1 Budget Set, Preferences, Consumer's Optimum

People choose the best things they can afford.

Every consumer makes a decision about how much to consume of each good. Without any sort of restrictions, we would want to eat as much as we could of everything. However, economics is about choice under scarcity. In particular, each of us has limited money to buy all the things that we like. Hence, we're going to introduce the budget set constraint.

1.1 The budget set

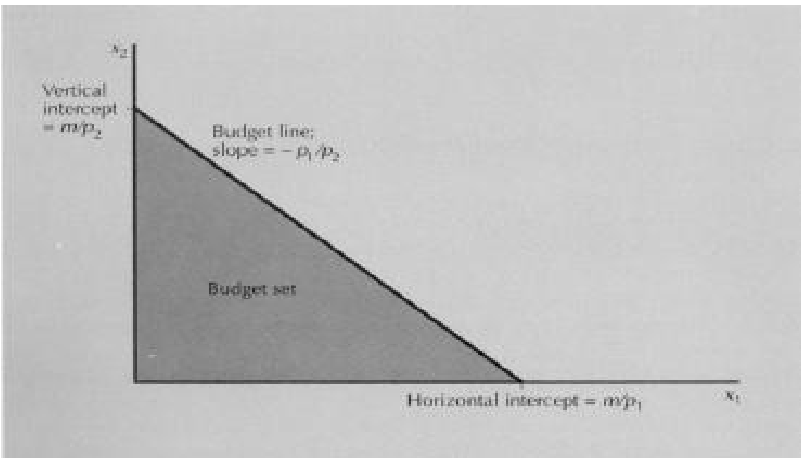
People choose things they can afford.

In general, we'll take prices and the consumer's income as given. The budget set consists of all bundles of goods that the consumer can afford at given prices and income. We can write down in an equation the amount of each good that I can consume.

Assume two goods X and Y, their prices P_X and P_Y and income I. Then the budget set is given by:

$$P_X X + P_Y Y \leq I \quad \Rightarrow \quad \text{Budget line is given by: } Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$$

The absolute easiest way to draw a budget set is to start by figuring out how much of each good I could buy if I spent all my money on it. To find this number, all you need to do is divide your income by the price of each good. You can then put these points on the graph and connect them with a line. That line is the budget set. (Why is the line straight?)



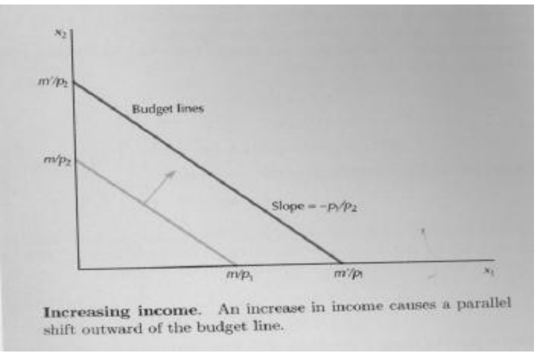
The budget set. The budget set consists of all bundles that are affordable at the given prices and income.

The formula tells us how many units of good 2 the consumer needs to consume in order to just satisfy the budget constraint if she is consuming X units of good 1. The slope of the budget line measures the rate at which the market is willing to substitute good 1 for good 2, or opportunity cost of consuming good 1.

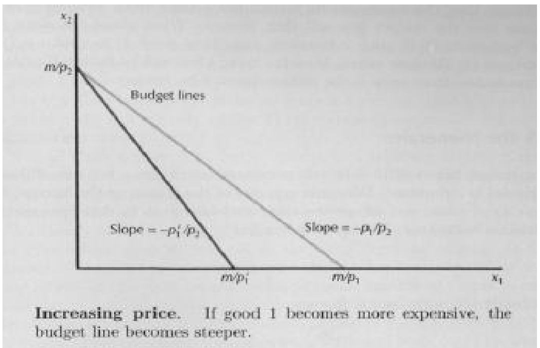
Changes in budget line:

When prices and incomes change, the set of goods that a consumer can afford changes as well. How?

- **Change in income:** Increase in income result in parallel shift outward of the budget line. The intercepts change, the slope remains the same.
- **Change in price:** Increase in price of good 1 result in shift of the horizontal intercept of the budget line inward.
- **Change in both prices and income by the same factor:** Budget set remains the same.



Increasing income. An increase in income causes a parallel shift outward of the budget line.



Increasing price. If good 1 becomes more expensive, the budget line becomes steeper.

Numeraire: Setting one of the prices equal to 1 and adjusting the other price appropriately does not change the budget set at all. If we set price of one good to 1 we refer to this good a numeraire and to this price as numeraire price. It is convenient to do that because there will be one variable less.

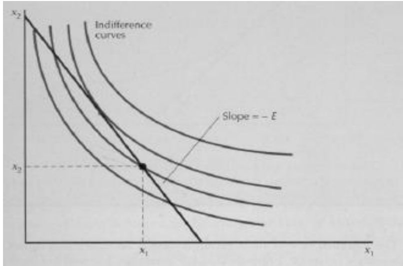
1.2 Preferences

People choose the best things.

We suppose that preferences are:

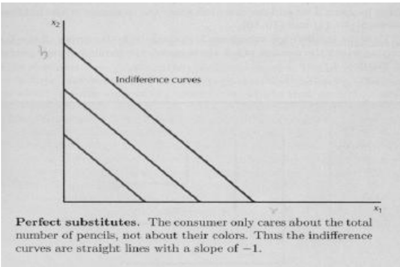
- **complete**, i.e. we suppose that given any two consumption bundles (x_1, x_2) and (y_1, y_2), the consumer can rank them as to their desirability. That is, the consumer can determine that one of the bundles is strictly better than the other, or decide that she is indifferent between the two
- **reflexive**, i.e. any bundle is at least as good as itself: $(x_1, x_2) \succeq (x_1, x_2)$
- **transitive**, i.e. if $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$

Indifference curve: is a graphical representation of preferences; it is a set of bundles for which the consumer is just indifferent. Start in some bundle. Now think about giving a little bit less of good 1 to the consumer. How much more of good 2 do you have to give him in order to keep his utility level?

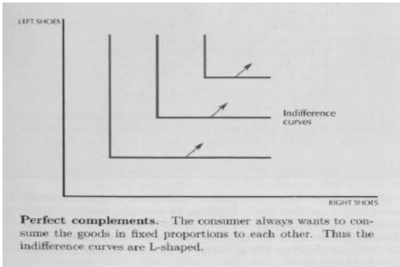


The shape of the indifference curves can be different:

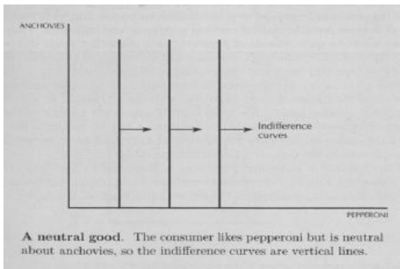
- perfect substitutes: if the consumer is willing to substitute one good for the other at a constant rate (linear utility function $u(x_1, x_2) = x_1 + x_2$)



Perfect substitutes. The consumer only cares about the total number of pencils, not about their colors. Thus the indifference curves are straight lines with a slope of −1.



Perfect complements. The consumer always wants to consume the goods in fixed proportions to each other. Thus the indifference curves are L-shaped.



A neutral good. The consumer likes pepperoni but is neutral about anchovies, so the indifference curves are vertical lines.

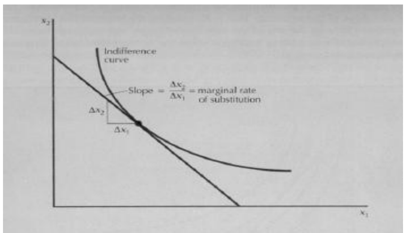
- perfect complements: goods that are always consumed together in fixed proportions (utility function $u(x_1, x_2) = \min\{x_1, x_2\}$)
- neutrals: a good that the consumer doesn't care about
- bads, satiation point

Well-behaved indifference curves: we assume that more is better (monotonicity of preferences, satiation is not possible, negative slope), indifference curves are convex. Concavity (ice-cream and olives) makes sense but if we look at monthly consumption, convexity is OK. (Cobb-Douglas utility function $u(x_1, x_2) = x_1^a x_2^b$)

Indifference curves representing distinct levels of preference cannot cross. If they cross, transitivity and "more is better" is violated.

Marginal rate of substitution (MRS): the slope of indifference curve at a particular point. It is the rate at which the consumer is willing to substitute one good for the other.

Suppose that we take a little of good 1, Δx_1 away from the consumer. Then we give him Δx_2 the amount that is sufficient to put him back on his indifference curve, so that he is just as well off after the substitution of x_2 for x_1 as he was before. We thing of the ratio of $\Delta x_2/\Delta x_1$ as being the rate at which the consumer is willing to substitute good 2 for good 1. If Δx_1 is very small change, marginal change, than $\Delta x_2/\Delta x_1$ measures the marginal rate of substitution of good 2 for good 1. As Δx_1 gets smaller, $\Delta x_2/\Delta x_1$ approaches the slope of the indifference curve.



The marginal rate of substitution (MRS). The marginal rate of substitution measures the slope of the indifference curve.

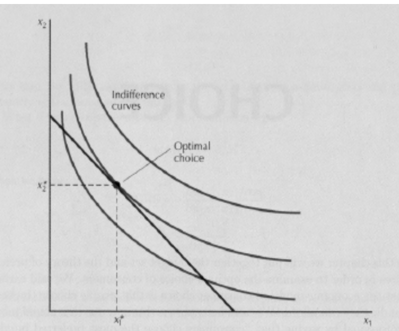
MRS is a negative number for well-behaved preferences, -1 for perfect substitutes, infinite for neutrals, zero or infinity for perfect complements. Moreover, for strictly convex indifference curves, the MRS - the slope - decreases (in absolute value) as we increase x_1. Thus the indifference curves exhibit a diminishing marginal rate of substitution. The more you have of one good, the more willing you are to give some of it up in exchange for the other good.

Marginal utility ($\mathbf{MU_1}$): measures change in utility associated with a small change in the amount of good 1. $MU_1 = \Delta U/\Delta x_1$. Similarly, $MU_2 = \Delta U/\Delta x_2$

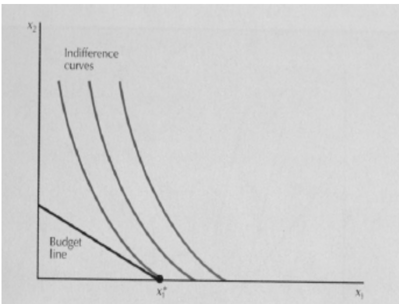
Note that: $MRS = \frac{\Delta x_2}{\Delta x_1} = \frac{MU_1}{MU_2}$

1.3 Consumer's optimum

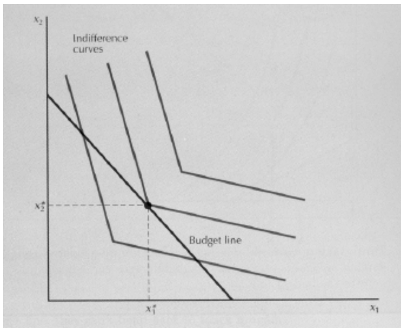
People choose the best things they can afford - People choose the most preferred bundle from their budget set.



Optimal choice. The optimal consumption position is where the indifference curve is tangent to the budget line.



Boundary optimum. The optimal consumption involves consuming zero units of good 2. The indifference curve is not tangent to the budget line.



Kinky tastes. Here is an optimal consumption bundle where the indifference curve doesn't have a tangent.

The optimal choice (x_1^*, x_2^*) is the bundle in which indifference curve is tangent to the budget line (interior optimum). This means that the slope of the indifference curve is equal to the slope of the budget line, i.e. $MRS = \frac{p_X}{p_Y}$. The optimality condition can also be given by:

$$\frac{MU_X}{p_X} = \frac{MU_Y}{p_Y} \quad \text{or} \quad \frac{MU_X}{p_X} = \frac{MU_Y}{p_Y}$$

(*Note:* if $\frac{MU_X}{p_X} > \frac{MU_Y}{p_Y}$, consumer could be better off by decreasing consumption of Y and increasing consumption of X.)

2 Theory of Demand, Slutsky Equation

2.1 Theory of Demand

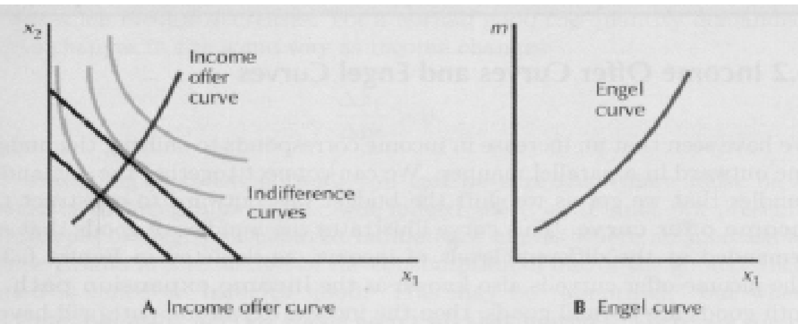
Based on the analysis of consumer's optimal consumption we know that the demand depends on individual preferences, prices, and income. Now we will analyze how the demand changes as prices and income change.

