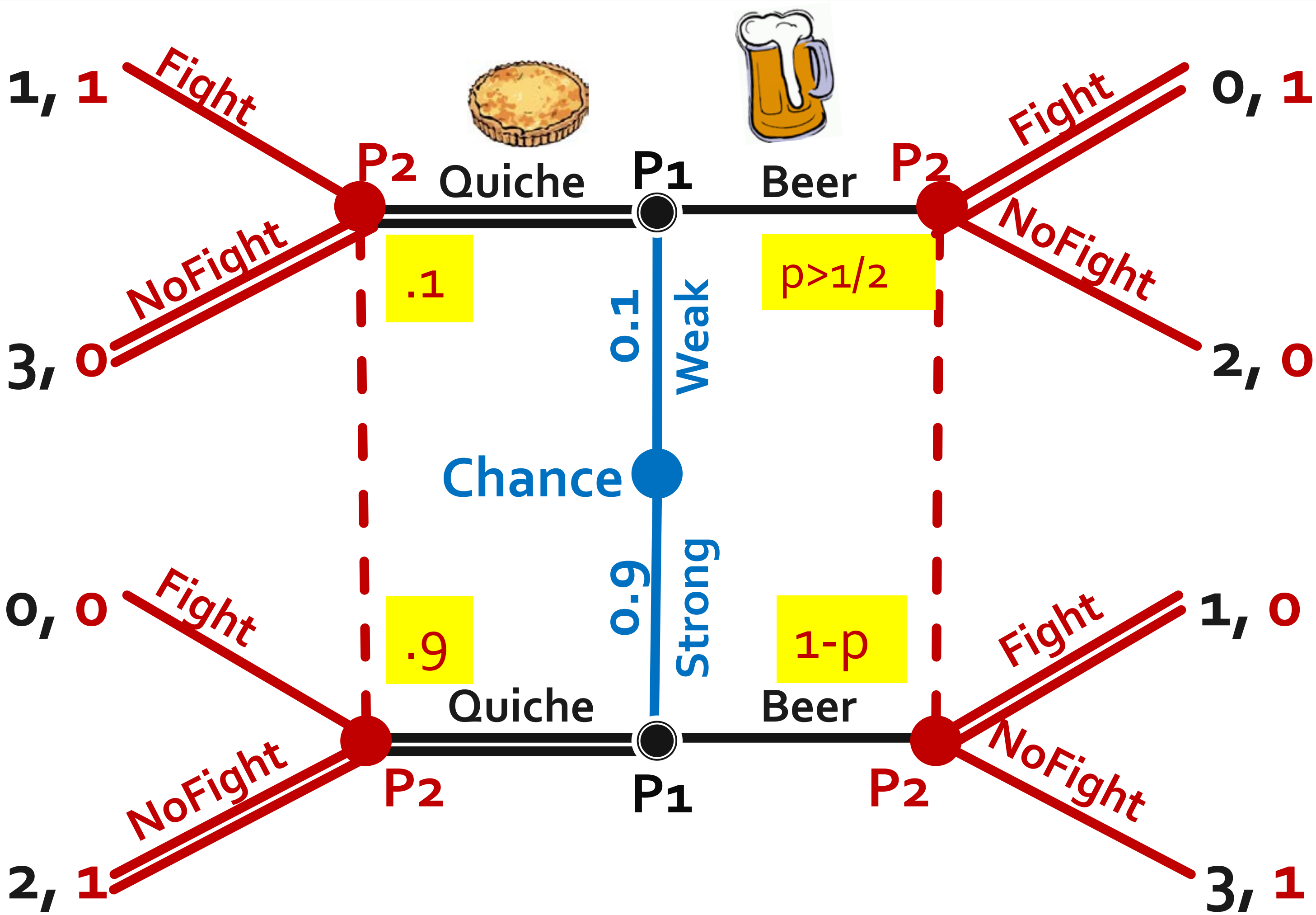
  NoFight if Quiche

  Fight if Beer (so P1-Strong does not deviate)

  if Quiche, Pr(Weak)=0.1; Pr(Strong)=0.9

  if Beer, Pr(Weak)=p; Pr(Strong)=1-p; where p>1/2

Pooling Equilibrium 2



Beer or Quiche

There are two pooling equilibria in this game:

