Beer or Quiche game

In this handout, we discuss signaling games of *Beer or Quiche* type and the way to find the plausible outcome of this type of games – weak perfect Bayesian equilibrium.

The basic idea is that we try all the combinations of actions of all players one by one see if any of them is an equilibrium. In general, we work in the following steps:

Step 1: Start with actions of Player 1 (both types)

Step 2: Find Player 2's beliefs based on actions of Player 1 (in both information sets)

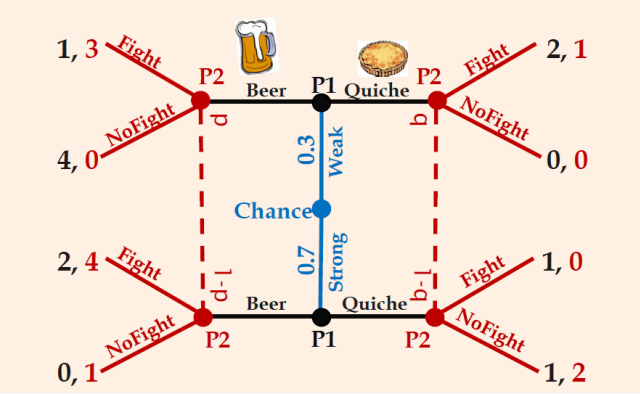
Step 3: Find optimal response of Player 2 (in both information sets)

Step 4: Check if both types of Player 1 play optimal response:

a. yes, no type of Player 1 wants to deviate => we have WPBE
b. no, at least one type of Player 1 wants to deviate => no WPBE

Step 5: Move to the next possible actions of Player 1

Example: consider the following game.



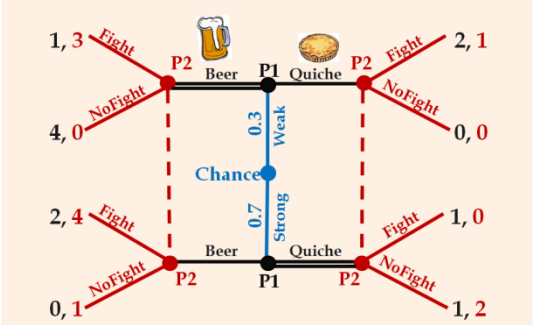
We will analyze these options:

1. Player 1: Beer if Weak, Quiche if Strong (Separating equilibrium 1)
2. Player 1: Quiche if Weak, Beer if Strong (Separating equilibrium 2)
3. Player 1: Quiche if Weak, Quiche if Strong (Pooling equilibrium 1)
4. Player 1: Beer if Weak, Beer if Strong (Pooling equilibrium 2)

Option 1: Player 1: Beer if Weak, Quiche if Strong (Separating equilibrium 1)

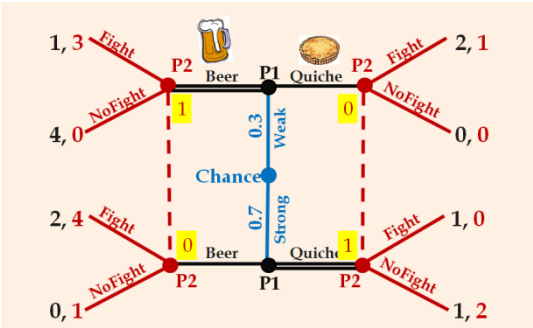
Step 1: Start with actions of Player 1 (both types)

Beer if Weak, Quiche if Strong



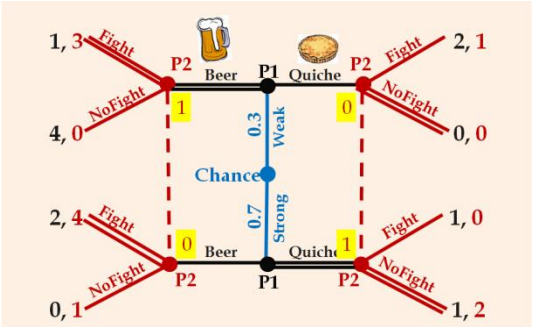
Step 2: Find Player 2's beliefs based on actions of Player 1 (in both information sets)

Information sets are reached, so beliefs are calculated based on actions of Player 1.



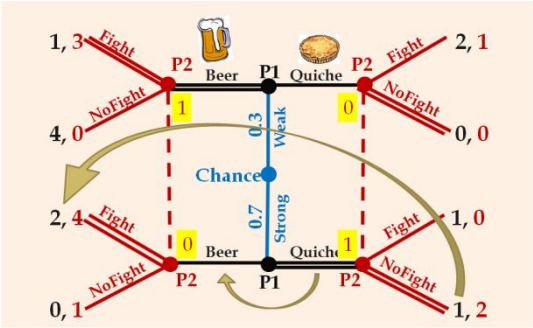
Step 3: Find optimal response of Player 2 (in both information sets)

In the left information set, Player 1 knows that Player 2 is of Weak type and Fighting is optimal (3>0).
In the right information set, Player 2 knows that Player 1 is of Strong type and Not Fighting is optimal (2>0).