Change in income: (keeping prices constant)

- Normal good: if income increase consumption increase as well
- Inferior good: increase in income causes reduction in consumption (low-quality good)

Set of bundles of goods that are demanded at the different levels of income is called **income offer curve (income expansion path).**

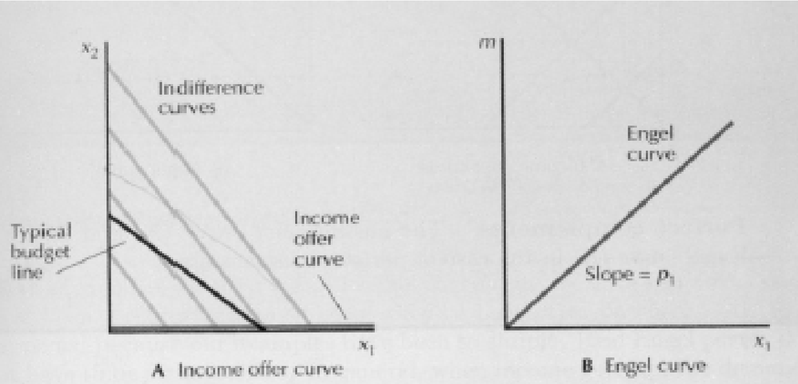
For each level of income, there is some optimal choice for each of the goods. The graph of the demand for one of the goods as a function of income with all prices being held constant is called the **Engel curve**.



How demand changes as income changes. The income offer curve (or income expansion path) shown in panel A depicts the optimal choice at different levels of income and constant prices. When we plot the optimal choice of good 1 against income, m, we get the Engel curve, depicted in panel B.

Two goods are perfect substitutes:

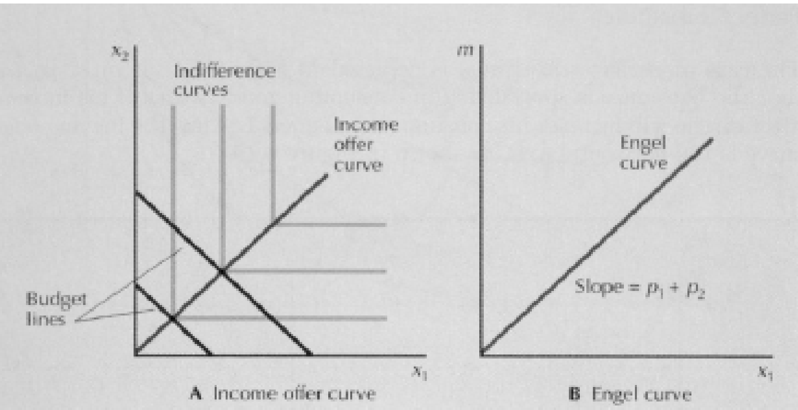
If $p_1 < p_2$, consumer spends entire budget on good 1 and therefore buys $x_1 = I/p_1$ units. Income offer curve is a horizontal axis. If income increases, consumption of good 1 increases as well. Engel curve is a linear line, because $I = p_1 x_1$.



Perfect substitutes. The income offer curve (A) and an Engel curve (B) in the case of perfect substitutes.

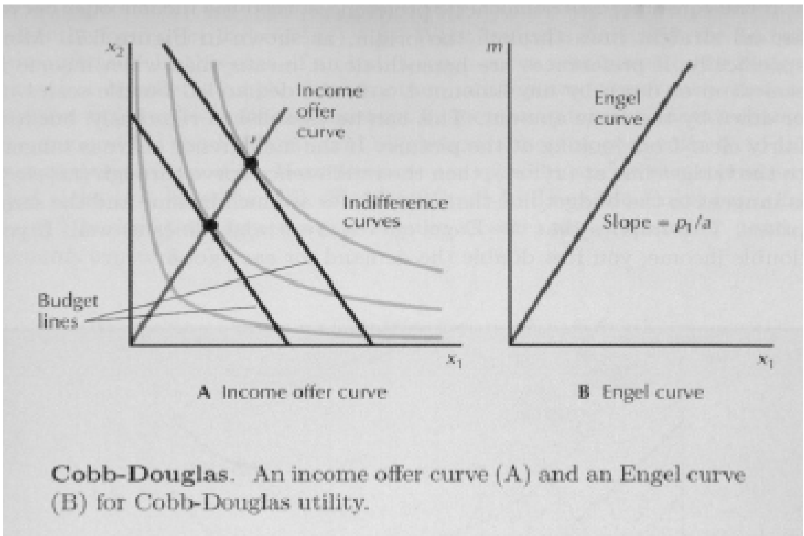
Two goods are perfect complements:

Consumer consumes the same amount of each good, the income curve is the diagonal line through the origin. The demand for good 1 is $x_1 = I/(p_1 + p_2)$ and Engel curve is a straight line with slope $(p_1 + p_2)$.



Perfect complements. The income offer curve (A) and an Engel curve (B) in the case of perfect complements.

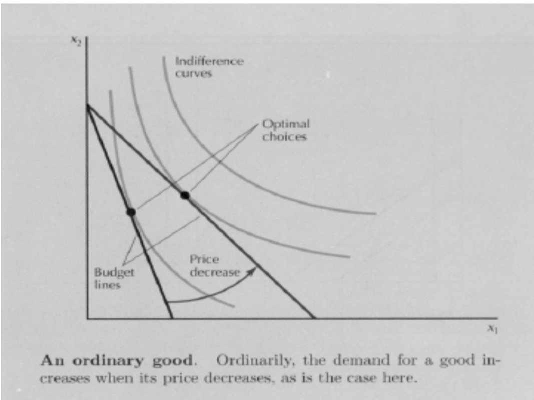
Cobb-Douglas utility function: (not covered on the lecture but useful example) $u(x_1, x_2) = x_1^a x_2^{(1-a)}$. The demand for good 1 is $x_1 = am/p_1$ and the demand for good 2 is $x_2 = (1-a)m/p_2$. For given prices the demand for both goods is a linear function of income. Doubling income causes double the consumption of both goods. Hence the income offer curve and Engel curve is a straight line.



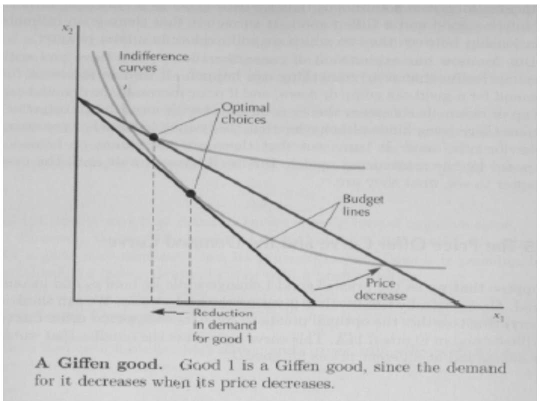
Cobb-Douglas. An income offer curve (A) and an Engel curve (B) for Cobb-Douglas utility.

Change in one price: (keeping the other price and income constant)

- Ordinary good: decrease in price causes increase in consumption of a given good
- Giffen god: decrease in price results in reduction in its demand



An ordinary good. Ordinarily, the demand for a good increases when its price decreases, as is the case here.

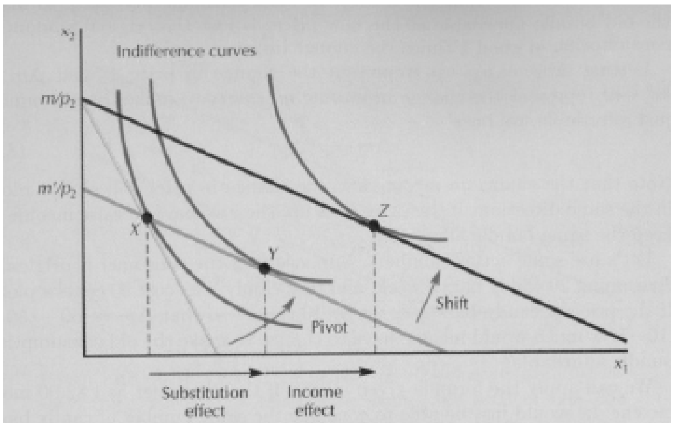


A Giffen good. Good 1 is a Giffen good, since the demand for it decreases when its price decreases.

2.2 Slutsky Equation

When the price of a good changes, there are two sorts of effects: the rate at which you can exchange on good for another changes, an the total purchasing power of your income is altered. The change in demand due to the change in the rate of exchange between the two goods is called **substitution effect**. The change in demand due to having more purchasing power is called the **income effect**.

Note: There are two approaches to decompose total effect into substitution and income effect. Below we use Slutsky's approach (pivot budget line goes through the original consumption bundle). Alternative approach is Hick's approach (pivot budget line is such that the consumer has the same level of utility as in the original point of consumption). These approaches are equivalent.



Substitution effect and income effect. The pivot gives the substitution effect, and the shift gives the income effect.

$x_1(p_1', m) - x_1(p_1, m) = [x_1(p_1', m') - x_1(p_1, m)] + [x_1(p_1', m) - x_1(p_1', m')]$
Change in demand = Substitution effect + Income effect

Substitution effect:

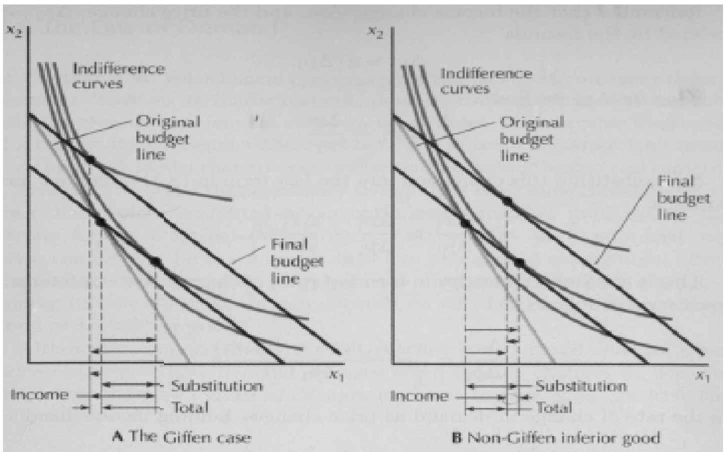
If price of good 1 decreases, new optimal choice must involve consuming at least as much of good 1 as originally. (The original bundle is preferred to all the bundles on pivot budget line to the left from original point of consumption - these were affordable but consumer did not choose them).

Income effect:

If the good is a normal good, then the decrease in income will lead to a decrease in demand. If the good is an inferior good, then the decrease in income will lead to an increase in demand. Income effect can be positive or negative.

$\Delta x_1 = \Delta x_1^s + \Delta_1^n$
$(-) = (-) + (-)$
$(?) = (-) + (+)$ If the income effect is large, ↗ in price ⇒ ↗ in consumption - Giffen good

This can only happen in case of very inferior goods, in case of only a little inferior good and normal good total effect is always negative.



A The Giffen case **B** Non-Giffen inferior good

Inferior goods. Panel A shows a good that is inferior enough to cause the Giffen case. Panel B shows a good that is inferior, but the effect is not strong enough to create a Giffen good.

2.3 Elasticity

Demand responds to changes in prices and/or income. Elasticity measures how responsive the demand is.

The price elasticity of demand, ϵ, is defined to be the percent change in quantity divided by the percent change in price:

$$\epsilon = \frac{\Delta q/q}{\Delta p/p}$$

- elastic demand: elasticity is greater than 1 in absolute value (if there are a lot of substitutes)
- inelastic demand: elasticity is less than 1 in absolute value (if there are very few substitutes)
- unit elastic demand: elasticity is equal to 1 in absolute value

Elasticity and revenue: If demand is very responsive to price, very elastic, then an increase in price will reduce demand so much that revenue will fall. If demand is very unresponsive to price, very inelastic, then an increase in price will not change demand very much and overall revenue will increase. If elasticity is equal to -1 and the price increases by 1 percent, the quantity will decrease by 1 percent, so overall revenue does not change.

The income elasticity of demand, ϵ_I, is defined to be the percent change in quantity divided by the percent change in income:

$$\epsilon_I = \frac{\Delta q/q}{\Delta I/I}$$

Income elasticity of demand is positive for normal good and negative for an inferior good.

The cross price elasticity, ϵ_c:

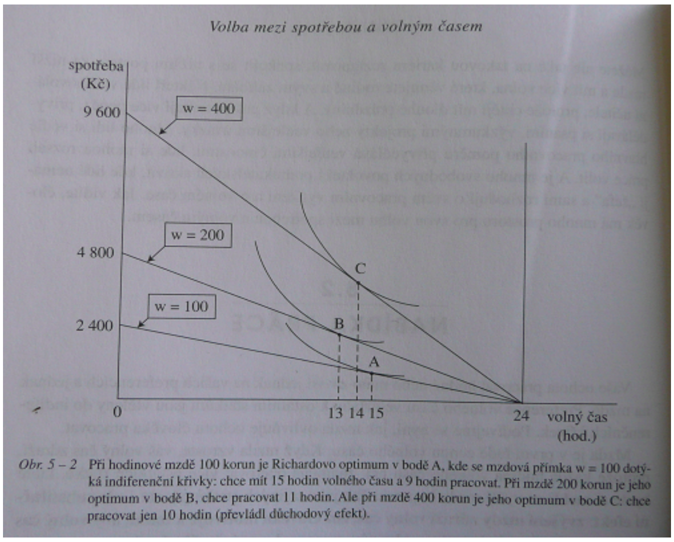
$$\epsilon_c = \frac{\Delta q_x/q_x}{\Delta p_y/p_y}$$

is positive if substitution effect is greater than income effect, negative otherwise.