- P1: Strong->Quiche; Weak->Quiche
- P2: if Beer->Fight; if Quiche->NoFight

  if Beer: Pr(Strong)=1-p; Pr(Weak)=p, p>1/2

  if Quiche: Pr(Strong)=0.9; Pr(Weak)=0.1

- P1: Strong->Beer; Weak->Beer
- P2: NoFight if Beer, Fight if Quiche

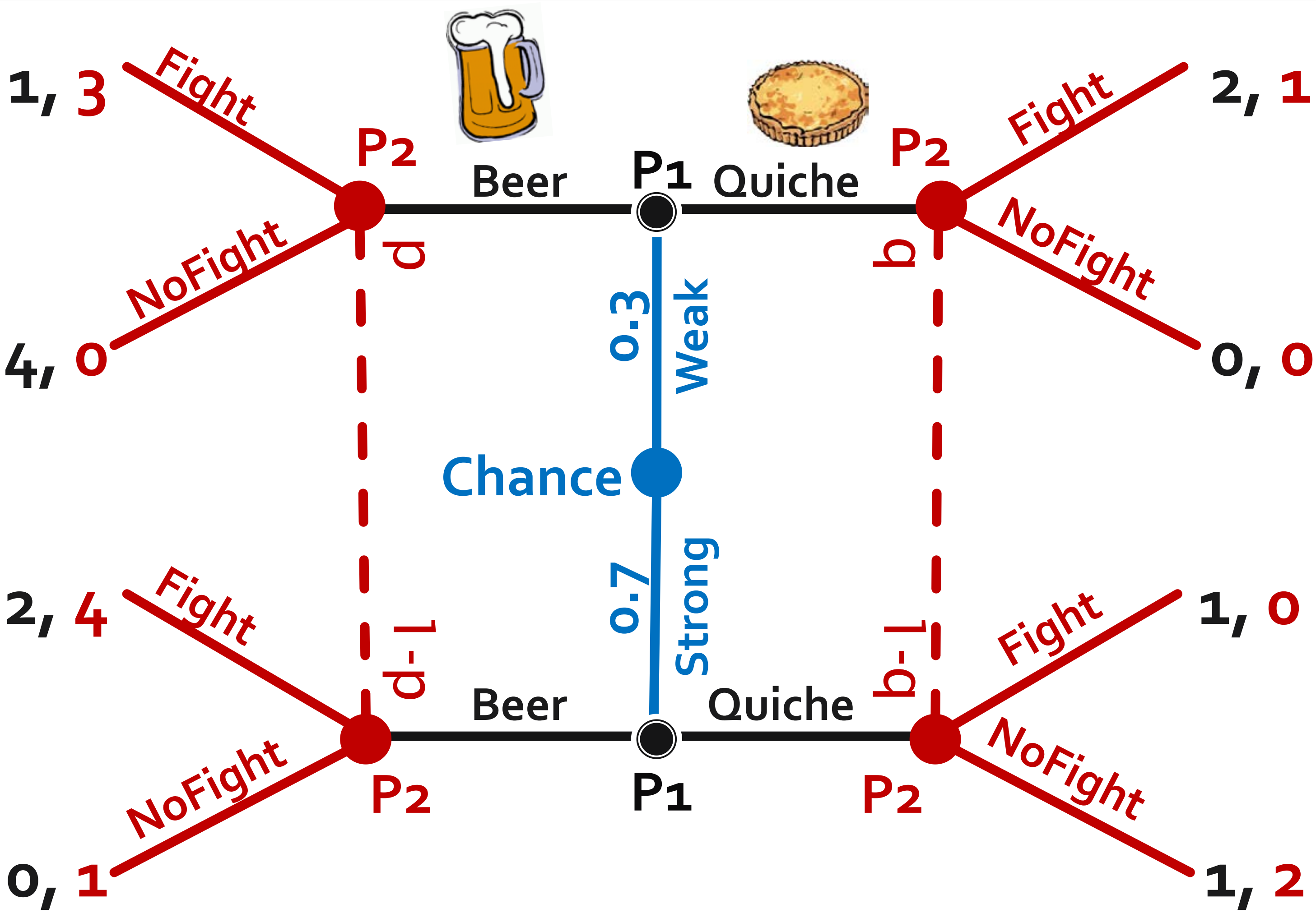
  if Beer, Pr(Weak)=0.1; Pr(Strong)=0.9

  if Quiche, Pr(Weak)=p; Pr(Strong)=1-p, p>1/2

Modified Beer or Quiche

- Beer or Quiche is a typical example of a signaling game: P1 sends a signal, P2 reacts
- nature chooses the type of P1: Weak, Strong with certain probabilities
- P1 chooses his breakfast: Beer or Quiche
- P2 does not know the type of P1 but observes what he had for breakfast. He then decides whether to pick up a fight with P1