Step 4: Check if both types of Player 1 play optimal response

Weak type of Player 1 is satisfied, but Strong type of Player 1 wants to deviate and get higher profit. So this is not WPBE.

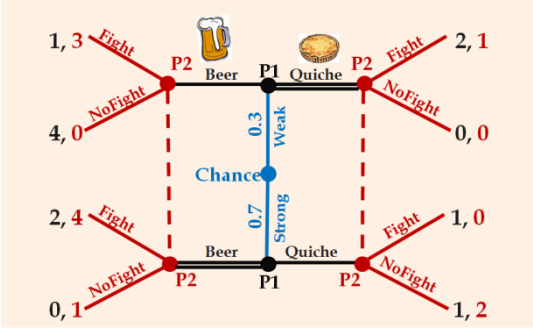


Step 5: Move to the next possible actions of Player 1

Option 2: Player 1: Quiche if Weak, Beer if Strong (Separating equilibrium 2)

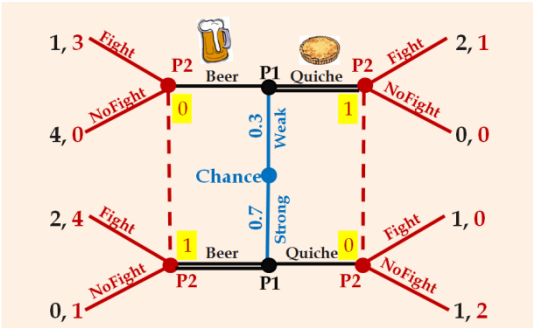
Step 1: Start with actions of Player 1 (both types)

Quiche if Weak, Beer if Strong



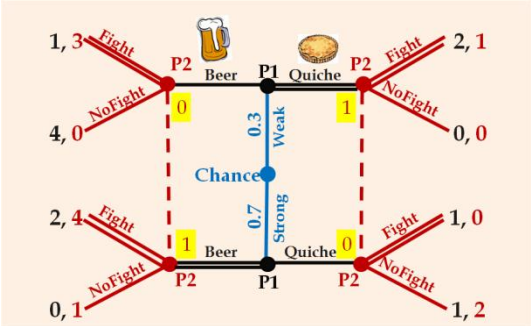
Step 2: Find Player 2's beliefs based on actions of Player 1 (in both information sets)

Information sets are reached, so beliefs are calculated based on actions of Player 1.



Step 3: Find optimal response of Player 2 (in both information sets)

In the left information set, Player 1 knows that Player 2 is of Strong type and Fighting is optima (4>1). In the right information set, Player 2 knows that Player 1 is of Weak type and Fighting is optimal (1>0).



Step 4: Check if both types of Player 1 play optimal response

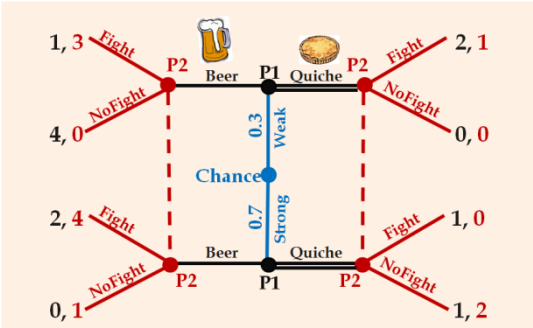
No type of Player 1 wants to deviate because the payoff would be lower. So this is WPBE.

Step 5: Move to the next possible actions of Player 1

Option 3: Player 1: Quiche if Weak, Quiche if Strong (Pooling equilibrium 1)

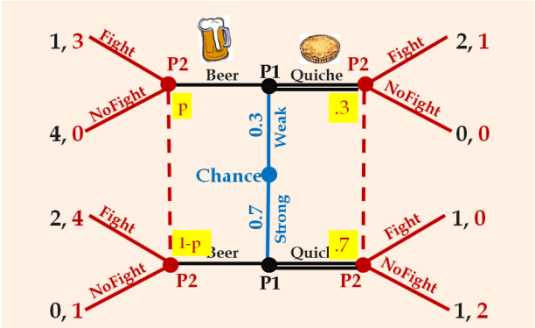
Step 1: Start with actions of Player 1 (both types)

Quiche if Weak, Quiche if Strong



Step 2: Find Player 2's beliefs based on actions of Player 1 (in both information sets)

Right information set is reached, so beliefs are calculated based on actions of Player 1. Left information set is not reached, so we are free to choose any beliefs, any set of beliefs is consistent with Player 1's actions. Denote beliefs p and 1-p.