3 Intertemporal Choice, Risk, Uncertainty

3.1 Choice between leisure and consumption

We discussed the choice between two goods. Similarly, we can illustrate the choice between leisure and consumption. Leisure can be considered as a normal good. Loosely speaking the price of leisure is forgone earnings, i.e. money that consumer could earn if he spent time working instead of enjoying leisure time. The optimal choice of consumption and leisure is a combination where the indifference curve touches the budget line. This choice depends on the wage. If wage increases we can observe substitution effect (leisure becomes more expensive relative to consumption and should be substituted by working time and hence higher consumption) and income effect (we assume that both "consumption" and "leisure" are normal goods ⇒ higher income leads to higher consumption of both goods and hence more leisure). The new optimal choice depends on which of the two effects is stronger:



3.2 Intertemporal choice (choice between current and future consumption)

So far we only studied static choices, but life is full of intertemporal choices (should I study for my test today or tomorrow; should I save or should I consume now, school, cigarettes, alcohol). When modeling intertemporal choice, economists treat one physical good consumed at two different times as two different goods.

We consider in this lecture the optimal allocation decision through time. In particular, we examine the optimal allocation of income to consumption through time. This is important as people often receive the income through time in a way that does not correspond to their preferred consumption stream through time. The individual needs to rearrange his or her income stream. This is achieved by borrowing and saving - through the use of capital markets. We assume here perfect capital markets, by which is meant that the individual can borrow and save as much as he or she wants at a constant and given rate of interest, which we shall denote by r . If the rate of interest is 10% then r = .1; if the rate of interest is 20% then r = .2; and so on. To keep our analysis simple we assume a two period world. Each individual gets current income I_a and expected future income I_b and we denote current consumption C_a and future consumption C_b. (For simplicity we assume a single consumption good and assume a price of 1 in both periods.) It may be the case that the individual is happy to currently consume his or her income I_a and to consume his or her income I_b in future. However, the individual may prefer to rearrange his or her consumption by borrowing or lending. If r is zero the possibilities are obvious: the maximum he or she could consume now is $I_a + I_b$ with zero consumption in future and the maximum he or she could consume in future is the same. More generally the choice of C_a and C_b must satisfy the budget constraint:

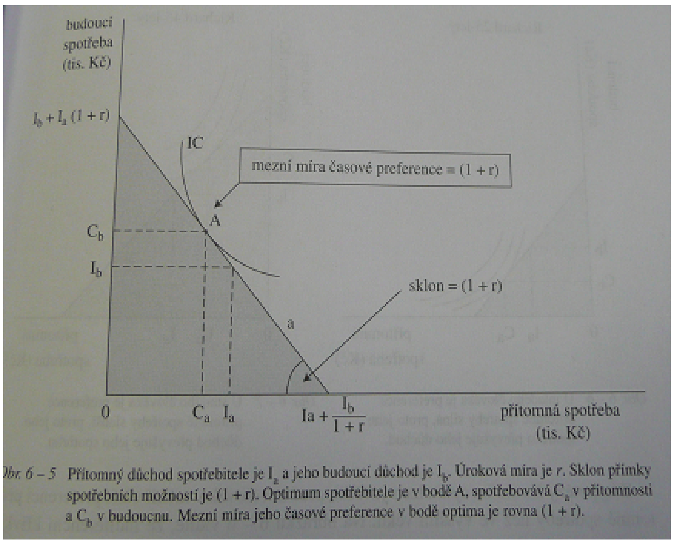
$$C_a + C_b = I_a + I_b$$

If there is a positive rate of interest things are a little more complicated. If the individual wanted to consume nothing now then he or she could save the income I_a, investing it at the rate of interest r, earning interest rI_a and thus in future having

$$I_a(1 + r) + I_b$$

Alternatively if he or she wanted to consume nothing in future, then now he or she could spend his current income plus what he or she could borrow on the strength of being able to pay back in future.

$$I_a + I_b/(1 + r)$$



Generally, the future value of the consumption stream must equal the future value of the income stream:

$$C_a(1 + r) + C_b = I_a(1 + r) + I_b$$

Alternatively, the present value of the consumption stream must equal the present value of the income stream:

$$C_a + C_b/(1 + r) = I_a + I_b/(1 + r)$$

Change of interest rate r: changes the slope of budget constraint. Substitution effect (current consumption is more expensive) $\Rightarrow$ higher savings. Income effect (increase in future income) $\Rightarrow$ higher current consumption. The total effect depends on the shape of indifference curves, i.e. on whether substitution effect is stronger than income effect or vice versa.

3.3 Risk, Uncertainty

So far we assumed that people's choices are not uncertain - once they decide how to spend their income, they get what they want. However, very often this is not true and people makes choices that involve incomplete information (unknown quality of used car, future value of investment,...) These events have not certain outcome but *expected outcome (expected value.)*

Example: Suppose that a consumer currently has \$10 of wealth and is contemplating a gamble that gives him a 50 percent probability of winning \$5 and a 50 percent probability of losing \$5. His wealth will therefore be random: he has a 50 percent probability of ending up with \$5 and a 50 percent probability of ending up with \$15. The *expected value* of his wealth is:

$$\frac{1}{2} * \$5 + \frac{1}{2} * \$15 = \$10$$

Generally,

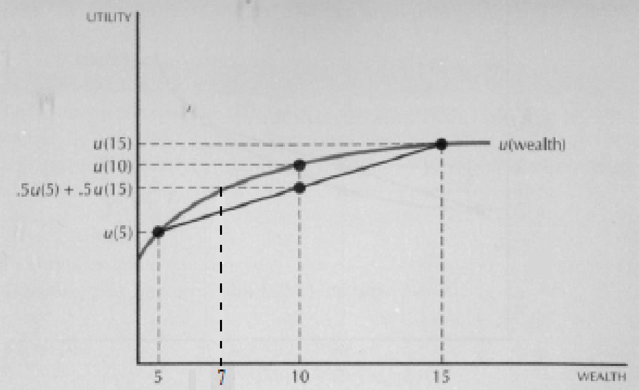
$$R_E = \sum \pi_i R_i,$$

where R_i are possible outcomes and π_i are their probabilities.

When there are several alternatives we could think that the individual should choose the alternative with the highest expected value. Is this always the case? Imagine a lottery where with 50% probability you win \$90 and with 50% probability you win \$110. The expected value is \$100. Now imagine a lottery where with 99% probability you win \$1 and with 1% probability you win \$10001. The expected value is \$101. So you should choose the second lottery. However, most of people would prefer the first lottery to the second one. The reason is that people do not compare expected values, they compare expected utilities and in general people are risk averse and the second lottery is much riskier than the first one.

In the example above the *expected utility* is:

$$\frac{1}{2}u(\$5) + \frac{1}{2}u(\$15)$$



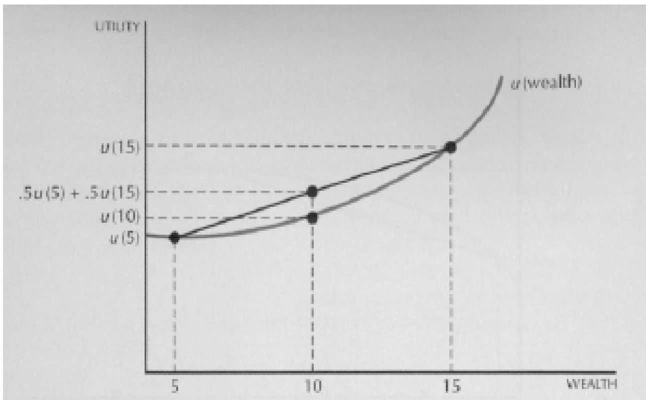
Risk aversion. For a risk-averse consumer the utility of the expected value of wealth, $u(10)$, is greater than the expected utility of wealth, $.5u(5) + .5u(15)$.

Note that in this diagram the expected utility of wealth is less than the utility of the expected wealth:

$$u\left(\frac{1}{2} * \$5 + \frac{1}{2} * \$15\right) = u(\$10) > \frac{1}{2}u(\$5) + \frac{1}{2}u(\$15)$$

In this case, consumer is **risk averse** since expected value is preferred to gamble.

It could happen that the consumer prefers a a random distribution of wealth to its expected value, in which case we say that the consumer is a **risk lover**.



Risk loving. For a risk-loving consumer the expected utility of wealth, $.5u(5) + .5u(15)$, is greater than the utility of the expected value of wealth, $u(10)$.

The intermediate case is that of a linear utility function. Here the consumer is **risk neutral**: the expected utility of wealth is the utility of its expected value. In this case the consumer doesn't care about the riskiness of his wealth at all-only about its expected value.

3.4 Insurance against risk

Risk premium: maximum amount of money that an individual is willing to pay in order to avoid the gamble. Risk-averse individual in example above is willing to pay up to \$3 to avoid the gamble, therefore the risk premium is \$3. If someone offers this individual insurance against the uncertainty; i.e. insurance company offers to keep individual's wealth constant and the price of this insurance is at most \$3 the individual buys this insurance.

The expected utility is the same whether the individual gets the insurance or not. Then why is the individual willing to buy it in the first place? Because when there is no insurance, marginal utility in the event of loss is higher than if no loss occurs (because of diminishing marginal utility of risk-averse individual). Hence, the transfer from no-loss to the loss situation must increase total utility.

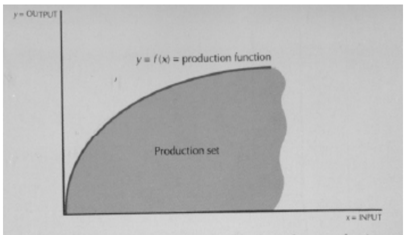
3.5 Diversification

Suppose, for example, that shares of the raincoat company and the sunglasses company currently sell for \$10 a piece. If it is a rainy summer, the raincoat company will be worth \$20 and the sunglasses company will be worth \$5. If it is a sunny summer, the payoffs are reversed: the sunglasses company will be worth \$20 and the raincoat company will be worth \$5. If you invest your entire \$100 in the sunglasses company, you are taking a gamble that has a 50 percent chance of giving you \$200 and a 50 percent chance of giving you \$50. The same magnitude of payoffs results if you invest all your money in the sunglasses company: ill either case you have an expected payoff of \$125. But look what happens if you put half of your money in each. Then, if it is sunny you get \$100 from the sunglasses investment and \$25 hom the raincoat investment. But if it is rainy, you get \$100 from the raincoat investment and \$25 hom the sunglasses investment. Either way, you end up with \$125 for sure. By diversifying your investment in the two companies, you have managed to reduce the overall risk of your investment, while keeping the expected payoff the same.

Diversification was quite easy in this example: the two assets were perfectly negatively correlated- when one went up, the other went down. Pairs of assets like this can be extremely valuable because they call reduce risk so dramatically. But, alas, they are also very hard to find. Most asset values move together: when GM stock is high, so is Ford stock, and so is Goodrich stock. But as long as asset price movements are not perfectly positively correlated, there will be some gains from diversification.

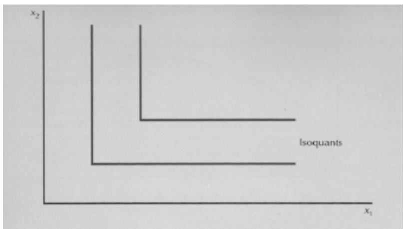
4 Optimum of the firm, Cost minimization

Firm's production is constrained by technology. Technology can by described by production set, or production function. In case of only one input (only one factor of production) we can illustrate production set and production function in the following way.

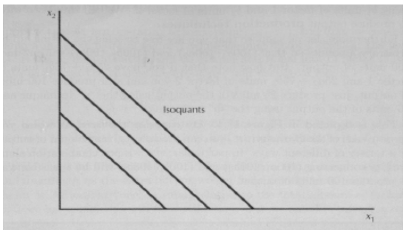


A production set. Here is a possible shape for a production set.

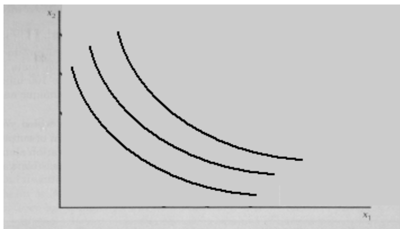
Usually, we assume that a firm uses two inputs. In this case we use **isoquants** to depict production relations. An isoquant is the set of all possible combinations of inputs 1 and 2 that are just sufficient to produce a given amount of output.



Fixed proportions. Isoquants for the case of fixed proportions.



Perfect substitutes. Isoquants for the case of perfect substitutes.


Perfect substitutes. Isoquants for the case of perfect substitutes.

$f\{x_1, x_2\} = \min\{x_1, x_2\}$ $f\{x_1, x_2\} = x_1 + x_2$ $f\{x_1, x_2\} = Ax_1^a x_2^b$

Properties of technology: we assume that the production function is:

- monotonic: if we increase the amount of at least one of the inputs, we will get at least as much output as originally (free disposal)
- convex: if we can produce output y in two different ways - using (x_1, x_2) or (z_1, z_2) units of inputs, then their weighted average will produce at least y units of output

Let's assume that we are operating at some point (x_1, x_2) and that we consider to use a little bit more of factor 1 while keeping factor 2 fixed at level x_2. The additional output that we get is called **marginal product of factor 1**. We typically expect that the marginal product of a factor will diminish as we get more and more of that factor. This is called the law of diminishing marginal product.

