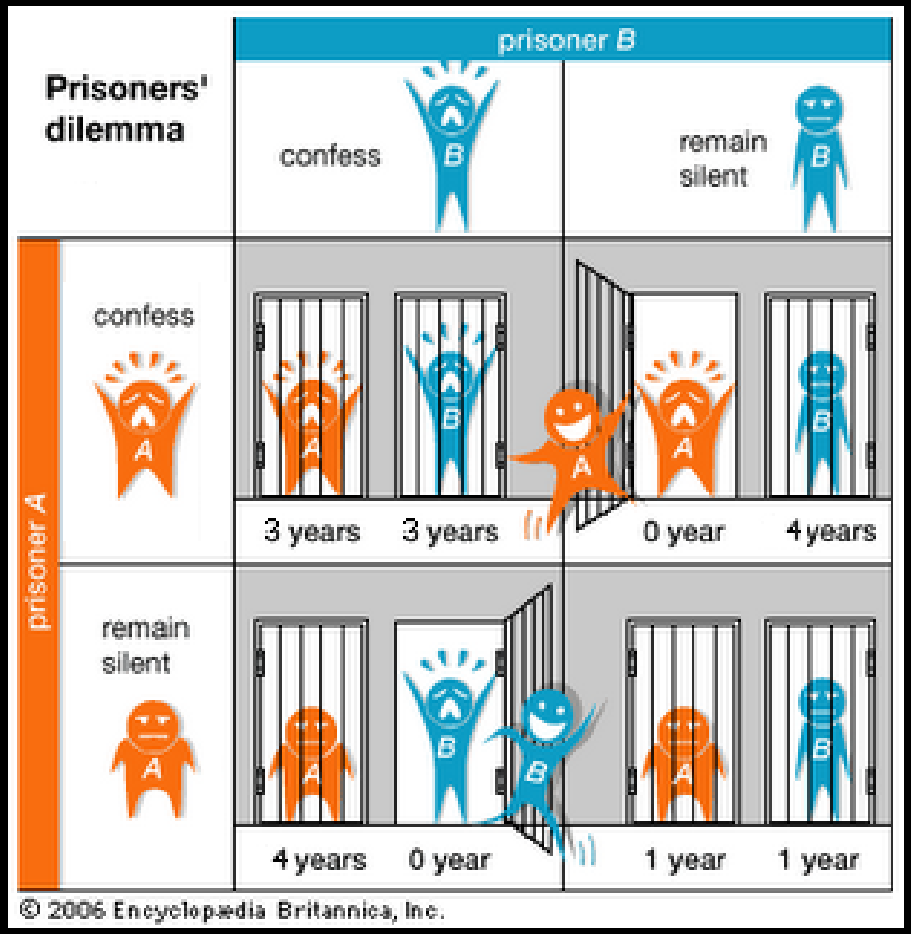


Introduction to Game Theory Lecture 1

Disclaimer: this presentation is only a supporting material and is not sufficient to master the topics covered during the lecture. Study of relevant books is strongly recommended.

Syllabus

- Contact: kalk00@vse.cz
  home.cerge-ei.cz/kalovcova/teaching.html
- Office hours: Wed 7.30pm – 8.00pm, NB339
  or by email appointment
- Osborne, M. J. – An Introduction to Game Theory
  Gibbons, R. – A Primer in Game Theory
  Suggested articles
- Important information on webpage
- Grading: Midterm 30%, Final 60%,
  Homework 10%, Experiments up to 5%

Game Theory is...