Modified Beer or Quiche

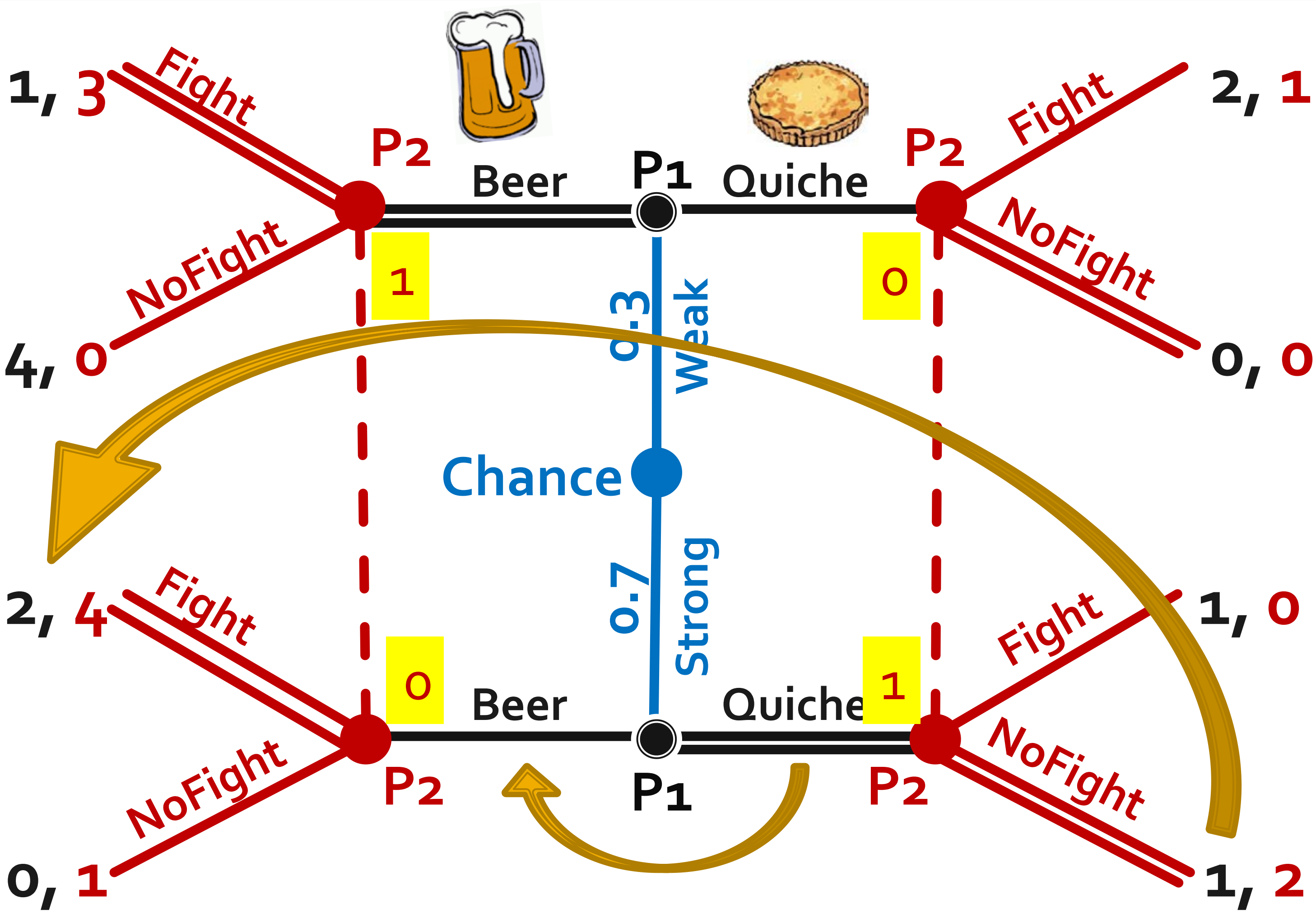


Separating Equilibrium 1

- P1: Strong->Quiche; Weak->Beer
- P2: optimal response to that is:
  if Beer->Fight; if Quiche->NoFight