Now again assume that we are operating at some point (x_1, x_2), and that we consider giving up a little bit of factor 1 and using just enough more of factor 2 to produce the same amount of output y. How much extra of factor 2, Δx_2, do we need if we are going to give up a little bit of factor 1, Δx_1? This is just the slope of the isoquant and we refer to if as the **technical rate of substitution**

$$TRS_{(x_1,x_2)} = \frac{\Delta x_1}{\Delta x_2} = -\frac{MP_1(x_1,x_2)}{MP_2(x_1,x_2)}$$

Returns to scale: What happens if instead of increasing the amount of 1 input we increase the amount of all the inputs in the same proportions? If, for example, we use twice as much of each input, how much output will we get? The most likely outcome is that we will get twice as much output (a firm should be able to replicate what it is doing right now). This is called **constant returns to scale**. Mathematically:

$$f(2x_1, 2x_2) = 2f(x_1, x_2)$$

Or generally,

$$f(tx_1, tx_2) = tf(x_1, x_2)$$

Note: it is possible to have constant returns to scale and diminishing marginal product at the same time. For example if $f(x_1,x_2) = \sqrt{x_1x_2}$. Then $f(2x_1,2x_2) = \sqrt{2x_1 2x_2} = 2\sqrt{x_1x_2}$. So this function has constant returns to scale. At the same time we observe diminishing marginal product:

$$MP_1 = \frac{\partial f(x_1,x_2)}{\partial x_1} = \frac{\partial \sqrt{(x_1x_2)}}{\partial x_1} = \frac{x_2}{2\sqrt{(x_1x_2)}}$$

This is decreasing function in x_1; i.e. if we keep increasing x_1 output will increase as well but slower and slower.

Similarly, we can have **increasing returns to scale**:

$$f(tx_1, tx_2) > tf(x_1, x_2)$$

(Ex: If we double the diameter of an oil pipe, we use twice as much material but are able to transport 4 times as much oil.)

and **decreasing returns to scale**:

$$f(tx_1, tx_2) < tf(x_1, x_2)$$

(Ex: Usually a short run case, where we can not double all the inputs. Otherwise something goes wrong, because it was possible just to replicate what a firm was doing and get constant returns to scale.)

4.1 Cost minimization

Suppose that we have two factors of production, x_1 and x_2, and their prices w_1 and w_2, and that we want to figure out the cheapest way to produce a given level of output, y. For a given amounts of two factors the cost of production is

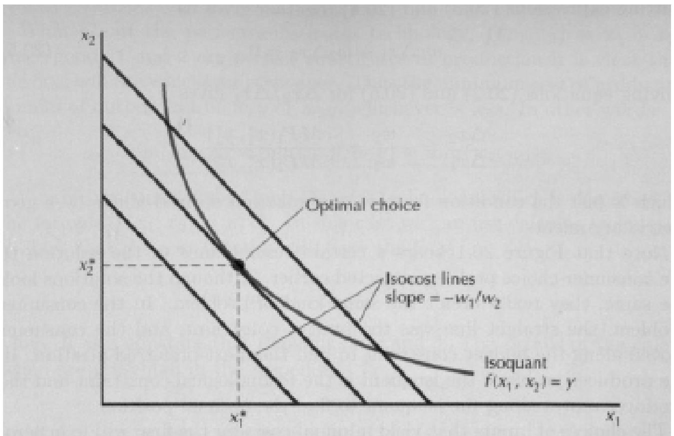
$$w_1 x_1 + w_2 x_2$$

Suppose that we want to plot all the combinations of inputs x_1 and x_2 that have some given level of cost, C. We can write this as

$$w_1 x_1 + w_2 x_2 = C \quad \Leftrightarrow \quad x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1$$

This is a straight line with a slope of $-\frac{w_1}{w_2}$ and a vertical intercept of $\frac{C}{w_2}$. As we let C change we get many different **isocost lines** - every point has the same cost and higher isocost lines are associated with higher costs.

So the cost-minimization problem of the firm is to find such point on given isoquant that lies on the lowest isocost line.



Cost minimization. The choice of factors that minimize production costs can be determined by finding the point on the isoquant that has the lowest associated isocost curve.

Note that if isoquant is a smooth curve then the cost-minimizing point will be characterized by a tangency condition: the slope of the isoquant must be equal to the slope of the isocost curve. Or the technical rate of substitution must equal the factor price ratio:

$$-\frac{MP_1}{MP_2} = TRS = -\frac{w_1}{w_2}$$

Mathematically, we denote this solution $c(w_1, w_2, y)$, where this function is called **cost function** and it measures the minimum cost of producing a given level of of output y at given factor prices w_1 and w_2.

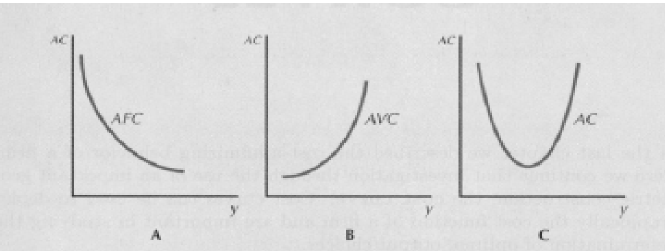
4.2 Cost curves

Types of costs:

- Fixed costs - independent of the level of output
- Variable costs - expenses that change in proportion to the amount of output produced
- Total costs = fixed costs + variable costs

Average costs: costs per unit of output. The average variable cost function measures the variable cost per unit of output, average fixed cost function measures the fixed costs per unit of output.

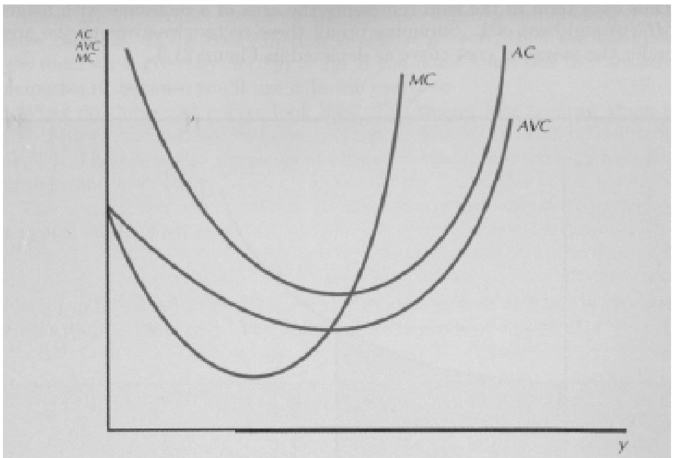
$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} = AVC(y) + AFC(y)$$



Construction of the average cost curve. (A) The average fixed costs decrease as output is increased. (B) The average variable costs eventually increase as output is increased. (C) The combination of these two effects produces a U-shaped average cost curve.

Marginal costs:

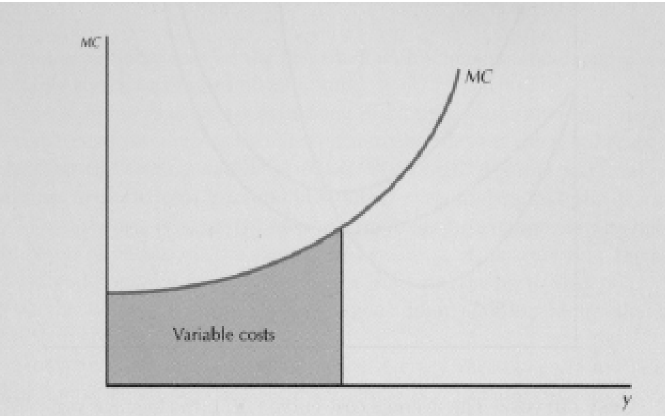
There is one more cost curve of interest: the marginal cost curve. The marginal cost curve measures the change in costs for a given change in output. That is, at any given level of output y, we can ask how costs will change if we change output by some amount Δy. Often we think of Δy as being one unit of output, so that marginal cost indicates the change in our costs if we consider producing one more discrete unit of output.



Cost curves. The average cost curve (AC), the average variable cost curve (AVC), and the marginal cost curve (MC).

Note that MC curve intercept AC and AVC curve in their minimum. If AC curve decreases it must be the case that marginal cost is lower than average cost. If AC increases that marginal cost must by higher than average cost. Thus we know that the marginal cost curve must lie below the average cost curve to the left of its minimum point and above it to the right. This implies that the marginal cost curve must intersect the average variable cost curve at its minimum point.

Also note that the area below the marginal cost curve up to point of output y gives variable cost of production. Marginal cost curve measures the cost of each additional unit of output. If we add up the cost of producing each unit of output we get total variable cost.



Marginal cost and variable costs. The area under the marginal cost curve gives the variable costs.

5 Profit maximization, Supply

We already described the technological possibilities now we analyze how the firm chooses the amount to produce so as to maximize its profits. Profits are defined as revenues minus cost. We assume that the firm faces fixed prices (is on a competitive market).

Example: A firm is on a competitive market, i.e. takes price of the output as given. Production function is given by $f(x_1, x_2) = x_1^{1/4}x_2^{1/4}$, prices of inputs are $w_1 = 4$, $w_2 = 4$ and price of output is p = 1.

- **Profit-maximization approach:** We maximize profit (revenues minus costs) of the firm.

$$\max_{\{x_1,x_2\}} py - w_1x_1 - w_2x_2 \quad \rightarrow \quad \max_{\{x_1,x_2\}} 1x_1^{1/4}x_2^{1/4} - 4x_1 - 4x_2$$
$$FOC[x_1] : \frac{x_2}{4(x_1x_2)^{3/4}} - 4 = 0$$
$$FOC[x_2] : \frac{x_1}{4(x_1x_2)^{3/4}} - 4 = 0$$

  Solving these two equations with two unknowns gives:

$$x_1 = x_2 = \frac{1}{256}$$

- **Cost-minimization approach:** Consists of two stages: First, we find minimum cost for producing any given level of output y. Second, we find optimal value of output y.

  First stage: find minimum cost for arbitrary level of output y:

$$\min_{\{x_1,x_2\}} w_1x_1 + w_2x_2 \quad \rightarrow \quad \min_{\{x_1,x_2\}} 4x_1 + 4x_2$$
$$\text{such that } \; x_1^{1/4}x_2^{1/4} = y \quad \Rightarrow \quad x_2 = \frac{y^4}{x_1}$$
$$\min_{x_1} 4x_1 + 4\frac{y^4}{x_1}$$
$$\text{FOC: } \; 4 - 4\frac{y^4}{x_1^2} = 0 \quad \Rightarrow \quad x_1 = y^2 \;\text{ and }\; x_2 = y^2$$

  So in this example, our cost function is:

$$c(y) = 4x_1 + 4x_2 = 4y^2 + 4y^2 = 8y^2$$

  Second stage: find optimal level of output y:

$$\max_y py - c(y) \quad \rightarrow \quad \max_y y - 8y^2$$
$$\text{FOC: } \; 1 - 16y = 0 \quad \Rightarrow \quad y = \frac{1}{16}$$
$$x_1 = x_2 = y^2 = \frac{1}{256}$$

Profit maximization $\leftrightarrow$ Cost minimization. If a firm is maximizing profits and if it chooses to supply some output y, then it must be minimizing the cost of producing y. If this were not so, then there would be some cheaper way of producing y units of output, which would mean that the firm was not maximizing profits in the first place. This simple observation turns out to be quite useful in examining firm behavior.

5.1 Profit maximization in short-run:

In short-run the amount of at least one inputs is fixed. In long-run all inputs can be changed.

$$\max_{x_1} pf(x_1, \overline{x_2}) - w_1x_1 - w_2\overline{x_2}$$

where:

- p - price of output
- $f(x_1, \overline{x_2})$ - production function
- $x_1, \overline{x_2}$ - inputs, x_2 is in short-run fixed at the level $\overline{x_2}$
- w_1, w_2 - prices of inputs x_1, x_2

For profit maximizing quantity the first order condition has to hold:

$$p\frac{\partial f(x_1, \overline{x_2})}{\partial x_1} = w_1 \quad \text{or} \quad pMP_1 = w_1$$

In other words, **the value of the marginal product of a factor should equal its price.**

In order to understand this rule, think about the decision to employ a little more of factor 1. As you add a little more of it, Δx_1, you produce $\Delta y = MP_1\Delta x_1$ more output that is worth $pMP_1\Delta x_1$. But this marginal output costs $w_1\Delta x_1$ to produce. If the value of marginal product exceeds its cost, then profits can be increased by increasing input 1. If the value of marginal product is less than its cost, then profits can be increased by decreasing the level of input 1.

Now we analyze profit maximization problem graphically. Profit of the firm is given by:

$$\pi = py - w_1x_1 - w_2\overline{x_2}$$

Rearranging terms we get:

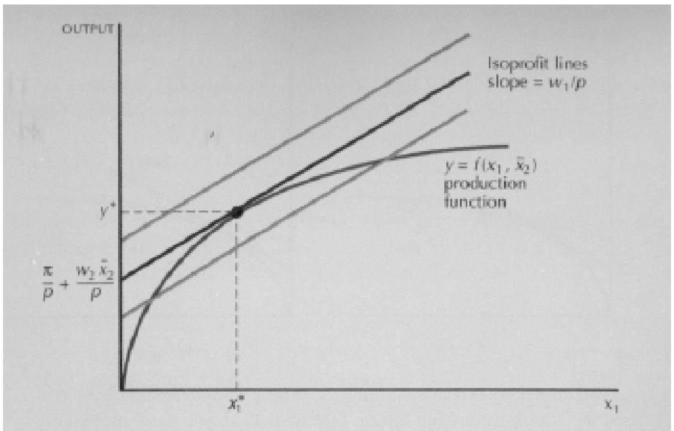
$$y = \frac{\pi}{p} + \frac{w_2}{p}\overline{x_2} + \frac{w_1}{p}x_1$$

The last equation describes **isoprofit lines** - all combinations of inputs and outputs that give a constant level of profit, π.

As π varies we get a set of parallel straight lines each with a slope of w_1/p and each having a vertical intercept of $\pi/p + w_2\overline{x_2}/p$, which measures the profits plus the fixed costs of the firm.

The profit-maximization problem is then to find the point on the production function that has the highest associated isoprofit line. Such a point is illustrated on the following picture. It's characterized by a tangency point - the slope of production function (MP_1) equals the slope of the isoprofit line (w_1/p). Hence,

$$MP_1 = \frac{w_1}{p}$$



Profit maximization. The firm chooses the input and output combination that lies on the highest isoprofit line. In this case the profit-maximizing point is (x_1^*, y^*).