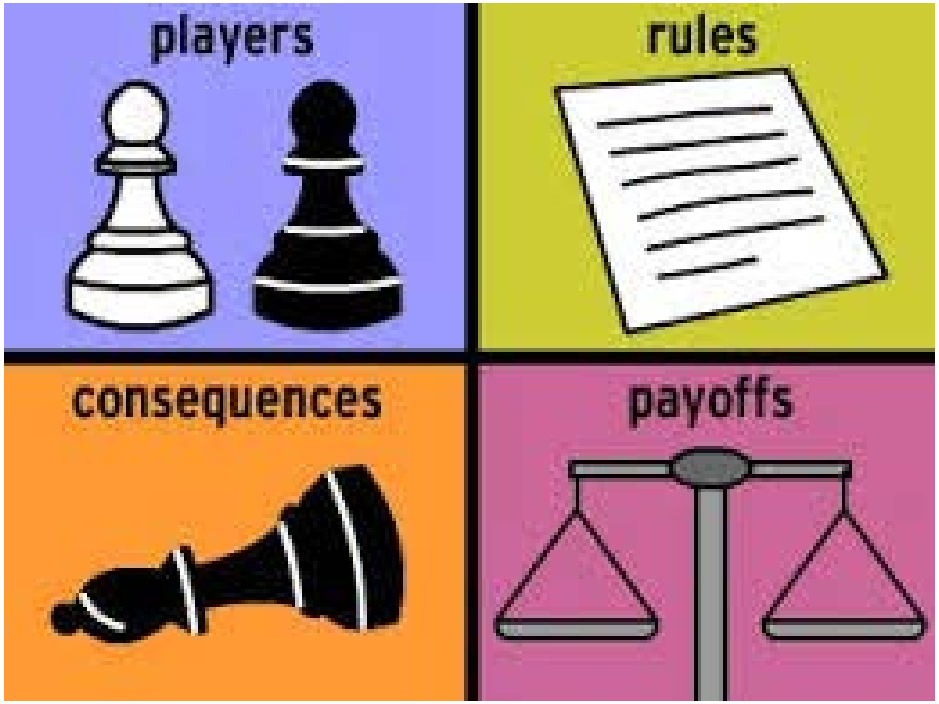
Game theory

"Game theory is the science of strategic thinking"

[A. K. Dixit, B. J. Nalebuff: Thinking Strategically: The Competitive Edge in Business, Politics, and Everyday Life]

- Strategic thinking is an art of trying to outsmart the others, knowing that they are trying to do the same to you
  - Corporations need to implement good strategies in order to survive competition
  - Politicians have to devise campaign strategies to get elected
  - Players plan strategies to win the match, …
- Strategic thinking in complex situations remains an art, however, its foundations consist of some simple basic principles embodied in the game theory

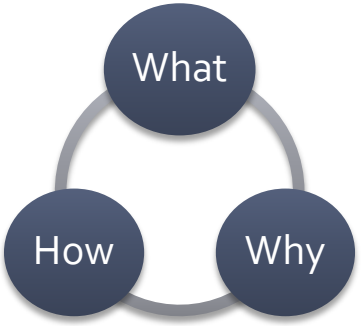
This course is about ...



... role of economic models
... models of strategic behavior
... models of social interactions

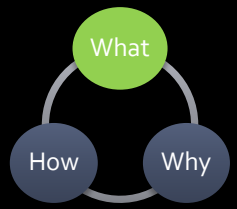
Plan for Today

- Economic models
  - What and Why?
- Game Theory Models
  - Examples
- Games
  - players, strategies, information, outcomes



Economic Models

Economic Models - What?



- an abstraction that we use to understand our observations and experience
- assumptions capture the essence, not irrelevant details => good enough approximation
- should be as simple as possible while keeping the purpose in mind (usefulness)
- Example: utility maximization (model of behavior of individuals)

Economic Models - Problem

- based on *Homo Rationalis*
- acts purposefully and logically
- has well-defined goals
- has motivation and ability to approach them
- does not exist!

- real-life cousin *Homo Sapiens*
- guided by (sub)conscious psychological (irrational) drives; affected by herd instincts
- not well-defined goals
- lack of ability to achieve them, lack of motivation
- guided by emotions rather than brains

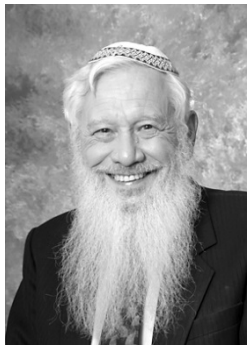
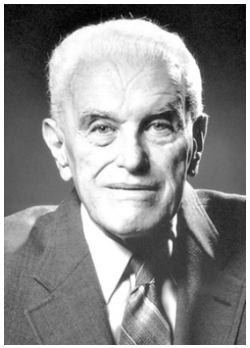
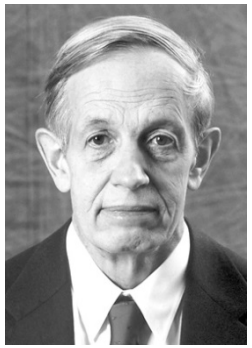
Economic Models - Problem

- models do not tell us how people truly behave
  people do not really maximize utility

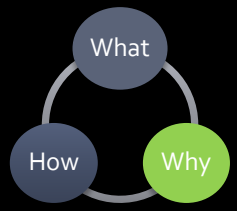
- no predictive power => models are far from being perfect one might even dare to say that they are “wrong”

Economic Models - “But”

… we must be doing something right, otherwise we would not be here…

Economic Models - Why?