- not an equilibrium, Strong wants to deviate

Separating Equilibrium 1



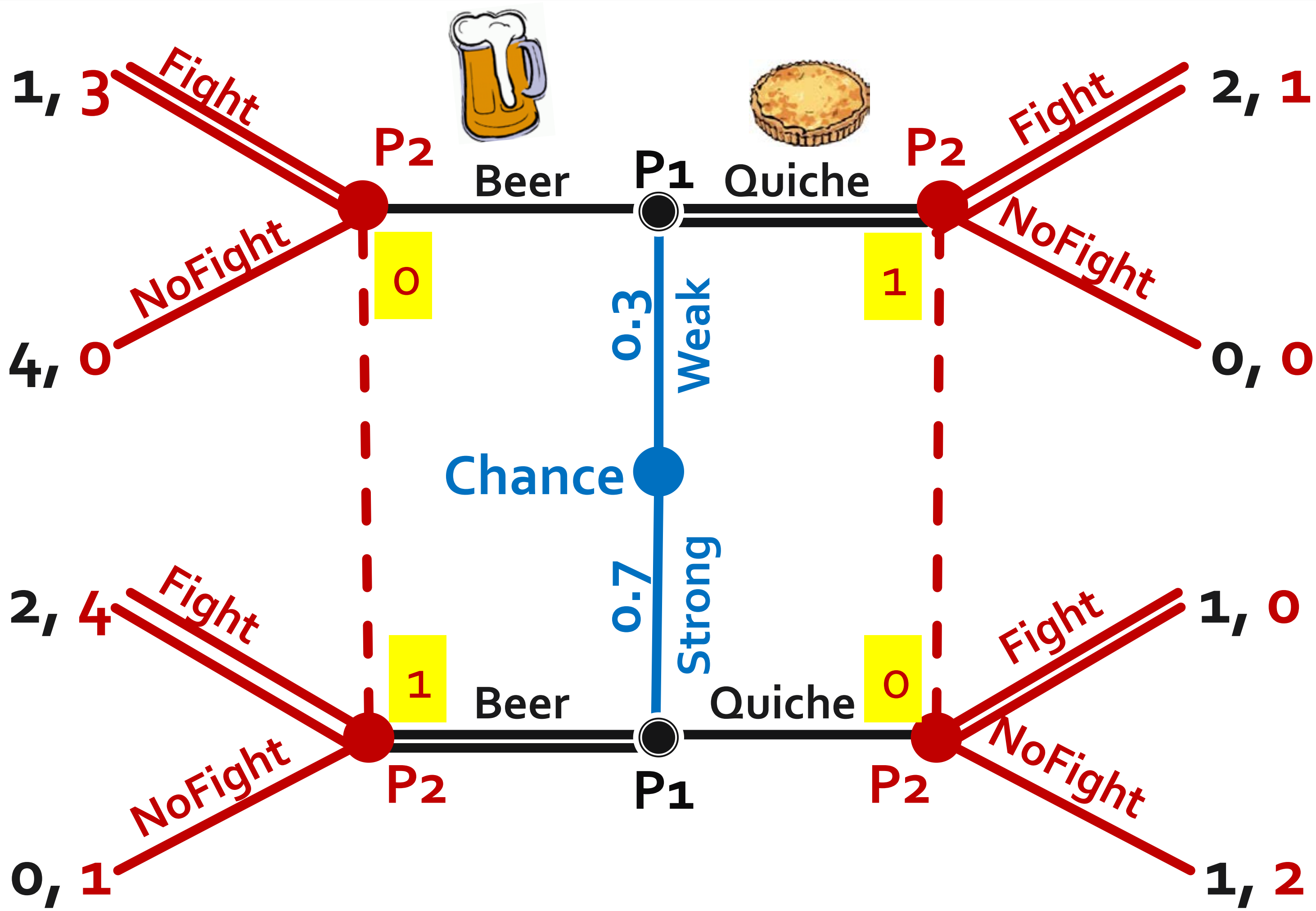
Separating Equilibrium 2

- P1: Strong->Beer; Weak->Quiche
- P2: optimal response to that is:
  if Beer->Fight; if Quiche->Fight

- No type of P1 wants to deviate

- wPBE:
  - P1: Strong->Beer; Weak->Quiche
  - P2: if Beer->Fight; if Quiche->Fight
    if Beer: Pr(Strong)=1; Pr(Weak)=0
    if Quiche: Pr(Strong)=0; Pr(Weak)=1