5.2 Profit maximization in long-run:

the level of all inputs can be chosen.

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

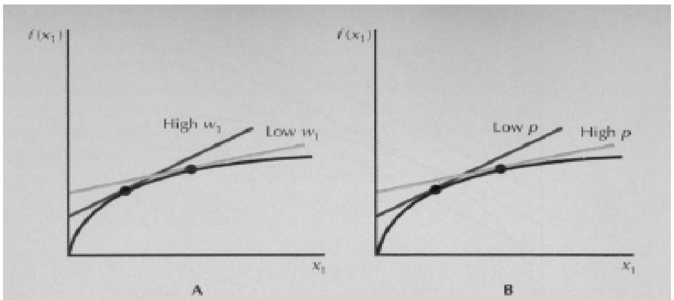
In this case, optimality conditions are:

$$pMP_1(x_1^*, x_2^*) = w_1$$
$$pMP_2(x_1^*, x_2^*) = w_2$$

The last two equations give solution to the profit-optimization problem - expressions for x_1 and x_2 - **factor demand curves**. These curves measure the relationship between the price of a factor and the profit-maximizing choice of the factor. **The inverse factor demand curve** measures the same relationship but from a different point of view. It measures what the factor prices must be for some given quantity of inputs to be demanded.

Comparative statics: we analyze how a firm's choice of inputs and outputs varies as the prices of inputs and outputs vary. How does the optimal choice of factor 1 vary as we vary its factor price w_1? Increasing w_1 will make the isoprofit line steeper. When the isoprofit line is steeper, the tangency must occur further to the left. Thus the optimal level of factor 1 must decrease. This simply means that as the price of factor 1 increases, the demand for factor 1 must decrease: factor demand curves must slope downward.

Similarly, if the output price decreases the isoprofit line must become steeper and the profit-maximizing choice of factor 1 will decrease.



Comparative statics. Panel A shows that increasing w_1 will reduce the demand for factor 1. Panel B shows that increasing the price of output will increase the demand for factor 1 and therefore increase the supply of output.

Finally, we can ask what will happen if the price of factor 2 changes? Because this is a short-run analysis, changing the price of factor 2 will not change the firm's choice of factor 2-in the short run, the level of factor 2 is fixed at $\overline{x_2}$. Changing the price of factor 2 has no effect on the slope of the isoprofit line. Thus the optimal choice of factor 1 will not change, nor will the supply of output.

Profit maximization and returns to scale: There is an important relationship between competitive profit maximization and returns to scale. Suppose that the firm's production function exhibits constant returns to scale and that it is making positive profits in equilibrium. Then consider what would happen if it doubled the level of its input usage. According to the constant returns to scale hypothesis, it would double its output level and its profits would also double. But this contradicts the assumption that its original choice was profit maximizing. This argument shows that the only reasonable long-run level of profits for a competitive firm that has constant returns to scale at all levels of output is a zero level of profits. (Of course if a firm has negative profits in the long run, it should go out of business.)

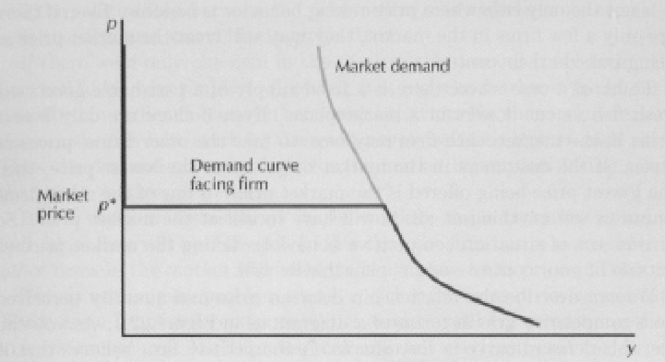
Firms exist to maximize profits so how can it be that they can only get zero profits in the long run? Think about what would happen to a firm that did try to expand indefinitely. Three things might occur.

- First, the firm could get so large that it could not really operate effectively. This is just saying that the firm really doesn't have constant returns to scale at all levels of output. Eventually, due to coordination problems, it might enter a region of decreasing returns to scale.
- Second, the firm might get so large that it would totally dominate the market for its product. In this case there is no reason for it to behave competitively-to take the price of output as given. Instead, it would make sense for such a firm to try to use its size to influence the market price. The model of competitive profit maximization would no longer be a sensible way for the firm to behave, since it would effectively have no competitors. We'll investigate more appropriate models of firm behavior in this situation when we discuss monopoly.
- Third, if one firm can make positive profits with a constant returns to scale technology, so can any other firm with access to the same technology. If one firm wants to expand its output, so would other firms. But if all firms expand their outputs, this will certainly push down

5.3 Firm supply

If the firm could choose quantity of production and the price freely it would choose arbitrarily large price and quantity. However, firm faces technological (production function) and market constraints (firm can set whatever price it wants, but it can only sell as much as people are willing to buy). We call the relationship between the price a firm sets and the amount that it sells the **demand curve** facing the firm. In this lecture we analyze a simple market environment - **pure competition**. In pure competition the price of a good is independent of firm's behavior. Prices are given and firms are **price takers** and only choose the level of quantity. This is plausible assumption if we imagine market with a very large number of small firms (wheat farmers in the USA, hot dog sellers on Vaclavske namesti, ...).

A competitive firm faces the following demand. If the price charged is higher than the market price the firm sells nothing. It the firm sells for the market price it can sell whatever amount it wants and if the price is lower than the market price it will get the entire market.



The demand curve facing a competitive firm. The firm's demand is horizontal at the market price. At higher prices, the firm sells nothing, and below the market price it faces the entire market demand curve.

Supply decision of a competitive firm:

Let us use the facts we have discovered about cost curves to figure out the supply curve of a competitive firm. Thus the maximization problem facing a competitive firm is

$$\max_y \{\text{revenues - costs}\} = \max_y py - c(y)$$

A profit maximizing firm chooses a level of output such that marginal revenue (extra revenue gained by one more unit of output) equals marginal (extra) cost. If this condition did not hold, the firm could always increase its profits by changing its level of output.

In the case of a competitive firm, marginal revenue is simply the price. To see this, ask how much extra revenue a competitive firm gets when it increases its output by Δy. We have

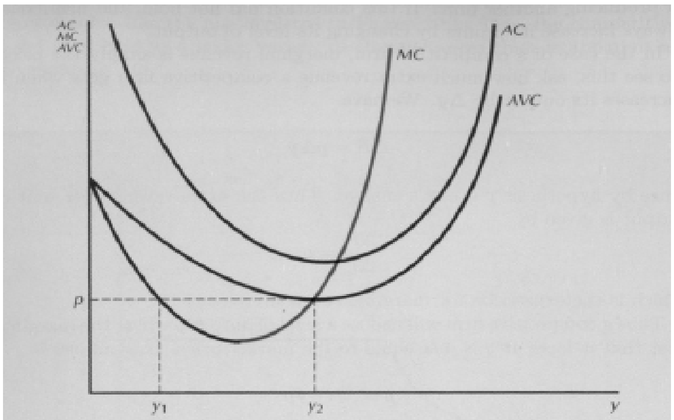
$$\Delta R = p\Delta y$$
$$\frac{\Delta R}{\Delta y} = p$$

Thus a competitive firm will choose a level of output y where the marginal cost that it faces at y is just equal to the market price:

$$p = MC(y)$$

Whatever the level of the market price p, the firm will choose a level of output y where $p = MC(y)$. Thus the marginal cost curve of competitive firm is precisely its supply curve. Or put another way, the market price is precisely marginal cost-as long as each firm is producing at its profit-maximizing level.

Graphical representation of the optimality condition that $p = MC$ is depicted on the graph below. Note that this condition holds for two different levels of output y_1 and y_2. But only y_2 is profit maximizing level of output. Marginal cost curve is decreasing on the part where $p = y_1$. The level of output where marginal cost curve is decreasing can never be optimal, because the revenue of producing one more unit of output would be higher than its cost.



Marginal cost and supply. Although there are two levels of output where price equals marginal cost, the profit-maximizing quantity supplied can lie only on the upward-sloping part of the marginal cost curve.

Shut-down condition: Sometimes a firm can be better off to stop production. This is the case if the price is so low that the it does not even cover the variable cost. Only the portions of the marginal cost curve that lie above the average variable cost curve are possible points on the supply curve. If a point where price equals marginal cost is beneath the average variable cost curve, the firm would optimally choose to produce zero units of output.

Since price equals marginal cost at each point on the supply curve, the market price must be a measure of marginal cost for every firm operating in the industry. A firm that produces a lot of output and a firm that produces only a little output must have the same marginal cost, if they are both maximizing profits. The total cost of production of each firm can be very different, but the marginal cost of production must be the same.

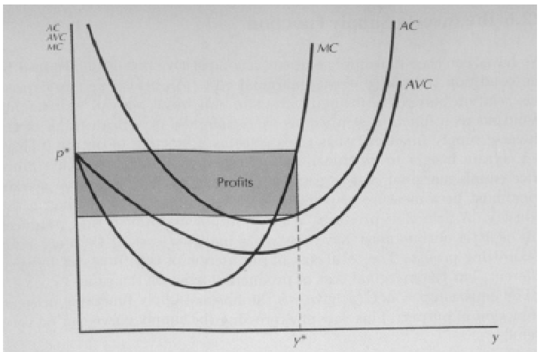
Given the market price we can now compute the optimal operating position for the firm from the condition that $p = MC(y)$. Given the optimal operating position we can compute the profits of the firm. Total revenue is given by

$$TR = p^* y^*$$

and total cost is given by

$$c(y) = y\frac{c(y)}{y} = yAC(y)$$

The profit is given by revenues minus costs and it is the shaded area on the picture below.

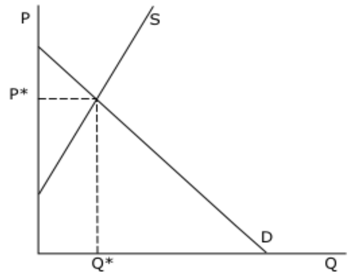


Profits. Profits are the difference between total revenue and total costs, as shown by the colored rectangle.

If a point where price equals marginal cost is beneath the average variable cost curve, the profit would be negative and the firm would optimally choose to produce zero units of output. If this point is on the average variable cost curve, the profit of the firm is zero.

6 Market equilibrium, Algebraic solution, Changes of equilibrium

Demand curve/function - for each price p we determine how much of a good will be demanded $D(p)$. **Supply curve/function** - for each price p we determine how much of a good will be supplied $S(p)$. Given individual demand curves we can add them up to get a market demand curve. Similarly, if we have a number of independent supply curves to get the market supply curve. Individual demanders and suppliers are assumed to take prices as given (feature of a competitive market). **The equilibrium price** of a good is that price where the supply of the good equals the demand, i.e. the price where the demand and supply curves cross.

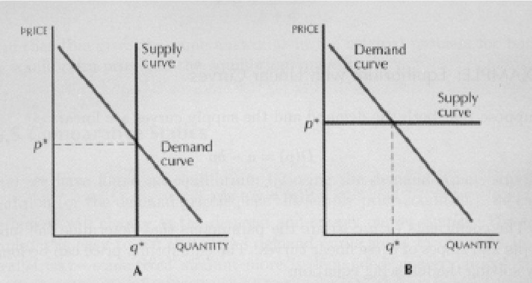


If $p < p^*$ the demand is greater than the supply. Then some product could be sold for higher price and hence the price is pushed up. If, on the other hand, $p > p^*$ demand is less than supply and some supplier will not sell what they would like to unless they lower the price. The equilibrium price and quantity is determined by both supply and demand. There are two special cases:

1. Vertical supply curve (perfectly inelastic supply). The amount supplied is fixed and independent of price. In this case the equilibrium quantity is determined entirely by the supply and the equilibrium price is determined entirely by demand conditions. When supply is perfectly inelastic, a shift in the demand curve has no effect on the equilibrium quantity supplied onto the market. Examples include the supply of tickets for sports or musical venues, and the short run supply of agricultural products (where the yield is fixed at harvest time) the elasticity of supply = zero when the supply curve is vertical.

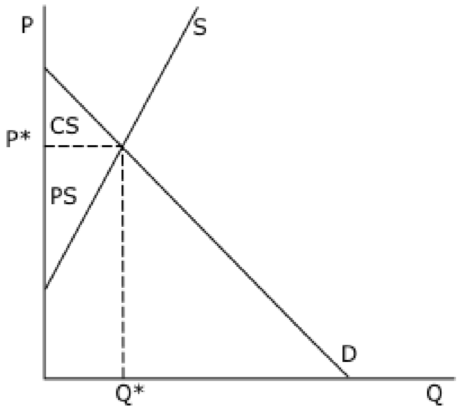
2. Horizontal supply curve (perfectly elastic supply). The industry will supply any amount of a good at a constant price. In this case the equilibrium price is determined entirely by the supply and the equilibrium quantity is determined entirely by demand curve. When supply is perfectly elastic a firm can supply any amount at the same price. This occurs when the firm can supply at a constant cost per unit and has no capacity limits to its production. A change in demand alters the equilibrium quantity but not the market clearing price.

When supply is relatively inelastic a change in demand affects the price more than the quantity supplied. The reverse is the case when supply is relatively elastic. A change in demand can be met without a change in market price.



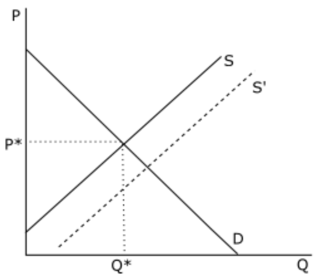
Special cases of equilibrium. Case A shows a vertical supply curve where the equilibrium price is determined solely by the demand curve. Case B depicts a horizontal supply curve where the equilibrium price is determined solely by the supply curve.