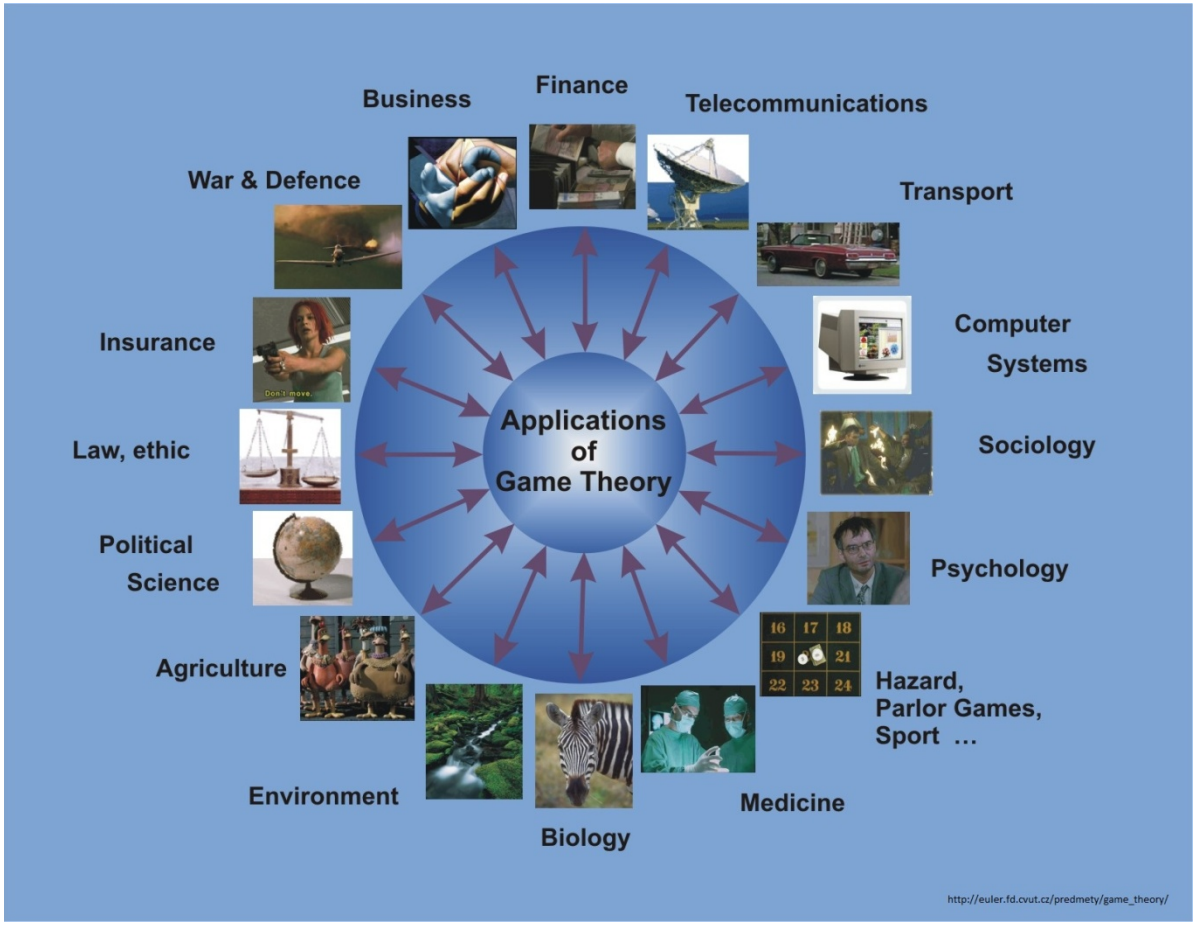
- models are NOT designed to tell us what people will do, they are not designed to be descriptive
- models are designed to provide some insight into behavior of Homo Sapiens by studying Homo Rationalis
- moreover, as setting gets familiar, people act more rationally in that setting (do not become more rational just act like that!)

  we know that people do not really maximize utility but in certain settings they behave as if they were maximizing utility

Economic Models - Why?

- organize thoughts and observations at a given time in a useful manner
- help to understand the relationships, recognize patterns
- provide non-trivial insights and common language
- analyze complex situations in a tractable way
- at the end of a day – provide practical implications
- Utility maximization - major component of a certain way of thinking, pulls together most of economic theory. More attractive and realistic alternatives failed because they did not have any interesting consequences
- For more information on this topic, please see Aumann, R. (1985). "What is Game Theory Trying to accomplish?“ – link on the website.