Separating Equilibrium 2

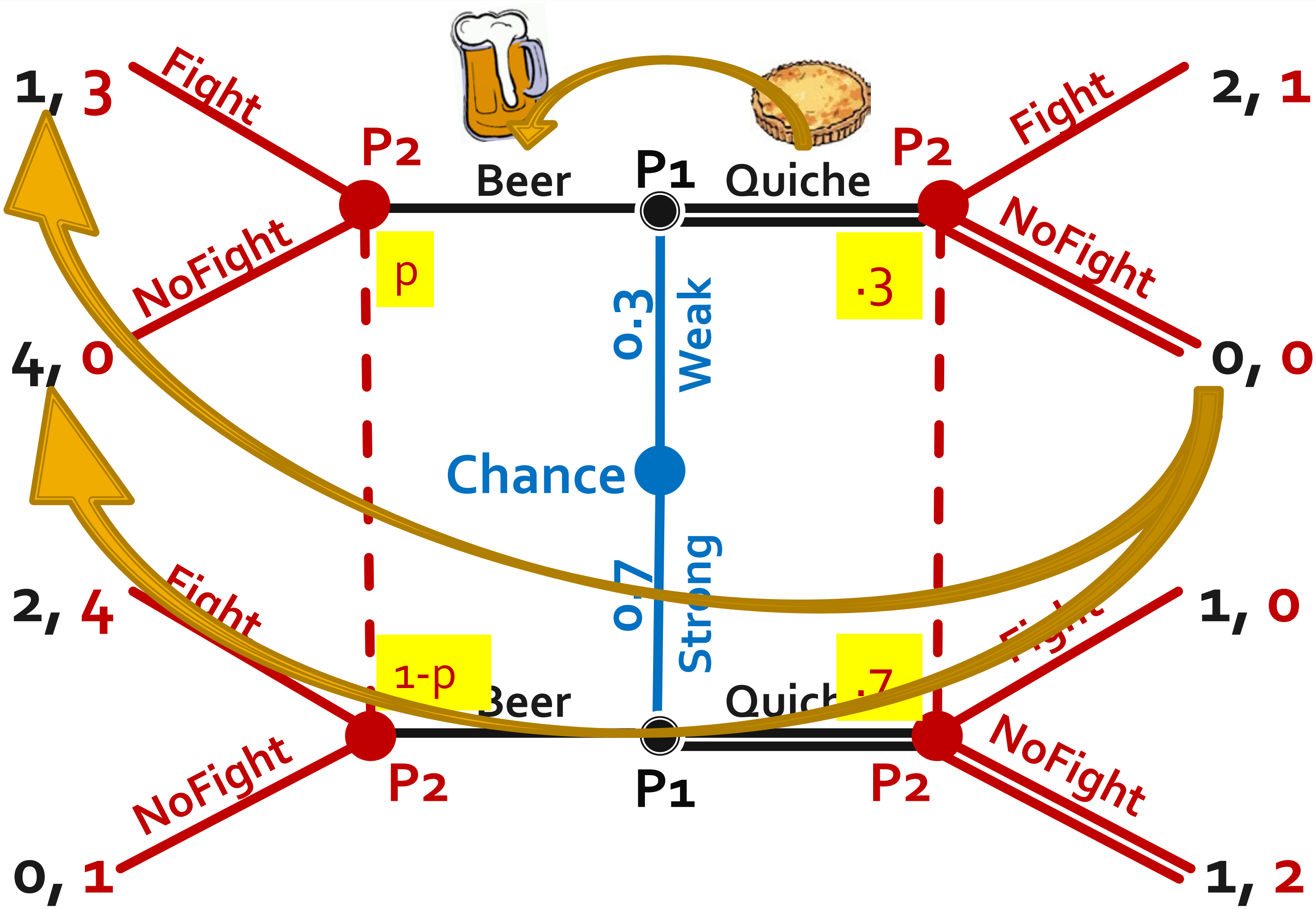


Pooling Equilibrium 1

- P1: Strong->Quiche; Weak->Quiche
- P2: optimal response to that:
  NoFight if Quiche

- not an equilibrium, Weak wants to deviate, irrespective of P2’s strategy after observing Beer

Pooling Equilibrium 1



Pooling Equilibrium 2

- P1: Strong->Beer; Weak->Beer
- P2: optimal response to that:

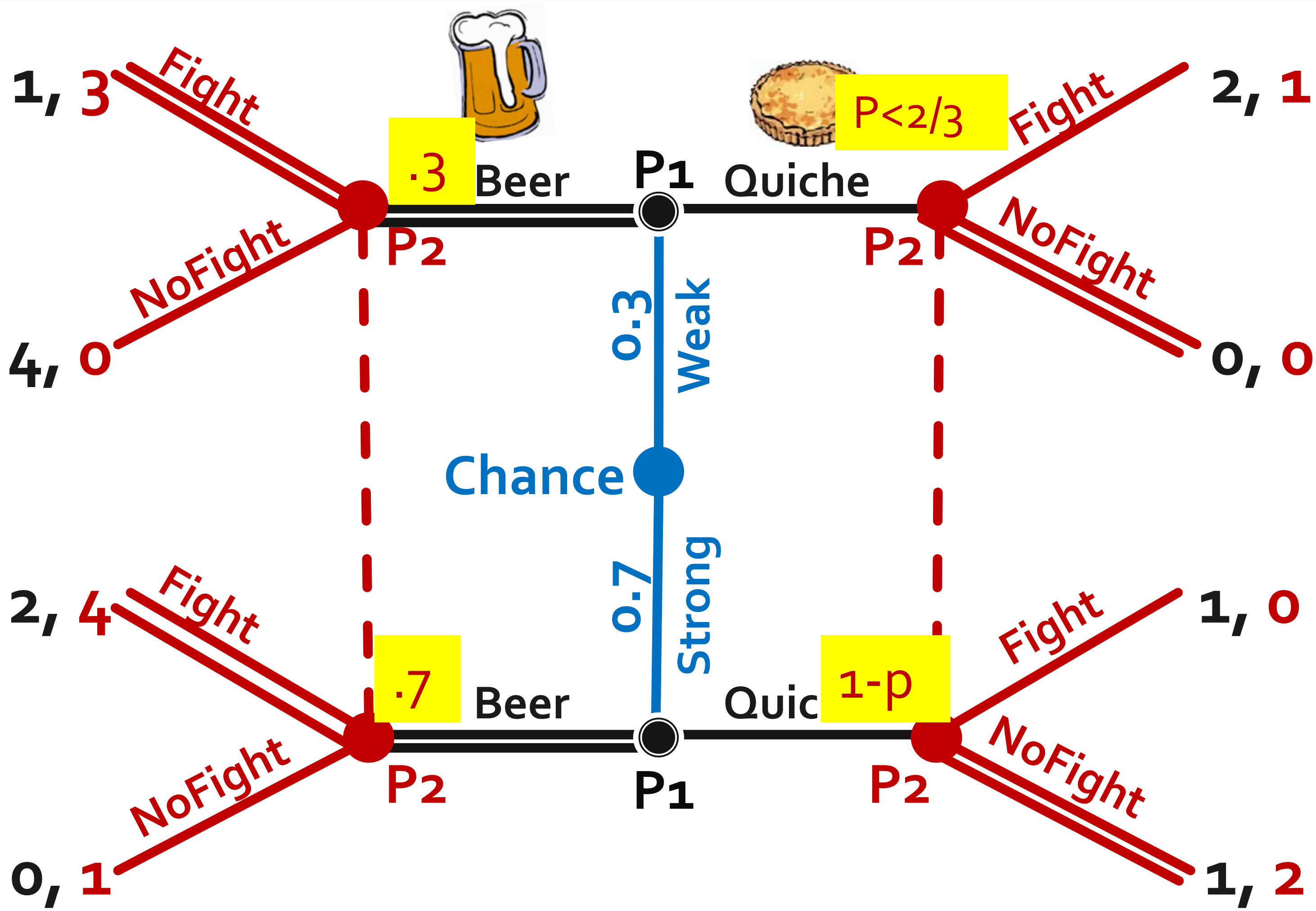
  Fight if Beer

  NoFight if Quiche (so P1-Weak does not deviate)

  if Beer, Pr(Weak)=0.3; Pr(Strong)=0.7

  if Quiche, Pr(Weak)=p; Pr(Strong)=1-p; where $p<2/3$

Pooling Equilibrium 2



Modified Beer or Quiche

There is one separating and one pooling equilibrium in this game:

- P1: Strong->Beer; Weak->Quiche
- P2: if Beer->Fight; if Quiche->Fight

  if Beer: Pr(Strong)=1; Pr(Weak)=0

  if Quiche: Pr(Strong)=0; Pr(Weak)=1

- P1: Strong->Beer; Weak->Beer
- P2: Fight if Beer, NoFight if Quiche

  if Beer, Pr(Weak)=0.3; Pr(Strong)=0.7

  if Quiche, Pr(Weak)=p; Pr(Strong)=1-p;

  where p<2/3

Summary

How to find wPBE?

- Analyze the game player by player, strategy by strategy (if game is simple enough)
  - start with one player, one strategy
  - compute beliefs and optimal responses of others
  - check seq. rationality and consistency of beliefs
- Find NE first, look for the system of beliefs that can support each NE
  - normal form game – NE
  - compute beliefs
  - check seq. rationality and consistency of beliefs
- In Signaling games
  - analyze all possible pooling equilibria
  - analyze all possible separating equilibria