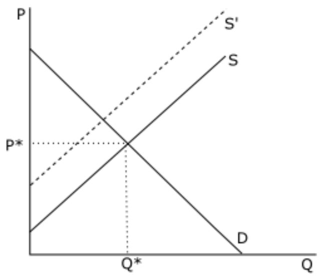
The consumer surplus is the amount that consumers benefit by being able to purchase a product for a price that is less than they would be willing to pay. **The producer surplus** is the amount that producers benefit by selling at a market price that is higher than they would be willing to sell for.



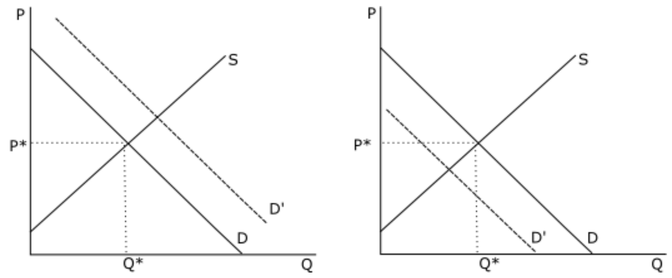
Changes in Equilibrium

Example: Imagine a company selling wheat. One year weather conditions, demand, and supply are such that the equilibrium price is P^* and equilibrium quantity is Q^*. The next year the company has a very large harvest. So the supply curve shifts to the right. As a result the equilibrium price decreases and equilibrium quantity increases. If, on the other hand, the harvest is very bad the whole supply curve shifts to the left. As a result the equilibrium price increases and equilibrium quantity decreases.





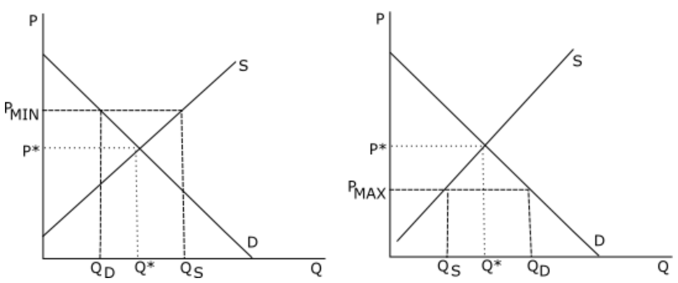
Example: Let's analyze the demand for ice cream. One summer the temperature is moderate and market demand and supply are such that the equilibrium price is P^* and equilibrium quantity is Q^*. The next summer is very hot, i.e. the whole demand curve shifts to the right and as a result both the equilibrium price and equilibrium quantity increase. If on the other hand summer is very cold, the demand curve shifts to the left and as a result both the equilibrium price and equilibrium quantity decrease.



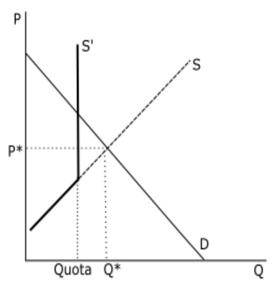
6.1 Price Caps and Quotas

Up to this point we analyzed situations where the equilibrium price and quantity where results of supply and demand curve only. Now we will look at what happens if government/local authority imposes some restrictions on price (price cap) or quantity (quota).

- Minimum price - if the minimum price is below $P*$ there is no change on the market. But if the minimum price is larger than $P*$ the result is that at this price supply exceeds demand (excess supply). There is a market inefficiency under such restriction. (left picture below)
- Maximum price - if the maximum price is above $P*$ there is no change on the market. But if the maximum price is lower than $P*$ the result is that at this price demand exceeds supply (excess demand). There is a market inefficiency under such restriction. (right picture below)

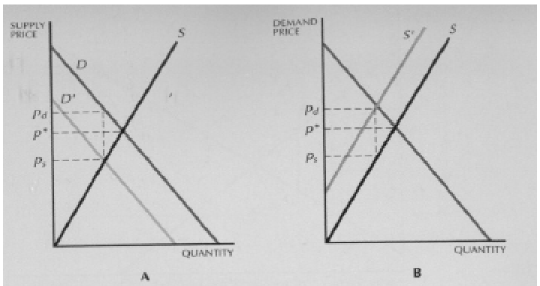


- Quota - if the quota imposed on production is larger than $Q*$ there is no change on the market. But if the quota is smaller than $Q*$ the result is that the equilibrium price increases and quantity sold decreases.



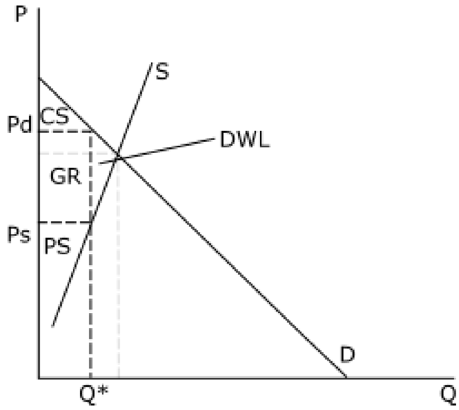
6.2 Taxes and Subventions

Unlike price caps and quotas taxes and subventions do not create inefficiencies in sense of excess demand or excess supply. In presence of taxes the price the demander pays and the price the supplier gets differ by he amount of the tax. If the tax is imposed on the suppliers, then the supply price plus the amount of the tax must equal the demand price: $P_D = P_S + t$ where t is quantity tax. If, on the other hand, the tax is imposed on the demanders, then $P_D - t = P_S$. These are the same equations which means that equilibrium price and quantity is the same in those two cases. Graphically, we either find the point where the curve $P_D - t$ crosses P_S or the point where the curve P_D crosses $P_S + t$.



The imposition of a tax. In order to study the impact of a tax, we can either shift the demand curve down, as in panel A, or shift the supply curve up, as in panel B. The equilibrium prices paid by the demanders and received by the suppliers will be the same either way.

In case of taxes the consumer and producer surplus is illustrated on the picture below. The rectangle denoted GR represent's government revenue from taxes and the triangle area DWL denotes the deadweight loss - the economic inefficiency caused by taxation. The taxing a good will typically increase the price paid by the demanders and decrease the price received by the suppliers. This certainly represents a cost to the demanders and suppliers, but from the economist's viewpoint, the real cost of the tax is that the output has been reduced.

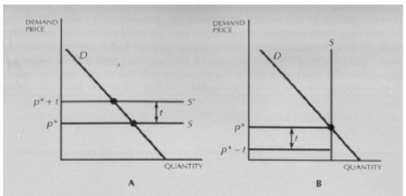


In general, a tax will both raise the price paid by consumers and lower the price received by firms. How much of a tax gets passed along will depend on the characteristics of demand and supply. To illustrate this we use two special cases of perfectly elastic (horizontal) and perfectly inelastic (vertical) supply.

In the case of a perfectly elastic supply curve the price to the consumers goes up by exactly the amount of the tax. The supply price is exactly the same as it was before the tax, and the demanders end up paying the entire tax. The horizontal supply curve means that the industry is willing to supply any amount of the good at some particular price, p', and zero amount at any lower price.

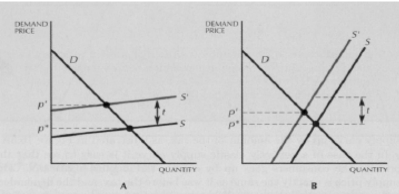
Thus, if any amount of the good is going to be sold at all in equilibrium, the suppliers must receive p^* for selling it. This effectively determines the equilibrium supply price, and the demand price is $p^* + t$. The entire tax is paid be demanders.

If the supply curve is vertical and we "shift the supply curve up," we don't change anything in the diagram. The supply curve just slides along itself, and we still have the same amount of the good supplied, with or without the tax. In this case, the demanders determine the equilibrium price of the good, and they are willing to pay a certain amount, p^*, for the supply of the good that is available, tax or no tax. Thus they end up paying p^*, and the suppliers end up receiving $p^* - t$. The entire amount of the tax is paid by the suppliers.



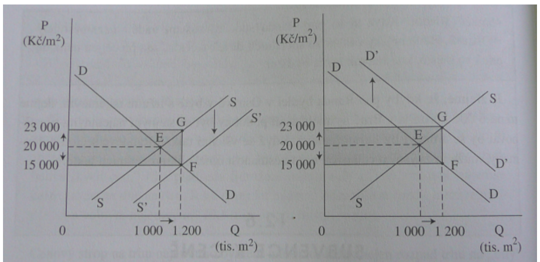
Special cases of taxation. (A) In the case of a perfectly elastic supply curve the tax gets completely passed along to the consumers. (B) In the case of a perfectly inelastic supply none of the tax gets passed along.

If the supply is not perfectly elastic or perfectly inelastic, the tax is paid by both supplier and demanders. The less elastic the supply curve the higher proportion of the tax is paid by suppliers.



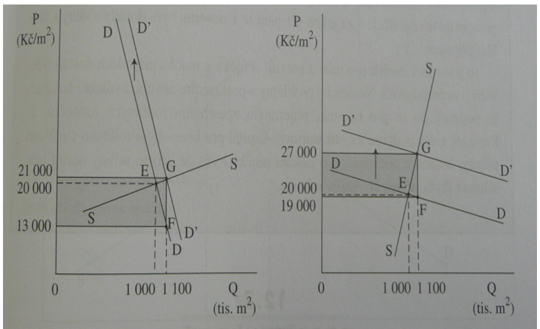
Passing along a tax. (A) If the supply curve is nearly horizontal, much of the tax can be passed along. (B) If it is nearly vertical, very little of the tax can be passed along.

Subventions: Similarly as in case of taxes - irrespective of who gets subvention (buyers or sellers) the new equilibrium price and quantity is the same. The case where sellers or buyers get subvention is depicted on left side of the picture below. Picture on the right hand side illustrates how buyers and sellers share subventions depending on the slope of demand and supply curves.



Obr. 12 – 8 Subvence prodávajícím – Původní rovnováha trhu byla v bodě E. Subvence prodávajícím ve výši 8 000 Kč/m² posune nabídkovou křivku vertikálně dolů o velikost subvence a nová rovnováha trhu je v bodě F. Prodávající docílí cenu 23 000 Kč/m² a kupující platí cenu 15 000 Kč/m².

Obr. 12 – 9 Subvence kupujícím – Původní rovnováha trhu byla v bodě E. Subvence kupujícím ve výši 8 000 Kč/m² posune poptávkovou křivku vertikálně nahoru o výši subvence, nová rovnováha je v bodě G. Prodávající docílí cenu 23 000 Kč/m² a kupující platí cenu 15 000 Kč/m².



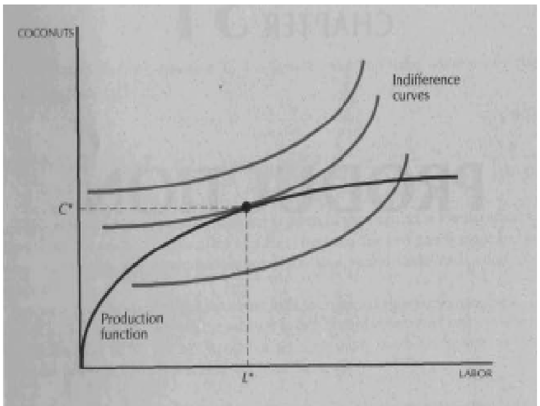
Obr. 12 – 10 Strmá poptávka a málo strmá nabídka způsobují, že kupující mají větší prospěch ze subvence než prodávající.

Obr. 12 – 11 Strmá nabídka a málo strmá poptávka způsobují, že prodávající mají větší prospěch ze subvence než kupující.

7 PPF, Comparative Advantages

7.1 The Robinson Crusoe economy

During this analysis Robinson plays two roles: he is both a consumer and a producer. Robinson can either lie on the beach and do nothing, i.e. consume leisure or he can spend time working, gathering coconuts. The more coconuts he gathers the more he has to eat, but the less time he has to relax. Robinson's preferences for leisure and coconuts are depicted on the picture below.



The Robinson Crusoe economy. The indifference curves depict Robinson's preferences for coconuts and leisure. The production function depicts the technological relationship between the amount he works and the amount of coconuts he produces.

We also illustrated typical Robinson's production function that describes the relationship between how much Robinson works and how many coconuts he gets. The more Robinson works the more coconuts he gets but due to diminishing returns to labor the marginal product of labor decreases as the hours of labor increase. How much will Robinson work and how much he will consume? The optimum combination of labor and consumption is a point where the highest indifference curve touches production function. The production function describes Production possibilities frontier (PPF) - maximum possible output for a given level of input(s). The set below this function is production set.

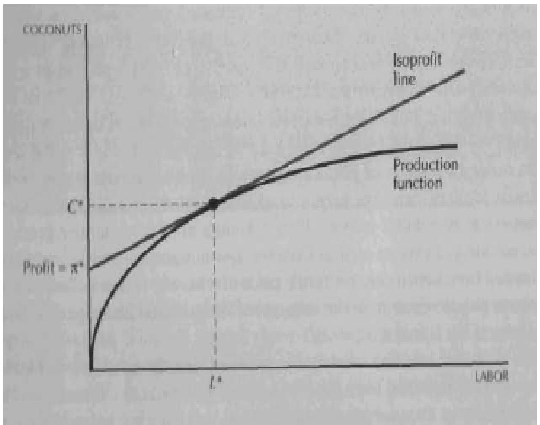
At this point, the slope of the indifference curve must equal the slope of the production function by the standard argument: if they crossed, there would be some other feasible point that was preferred. This means that the marginal product of an extra hour of labor must equal the marginal rate of substitution between leisure and coconuts. If the marginal product were greater than the marginal rate of substitution, it would pay for Robinson to give up a little leisure in order to get the extra coconuts. If the marginal product were less than the marginal rate of substitution, it would pay for Robinson to work a little less.