Games


Business
Finance
Telecommunications
War & Defence
Transport
Insurance
Computer Systems
Applications of Game Theory
Law, ethic
Sociology
Political Science
Psychology
Agriculture
Hazard, Parlor Games, Sport ...
Environment
Medicine
Biology
http://euler.fd.cvut.cz/predmety/game_theory/

Game theory is about …

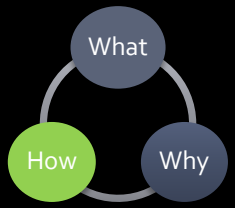
Take following situations:

- tennis players deciding whether to serve to the left or right side of the court
- employees deciding how hard to work when the boss is away
- rival drug firms investing in a race to reach patent
- all these examples have something in common: strategic (interactive) nature
- *Note: as opposed to situations where an individual or firm makes decisions which do not affect others (which TV channel you watch when you are home alone, what furniture a firm chooses, etc.)*

Game Theoretic Models

- Model situations in which decision-makers interact / strategic situations
  - firms competing for business
  - political candidates competing for votes
  - bidders competing in an auction
  - firm and union negotiating next year's wage contract
  - legislators negotiating the date of next elections or change of constitution
- all these examples illustrate different types of games (ultimatum games, gift exchange, mixed equilibrium, patent race games)

Economic Models - How?