Step 3: Find optimal response of Player 2 (in both information sets)

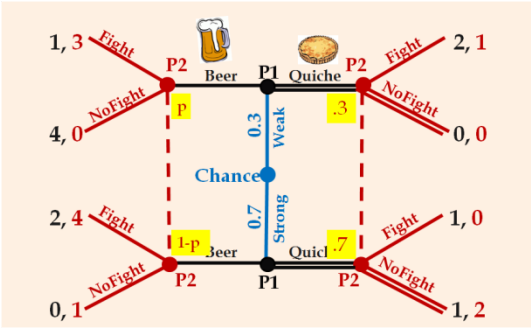
In the right information set, Player 2 calculates expected payoff of playing Fight and NotFigt:

EP(Fight) = 0.3*1+0.7*0 = 0.3

EP(NotFight) = 0.3*0+0.7*2 = 1.4

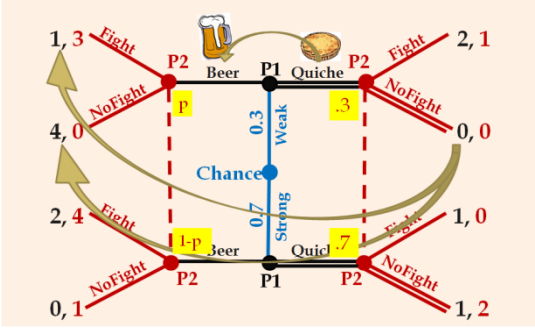
So NotFight is optimal response.

In the left information set, optimal response of Player 2 generally depends on value of p (beliefs). However, in this case, Fight dominates NotFight in the left information set and hence Fight is optimal.



Step 4: Check if both types of Player 1 play optimal response

Actually, no matter what action Player 2 chooses in the left information set, Weak type of Player 1 will always want to deviate. So this is not WPBE.

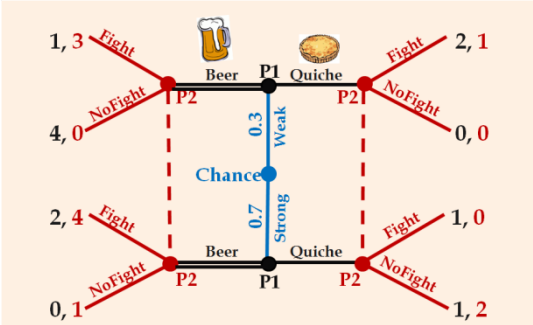


Step 5: Move to the next possible actions of Player 1

Option 4: Player 1: Beer if Weak, Beer if Strong (Pooling equilibrium 2)

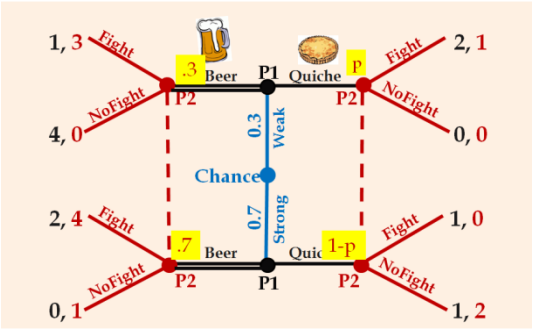
Step 1: Start with actions of Player 1 (both types)

Beer if Weak, Beer if Strong



Step 2: Find Player 2's beliefs based on actions of Player 1 (in both information sets)

Left information set is reached, so beliefs are calculated based on actions of Player 1. Beliefs in the right information set are denoted p and 1-p because this information set is not reached and hence any set of beliefs is consistent with actions of P1..



Step 3: Find optimal response of Player 2 (in both information sets)

In the left information set, Player 2 calculates expected payoff of playing Fight and NotFigt:

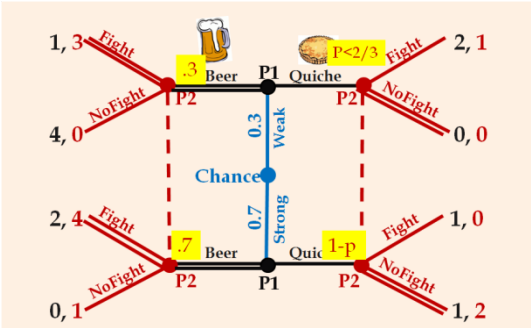
EP(Fight) = 0.3*3+0.7*4 = 3.7

EP(NotFight) = 0.3*0+0.7*1 = 0.7

So Fight is optimal response.

In the right information set, optimal response of Player 2 generally depends on value of p (beliefs). There are two options, either the optimal response is Fight, but then Weak Player 1 would want to deviate; or the optimal response is NotFight. In this case no type of Player 1 would want to deviate. So the beliefs have to be set such that expected payoff from NotFight is larger than from Fight.

EP(Fight) < EP(NotFight) => 1*p + 0*(1-p) < 0*p + 2*(1-p) => p<2/3



Step 4: Check if both types of Player 1 play optimal response

No type of Player 1 wants to deviate because the payoff would be lower. So this is WPBE.

Step 5: Move to the next possible actions of Player 1. There are no more possibilities.

There are two WPBE in this game:

1. Weak P1 – Quiche, Strong P1 – Beer; P2 Fight if Beer and Fight if Quiche and p=0 in the left information set and p=1 in the right information set.
2. Weak P1 – Beer, Strong P1 – Beer; P2 – Fight if Beer and NotFight if Quiche and p<2/3.