Market approach: Robinson the firm

Each evening, Crusoe decides how much labor it wants to hire the next day, and how many coconuts he wants to produce. Given a price of coconuts of 1 and a wage rate of labor of w, we can solve the firm's profit- maximization problem:

$$\max_{C} \pi = C - wL$$

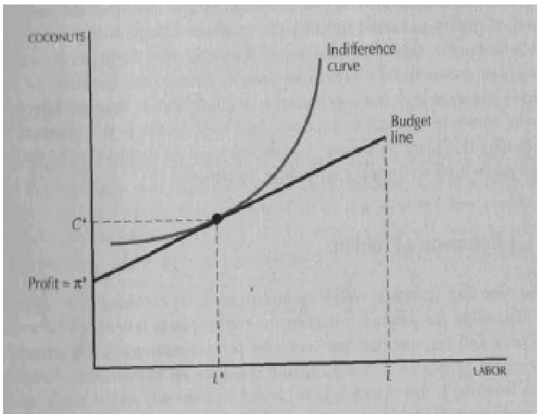
For a given level of profit π, the formula $\pi = C - wL$ or $C = \pi + wL$ describes the isoprofit lines - all combinations of labor and coconuts that yield profits of π. Crusoe will choose a point where the profits are maximized. As usual, this implies a tangency condition: the slope of the production function - the marginal product of labor - must equal w, as illustrated in the picture below.



Profit maximization. Crusoe, Inc. chooses a production plan that maximizes profits. At the optimal point the production function must be tangent to an isoprofit line.

Market approach: Robinson the consumer

The next day Robinson wakes up and receives his dividend of π dollars. While eating his coconut breakfast, he decides how much he wants to work and consume. He may consider just consuming his endowments - spend his profits on π coconuts (the price of coconuts is normalized to 1) and consume his endowment of leisure. But he also might decide to work for a few hours. We can describe Robinson's labor-consumption choice using standard indifference curve analysis. Plotting labor on the horizontal axis and coconuts on the vertical axis, we can draw in an indifference curve as illustrated in the following picture:

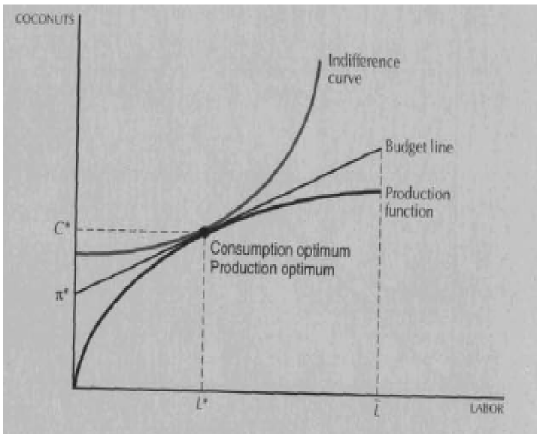


Robinson's maximization problem. Robinson the consumer decides how much to work and consume given the prices and wages. The optimal point will occur where the indifference curve is tangent to the budget line.

Note that the indifference curves have a positive slope. This is because coconuts are a good and labor is a "bad". If we indicate the maximum amount of labor by $\overline{L}$, then the distance from $\overline{L}$ to the chosen supply of labor gives Robinson's demand for leisure. Robinson's budget line is also illustrated in the picture above. It has a slope of w and passes through his endowment point $(\pi^*, 0)$ (Robinson has a zero endowment of labor and a π^* endowment of coconuts). At his optimal consumption, the marginal rate of substitution between consumption and leisure must equal the wage rate.

Robinson the consumer meets Robinson the producer

Using the market system (solving separately consumer's and producer's problem) gives exactly the same result as choosing between leisure and consumption directly.



Equilibrium in both consumption and production. The amount of coconuts demanded by the consumer Robinson equals the amount of coconuts supplied by Crusoe, Inc.

Since the marginal rate of substitution between leisure and consumption equals the wage, and the marginal product of labor equals the wage as well, it follows that marginal rate of substitution between leisure and consumption equals the marginal product and hence the slope of indifference curve and production function is the same.

In case on one-person economy using the markets, i.e. dividing the decision onto two parts is unnecessary. However, in economy with many consumers it makes sense - firms and consumers make decisions based on the prices.

Robinson Crusoe Economy helps to illustrate:

- closed economy - no trade
- labor choices under changing environments, consumption-leisure choice
- trade offs, opportunity cost

7.2 Preview to absolute/comparative advantages:

Opportunity cost or economic opportunity loss is the value of the next best alternative foregone as the result of making a decision. Opportunity costs are not restricted to monetary or financial costs: the real cost of output forgone, lost time, pleasure or any other benefit that provides utility should also be considered. There is always an opportunity cost in a decision that is made either in economics or everyday life.

Example: You won a free ticket to see an Eric Clapton concert (which has no resale value). Bob Dylan is performing on the same night and is your next-best alternative activity. Tickets to see Dylan cost $40. On any given day, you would be willing to pay up to $50 to see Dylan. Assume there are no other costs of seeing either performer. Based on this information, what is the opportunity cost of seeing Eric Clapton? (a) $0, (b) $10, (c) $40, or (d) $50.

In other words - what is the minimum amount (in dollars) you would have to value seeing Eric Clapton for you to choose his concert? The correct answer is $10.

7.3 Absolute and comparative advantages

Absolute advantage refers to the ability of a particular person or a country to produce a particular good with less resources than another person or country. It can be contrasted with the concept of **comparative advantage** which refers to the ability to produce a particular good at a lower opportunity cost. It is not necessary to have an absolute advantage to gain from trade, only a comparative advantage.

Example: Imagine that Robinson can produce 10 pounds of fish per hour or 20 coconuts per hour of work. Friday can produce 20 pounds of fish per hour or 10 coconuts per hour. In this example Robinson has absolute advantage in producing coconuts and Friday has absolute advantage in producing fish.

Opportunity cost: Robinson's opportunity cost of producing 1 fish is 2 coconuts, Friday's opportunity cost of producing 1 fish is 0.5 coconut. In other words, for each fish produced Robinson has to give up 2 coconuts. Friday has lower opportunity cost of producing fishing. We say that he has **comparative advantage** in fishing and hence he should specialize in fishing. Similarly, Robinson has comparative advantage in coconut production. Robinson can get more fish by specializing in coconut production and trading coconuts for fish while Friday can benefit by trading fish for coconuts.

Comparative advantage explains how trade can create value for both parties even when one can produce all goods with less resources than the other. The net benefits of such an outcome are called gains from trade. Having a comparative advantage is not the same as being the best at something. In fact, someone can be completely unskilled at doing something, yet still have a comparative advantage at doing it! How can that happen?

Example: Now imagine that Robinson can produce 25 pounds of fish per hour or 25 coconuts per hour of work. Robinson only likes coconuts so he will spend all the time picking them. Friday can only produce 10 pounds of fish per hour or 20 coconuts per hour. Friday only likes fish so he will spend all the time catching fish. In this example Robinson has absolute advantage in both coconuts and fish production. So it could seem that Robinson has no interest in trade with Friday. However, there is some space for trade that will be beneficial for both Robinson and Friday. Without any trade Robinson's and Friday's consumption is:

$$(F_R, C_R) = (0, 25)$$
$$(F_F, C_F) = (10, 0)$$

Robinson's opportunity cost of producing 1 pound of fish is 1 coconut (recall that opportunity cost is how many coconuts Robinson gives up by producing 1 pound of fish). Friday's opportunity cost of producing 1 pound of fish is 2 coconuts. Since Robinson's opportunity cost is lower he should specialize in fish production. Similarly, Robinson's opportunity cost of producing 1 coconut is 1 pound of fish and Friday's opportunity cost of producing 1 coconut is 0.5 pounds of fish. Friday's opportunity cost is lower and hence he should specialize in coconut production.

If Robinson and Friday agree on mutually beneficial exchange rate 1 pound of fish for 1.25 coconuts (or equivalently 1 coconut for 0.8 pound of fish) they can do the following: Friday produces 20 coconuts and exchange them for $0.8 * 20 = 16$ pounds of fish. Robinson produces 16 pounds of fish and $25 - 16 = 9$ coconuts. He exchanges 16 pounds of fish for 20 coconuts and as a result their consumption after trade is:

$$(F_R, C_R) = (0, 29)$$
$$(F_F, C_F) = (16, 0)$$

Despite the fact that Robinson has absolute advantage in production of both fish and coconuts they still can both benefit from the trade.

NOTE: To find people's comparative advantages, do not compare their absolute advantages. Compare their opportunity costs.

8 The Optimum of Monopoly, Price Discrimination

8.1 Monopoly

Up to now we have analyzed the behavior of a competitive industry, a market structure that is most likely when there are a large number of small firms. In that case all firms took price as given. In this chapter we turn to the opposite extreme and consider an industry structure when there is only one firm in the industry-a monopoly. A monopolist does not take the price as given. The monopolist will choose the price to maximize the profit.

Profit maximization:

Recall, that in case of perfect competition the optimality condition is $MR = MC$. In case of perfect competition marginal revenue is equal to price and hence in the optimum price equals marginal cost ($p = MC$). This holds because in perfect competition firms take price as given and hence the price is not function of output y. However, in case of monopoly the price is a function of output (the higher the price the lower the demand and vice versa) and hence the term for marginal revenue is a little more complicated.

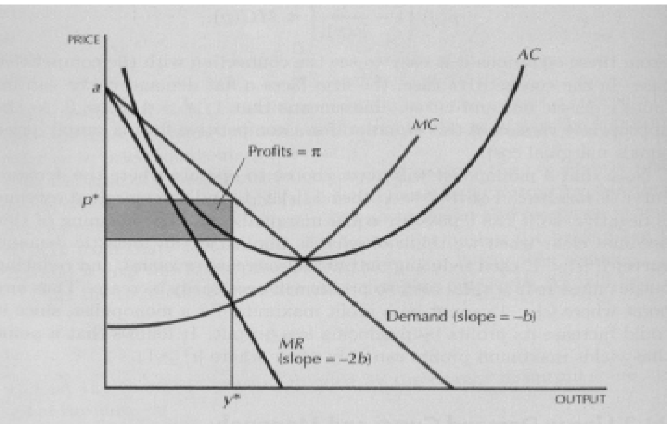
Let's use $p(y)$ to denote the market inverse demand curve and $c(y)$ to denote the cost function. Let $r(y) = p(y)y$ denote the revenue function of the monopolist. The monopolist's profit-maximization problem then takes the form

$$\max_y r(y) - c(y) = \max_y p(y)y - c(y)$$

Optimality condition: at the optimal choice of output we must have marginal revenue equal to marginal cost ($MR = MC$ or $p'(y)y + p(y) = c'(y)$). If marginal revenue were less than marginal cost it would pay the firm to decrease output, since the savings in cost would more than make up for the loss in revenue. If the marginal revenue were greater than the marginal cost, it would pay the firm to increase output. The only point where the firm has no incentive to change output is where marginal revenue equals marginal cost.

Another way to think about this is to think of the monopolist as choosing its output and price simultaneously. If the monopolist wants to sell more output it has to lower its price. But this lower price will mean a lower price for all of the units it is selling, not just the new units. In the competitive case, a firm that could lower its price below the price charged by other firms would immediately capture the entire market from its competitors. But in the monopolistic case, the monopoly already has the entire market; when it lowers its price, it has to take into account the effect of the price reduction on all the units it sells.

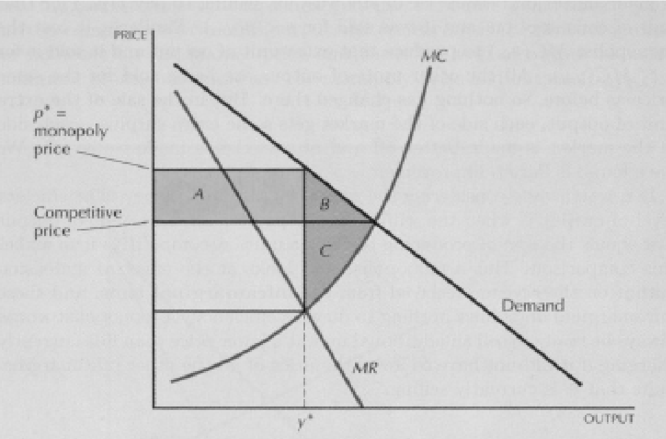
The optimal choice of output and price is depicted on the picture below. The optimal output y^* is where the marginal revenue curve intersects the marginal cost curve. The monopolist will charge the maximal possible price for this level of output. This gives the monopolist a revenue $p(y^*)y^*$ from which we subtract the total cost $c(y^*) = AC(y^*)y^*$ to get the profit (shaded area on the picture below).



Monopoly with a linear demand curve. The monopolist's profit-maximizing output occurs where marginal revenue equals marginal cost.

On this picture we have a case of linear demand function of a form $p(y) = a - by$. In this case the revenue function is given by $r(y) = p(y)y = (a - by)y = a - by^2$. And hence marginal revenue is $MR = r'(y) = a - 2by$. Hence it is also a straight line, it has the same vertical intercept and the slope is twice the slope of demand function.