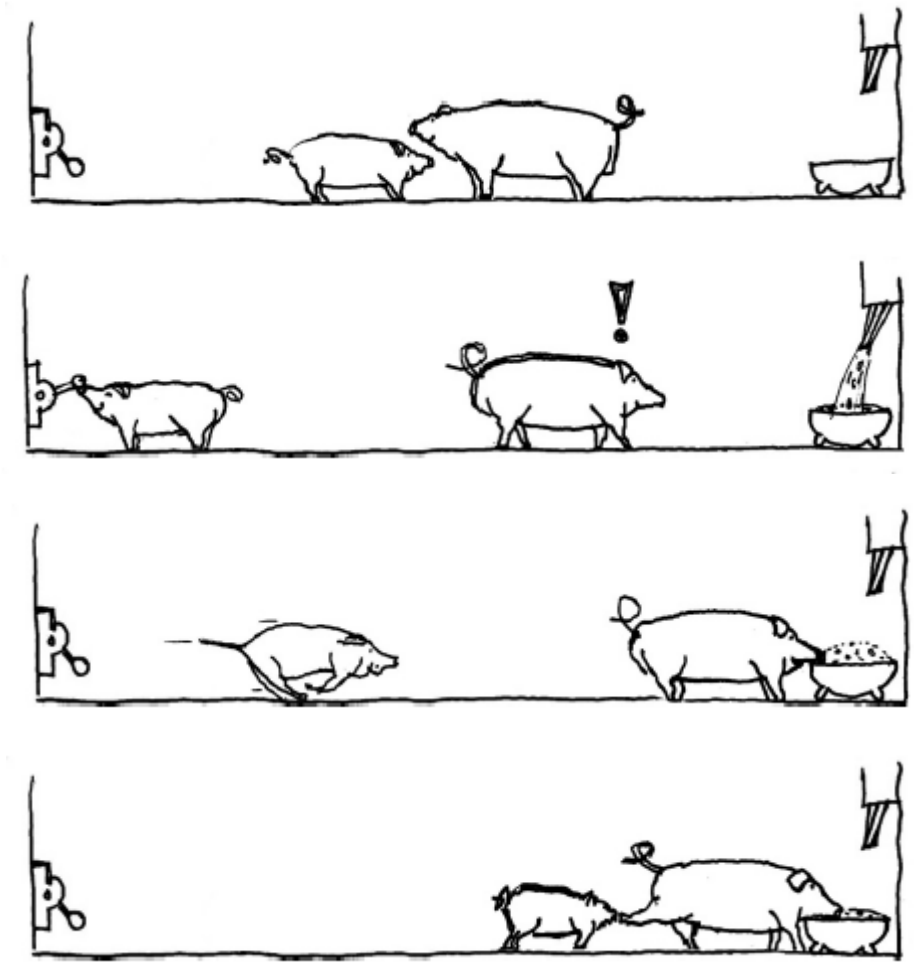
Boxed Pigs Example

Staring: Small pig and Big pig

Boxed Pigs Example

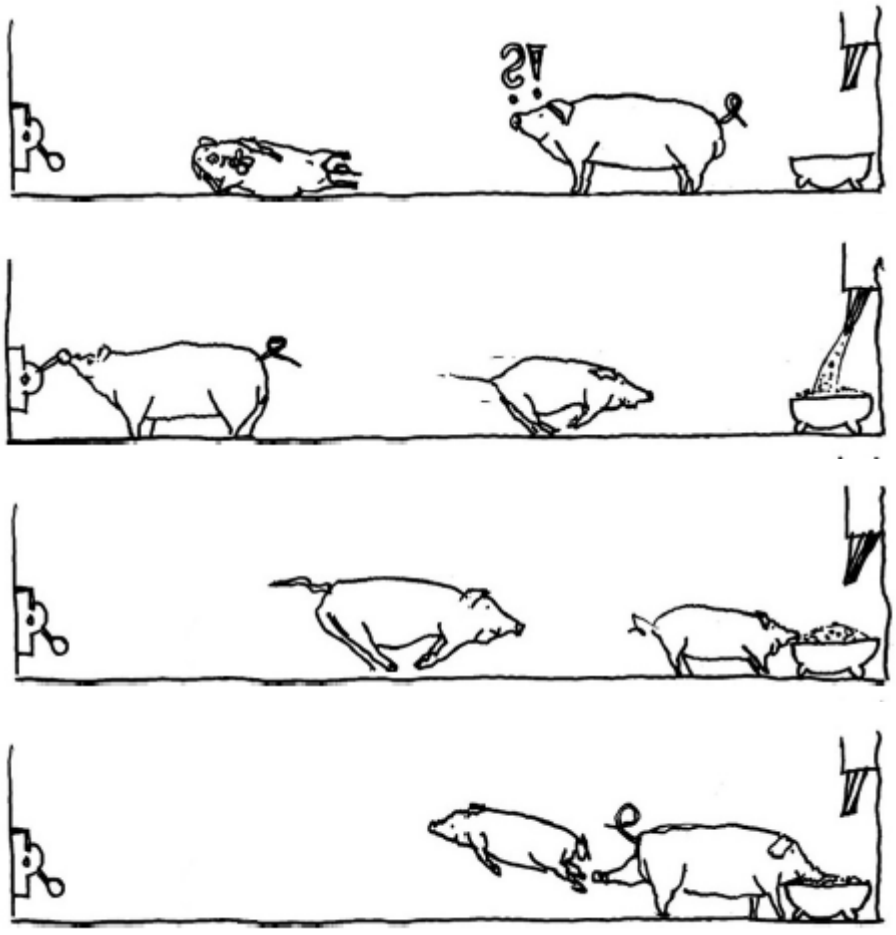
- lever is at one end of the box
- food dispenser is at the other end
- at least one pig has to push lever for food dispenser to work and then go to the other end to get the food
- place two pigs in the box – it creates opportunity for one pig to exploit the other; experimental evidence shows, that indeed eventually one (*master*) will sit and eat and the other (*slave*) will run back and forth

Boxed Pigs Example



http://euler.fd.cvut.cz/predmety/teorie_her/

Boxed Pigs Example



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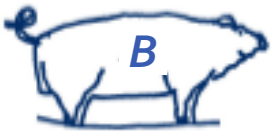
Boxed Pigs Example

- Result of experiment – Big pig presses lever, Small pig eats a little and is pushed away
- How can we model this situation
  - capturing the essence
  - in a tractable way
  - as simply as possible
- *Important note*: we do not judge whether pigs behave rationally or not. We merely take real situation and try to model it in a simple way

Boxed Pigs Example



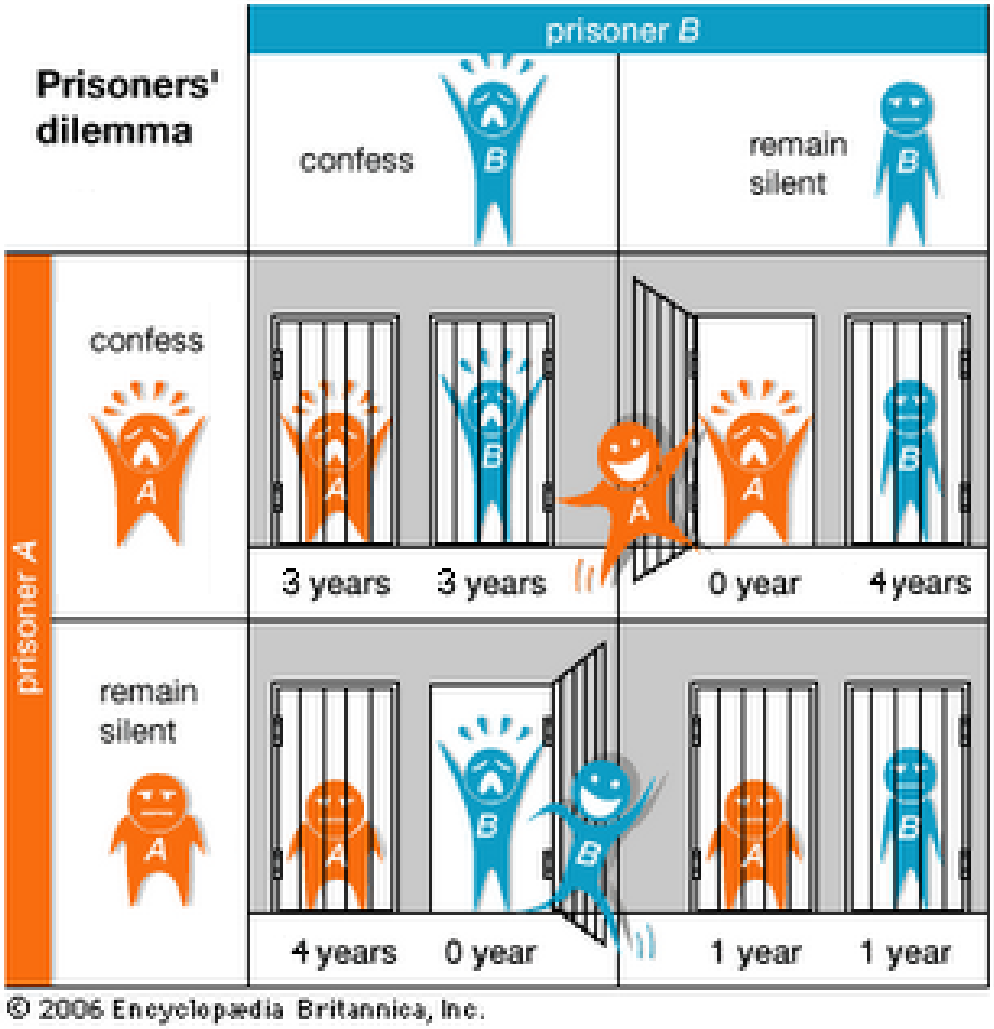
Strategy	Press lever		Sit & wait
Press lever	$(8, -2)$	$\rightarrow$	$\boxed{(5, 3)}$
Sit & wait	$(10, -2)$	$\rightarrow$	$(0, 0)$

- in this representation:
  - food is worth 10 units
  - cost of pressing lever is 2 units
  - small pig eats 3 units before it is pushed away
- (there are many possible representations as long as we keep them in line with the story)

Basic Games

- The most commonly presented games include:
  - Prisoner's Dilemma
  - Battle of Sexes
  - Matching Pennies
  - Stag hunt

Game 1 - Prisoner's dilemma



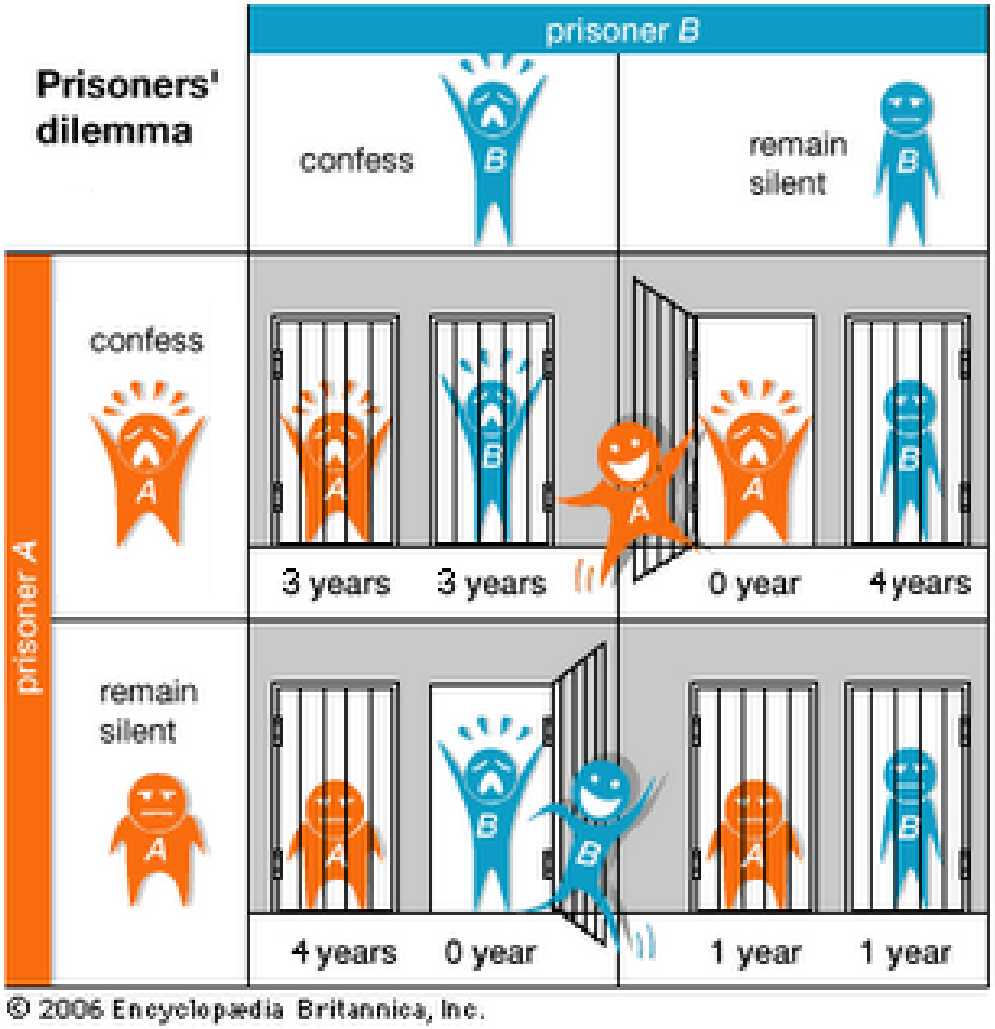
Games - Elements

- (strategic) game is a model of interacting decision-makers
- normal-form representation (table)
- decision-makers are called players
- each player has a possible set of actions
- each player has preferences about action profiles – list of all players' actions
- payoffs are results of individual outcomes

Games - Assumptions

- for each pair of actions, player knows which action is better (or is indifferent between them) - completeness
- if action *a* is preferred to *b*, and *b* is preferred to *c*, then also *a* is preferred to *c* - transitivity
- *the action chosen by a decision-maker is at least as good, according to her preferences, as every other available action* - Rational Choice
  (most preferred bundle of all possible bundles)
- consistency: if {a,b} -> a, then NOT {a,b,c}->b

Games - Elements



players:

prisoner A, prisoner B

actions for A and B (symmetric game):

confess (C), remain silent (Rs)

action profiles A,B:

CC, CRs, RsC, RsRs

preferences:

“less is better”

Games - Elements

- preferences are often represented by payoff functions

- payoff functions have only ordinal (not cardinal) significance; i.e. only order of outcomes (payoff from a certain action profile) matters

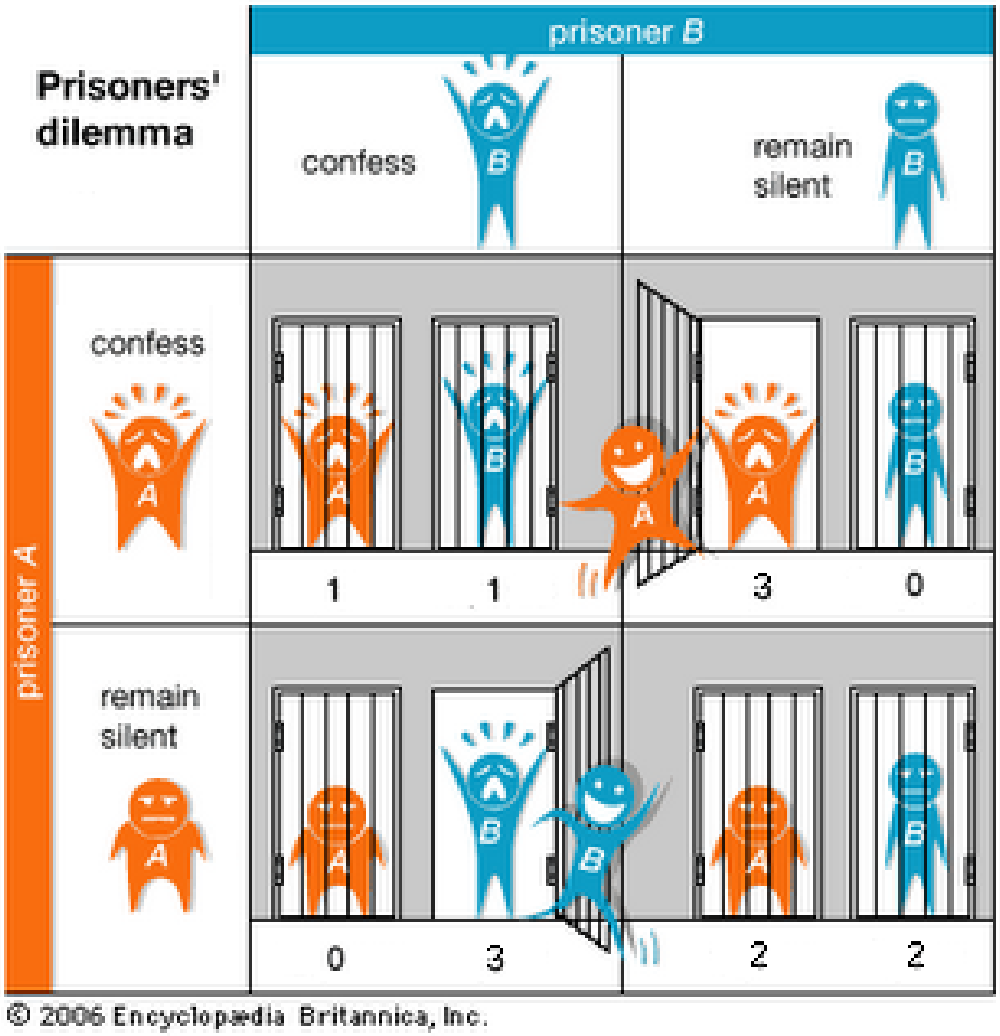
Games - Elements

- Prisoner's dilemma:

  $u_1(C,Rs) > u_1(Rs,Rs) > u_1(C,C) > u_1(Rs,C)$

- obvious candidate for a payoff function:
  - $u_1(C,Rs) = 3, \quad u_1(Rs,Rs) = 2$
  - $u_1(C,C) = 1, \quad u_1(Rs,C) = 0$
- any other combination of numbers is possible as long as the order is the same

Games - Elements



players:

prisoner A, prisoner B

actions for A and B

confess (C), remain silent (Rs)

action profiles A,B:

CC, CRs, RsC, RsRs

~~preferences~~:

payoff function:

"more is better"

Game 1 - Prisoner's Dilemma

1 \ 2	A Lot	Little
A Lot	1,1	3,0
Little	0,3	2,2

- Working on joint project
- Duopoly
- Arms race
- Common property

Game 2 - Battle of Sexes

1 \ 2	Boxing	Shopping
Boxing	2,1	0,0
Shopping	0,0	1,2

- Negotiations of political parties
  - they disagree about the best stand but are both better off if they choose the same stand
- Two merging firms using different technologies

Game 2 - Battle of Sexes

- Set of players
  - two friends deciding about where to spend evening
- Set of actions
  - Boxing, Shopping
- Profiles of actions
  - {B,B}, {B,S}, {S,B}, {S,S}
- Preferences / payoff functions
  - 1: {B,B} > {S,S} > {B,S} = {S,B}
  - 2: {S,S} > {B,B} > {B,S} = {S,B}

Game 3 - Matching Pennies

1 \ 2	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

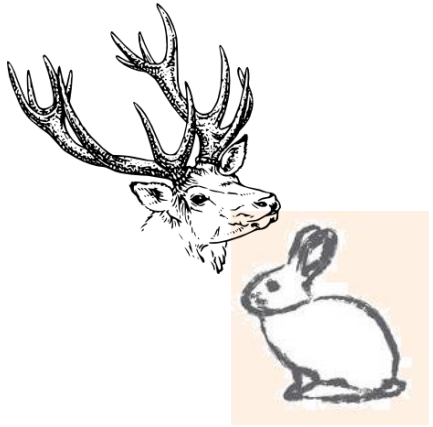
- Choice of appearances for new product
  - established producer prefers a new firm with different appearance, new firm prefers the same
- Relationship between two people
  - one person wants to be like the other, the other wants to be different

Game 3 - Matching Pennies

- Set of players
  - two persons/ two producers
- Set of actions
  - Head, Tail
- Profiles of actions
  - {H,H}, {H,T}, {T,H}, {T,T}
- Preferences / payoff functions
  - 1: {H,H} = {T,T} > {H,T} = {T,H}
  - 2: {H,T} = {T,H} > {H,H} = {T,T}

Game 4 - Stag Hunt

1 \ 2	No Bomb	Bomb
No Bomb	2,2	0,1
Bomb	1,0	1,1



- Alternative to Prisoner's dilemma (arms race)
  - Cost of arming outweighs benefit of other country not arming

Game 4 - Stag Hunt

- Set of players
  - two hunters
- Set of actions
  - Stag, Hare
- Profiles of actions
  - {S,S}, {S,H}, {H,S}, {H,H}
- Preferences / payoff functions
  - 1: {S,S} > {H,S} = {H,H} > {S,H}
  - 2: {S,S} > {S,H} = {H,H} > {H,S}

Games - Classification

- Prisoner's dilemma, Battle of sexes, Matching pennies, Stag hunt - examples of Static Games of Complete and Perfect Information

- static games: players simultaneously chose actions
- complete information: actions and payoff functions are known to all players (structure is known)
- perfect information: players know actions taken by other players (actions are known)

Games - Classification

Preview of the course:

- Games of complete and perfect information
  - Static Games (Nash Equilibrium)
  - Dynamic Games (Backward Induction)
- Games of complete but imperfect information
  - Dynamic Games (Subgame perfect NE)
- Games of incomplete information
  - Static Games (Auctions)
  - Dynamic Games (Signaling)

Summary I

- Economic Models
  - good enough approximation of real world with many useful purposes
- Game Theory Models
  - subgroup of economic models
  - situations where decision-makers interact

Summary II

- Strategic game consists of
  - set of players
  - for each player set of actions
  - for each player set of preferences over the set of action profiles
  - preferences represented by payoff function
- Solving games
  - iterative elimination of strictly dominated strategies (next lecture)
  - Nash Equilibrium (next lecture)
  - Other methods (later in the course)