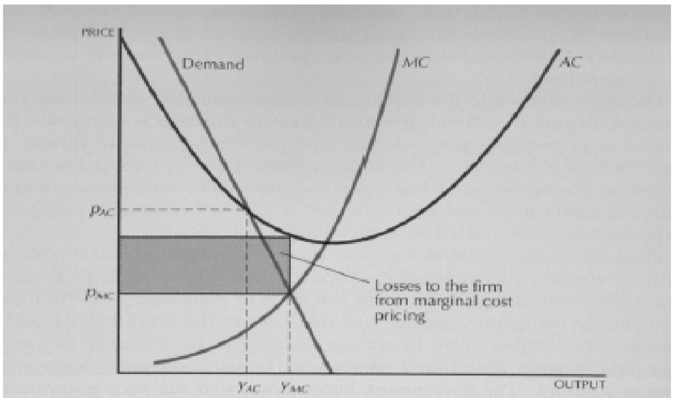
A competitive industry operates at a point where price equals marginal cost. A monopolized industry operates where price is greater than marginal cost. Thus in general the price will be higher and the output lower. For this reason, consumers will typically be worse off in an industry organized as a monopoly than in one organized competitively. The price and output choice of monopolist is not Pareto efficient. This pricing also creates an inefficiency on the market. The inefficiency is caused by the following fact: there are people that would be willing to pay some additional units for price higher than marginal cost. But monopoly will not supply these because by selling an additional unit of product would decrease the price of all units sold and overall revenue would decrease.



Deadweight loss of monopoly. The deadweight loss due to the monopoly is given by the area $B + C$.

The deadweight loss due to monopoly, like deadweight loss due to tax, measures the value of output that is lost by valuing each unit at the price that people are willing to pay for it.

Now we know that Pareto efficient output is where price is equal to marginal cost. Monopolist produces where marginal revenue is equal to marginal cost and thus produces too little output. It seems that regulation authority could restore efficiency by setting the price equal to marginal cost. However, in some cases this could mean that the monopolist is making negative profit and prefers to leave the market. This situation is illustrated on the picture below.



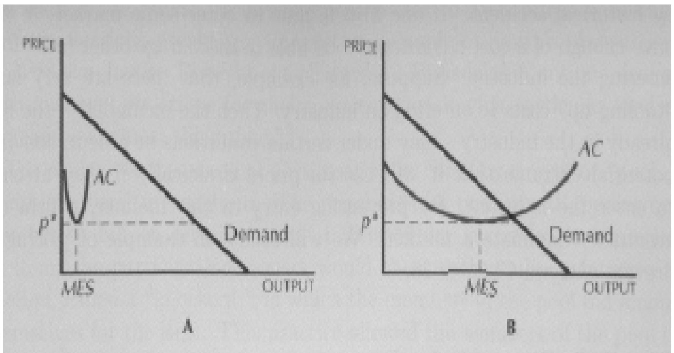
A natural monopoly. If a natural monopolist operates where price equals marginal cost, then it will produce an efficient level of output, y_{MC}, but it will be unable to cover its costs. If it is required to produce an output where price equals average cost, y_{AC}, then it will cover its costs, but will produce too little output relative to the efficient amount.

The minimum point of the average cost curve is to the right of the demand curve, and the intersection of demand and marginal cost lies underneath the average cost curve. Even though the level of output Y_{MC} is efficient, it is not profitable. If a regulator set this level of output, the monopolist would prefer to go out of business. This situation often arises when fixed cost of production is very large and marginal cost relatively small (gas company, telephone company, etc.). This situation is referred to as natural **natural monopoly**.

This situation is solved differently in different countries. Sometimes this type of business is run by government sometimes it is run by private firms and regulated by government. In this case the price has to be such that the firm does not make negative profit (it must operate on or above the average cost curve) and also it has to provide service to all who are willing to pay for it (it has to operate on the demand curve). So the natural solution for the firm is to charge price P_{AC} and produce quantity Y_{AC}. This is the usual price set by government regulators. Note: sometimes it might be difficult to estimate the true cost of the firm.

The other solution to the problem of natural monopoly is to let the government operate it. The ideal solution here in this case is to operate the service at price equals marginal cost and provide a lump-sum subsidy to keep the firm in operation. This is often the practice for local public transportation systems such as buses and subways. The lump-sum subsidies may not reflect inefficient operation per se but rather, simply reflect the large fixed costs associated with such public utilities.

Cause of monopoly: Given information on costs and demand, when would we predict that an industry would be competitive and when would we predict that it would be monopolized? In general the answer depends on the relationship between the average cost curve and the demand curve. The crucial factor is the size of the minimum efficient scale (MES), the level of output that minimizes average cost, relative to the size of demand.



Demand relative to minimum efficient scale. (A) If demand is large relative to the minimum efficient scale, a competitive market is likely to result. (B) If it is small, a monopolistic industry structure is possible.

Thus the shape of the average cost curve, which in turn is determined by the underlying technology, is one important aspect that determines whether a market will operate competitively or monopolistically. If the minimum efficient scale of production-the level of output that minimizes average costs-is small relative to the size of the market, we might expect that competitive conditions will prevail.

A second reason why monopoly might occur is that several different firms in an industry might be able to collude and restrict output in order to raise prices and thereby increase their profits. When firms collude in this way and attempt to reduce output and increase price, we say the industry is organized as a cartel. Cartels are illegal and firms engaged in this behavior have to pay heavy fines.

8.2 Price Discrimination

We have argued earlier that a monopoly operates at an inefficient level of output since it restricts output to a point where people are willing to pay more for extra output than it costs to produce it. The monopolist doesn't want to produce this extra output, because it would force down the price that it would be able to get for all of its output. But if the monopolist could sell different units of output at different prices, then we have another story. Selling different units of output at different prices is called price discrimination. Economists generally consider the following three kinds of price discrimination:

- **First-degree price discrimination** means that the monopolist sells different units of output for different prices and these prices may differ from person to person. This is sometimes known as the case of perfect price discrimination.
- **Second-degree price discrimination** means that the monopolist sells different units of output for different prices, but every individual who buys the same amount of the good pays the same price. Thus prices differ across the units of the good, but not across people. The most common example of this is bulk discounts.
- **Third-degree price discrimination** occurs when the monopolist sells output to different people for different prices, but every unit of output sold to a given person sells for the same price. This is the most common form of price discrimination, and examples include senior citizens' discounts, student discounts, and so on.

Price discrimination transfers some of this surplus from the consumer to the producer/marketer. Strictly, a consumer surplus need not exist, for example where price discrimination is necessary merely to pay the costs of production. An example is a high-speed internet connection shared by two consumers in a single building; if one is willing to pay less than half the cost, and the other willing to make up the rest but not to pay the entire cost, then price discrimination is necessary for the purchase to take place.

Examples of price discrimination: Airlines (business class, economy class), employees discount, segmentation by age group, premium pricing (company charges two different prices for low quality and premium product - tesco products), bundling.

9 The optimum of Oligopoly

During previous lectures we have investigated two important forms of market structure: pure competition, where there are typically many small competitors, and pure monopoly, where there is only one large firm in the market. However, much of the world lies between these two extremes. Often there are a number of competitors in the market, but not so many as to regard each of them as having a negligible effect on price. This is the situation known as **oligopoly**. Behavior of firms in case of oligopoly can be either non-cooperative (individual profit maximization) or cooperative (cartel). It is unreasonable to expect one grand model since many different behavior patterns can be observed in the real world. What we want is a guide to some of the possible patterns of behavior and some indication of what factors might be important in deciding when the various models are applicable.

9.1 Bertrand model of oligopoly

Bertrand oligopoly is an example of non-cooperative competition in prices. In Bertrand model firm chooses own price while taking the price of competition as given.

First let's start with a single firm on a market - Arkus. Arkus is facing an inverse demand function:

$$P = 60 - 0.6 * Q$$

Profit is maximized where marginal revenue equals marginal cost (equal to zero for simplicity). In our example the revenue is:

$$R = P * Q = (60 - 0.6 * Q)Q = 60Q - 0.6 * Q^2$$

and hence marginal revenue is:

$$MR = R' = 60 - 1.2 * Q$$

The profit maximizing (monopolistic) quantity is where marginal revenue equals zero which holds for quantity equal to 50 and corresponding profit maximizing price is equal to 30.

Then the second firm, Bona, comes to the market and chooses the price slightly lower in order to get the whole market (Bona can afford this because marginal cost is zero so any price is profitable). Arkus has no customers and hence makes zero profit so it's better to undercut Bona's price to get the whole market and make positive profit. This price war leads to both firms charging basically zero price and earning zero profit.

9.2 Cournot model of oligopoly

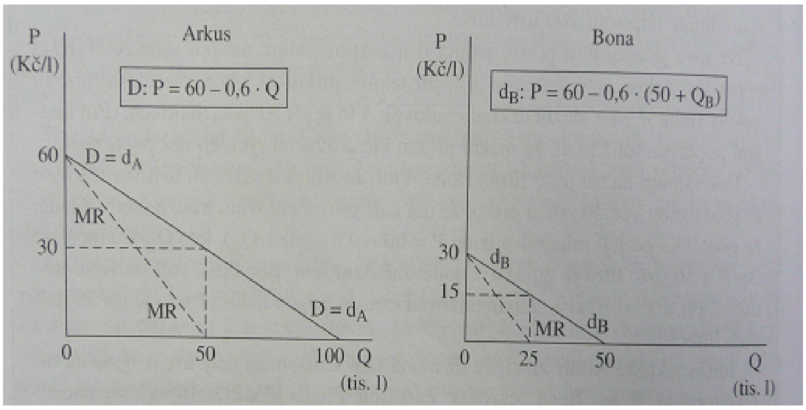
Cournot oligopoly is an example of non-cooperative competition in quantities. In Bertrand competition a firm makes decision about own price while taking the price of competition as given. In Cournot competition a firm chooses own quantity to be produced and takes the quantity produced by a competitor as given. First let start with a single firm on a market - Arkus. Arkus is facing an inverse demand function $P = 60 - 0.6 * Q$. Profit is maximized where marginal revenue equals marginal cost (equal to zero for simplicity) and profit maximizing (monopolistic) quantity is 50. This situation is depicted on the left picture below. Later the second firm, Bona, enters the market. Bona takes Arkus' production of 50 as given and hence the inverse demand function is:

$$P = 60 - 0.6(50 + Q_B)$$

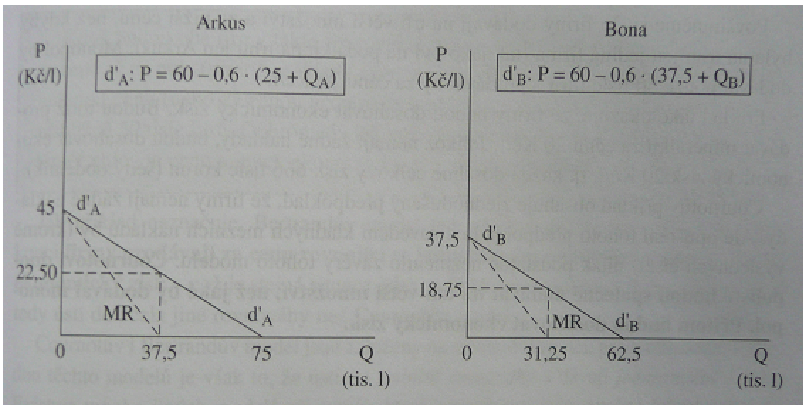
Now the revenues and marginal revenues are:

$$R = [60 - 0.6(50 + Q_B)]Q_B = [30 - 0.6Q_B]Q_B, \quad MR = 30 - 1.2Q_B$$

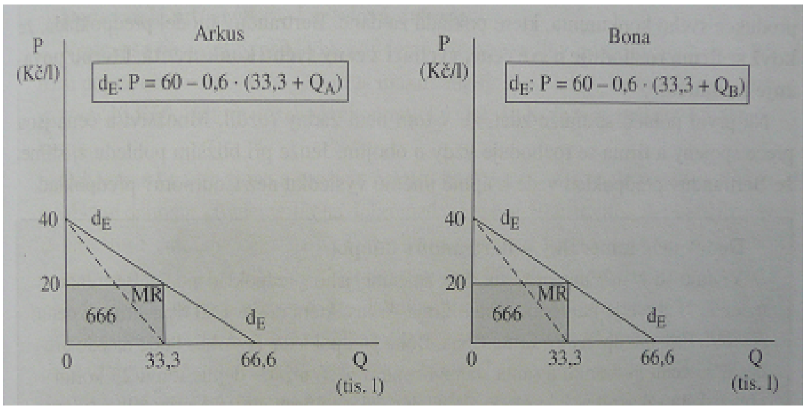
As a result (if we put marginal revenues equal to zero marginal cost) the profit maximizing quantity for Bona is 25. This situation is depicted on the right picture below.



After Bona enters the market and produces 25 the situation changes for Arkus. Now, Arkus takes Bona's production as given and hence the new demand function is $P = 60 - 0.6(25 + Q_A)$ and the new profit maximization quantity will be 37.5. And after this change the new profit maximizing quantity for Bona is 31.25 (look at the picture below).



This behavior will continue till two firms on the market reach equilibrium. Since both firms have zero marginal cost (are the same) the equilibrium will be symmetric, i.e. both firms will produce the same quantity and it will be equal to 33.3. (Note that if Arkus takes Bona's production of 33.3 as given then Arkus' profit maximizing quantity is 33.3 as well and vice versa).



To solve for Cournot outcome we solve maximization problem of the two firms separately:

$$\max_{Q_A} PQ_A = [60 - 0.6(Q_A + Q_B)]Q_A$$
$$\max_{Q_B} PQ_B = [60 - 0.6(Q_A + Q_B)]Q_B$$

FOC (first order conditions) give us:

$$60 - 1.2Q_A - 0.6Q_B = 0$$
$$60 - 1.2Q_B - 0.6Q_A = 0$$

Solving these two equations we get:

$$Q_A = Q_B = 33.\overline{3}$$

This situation can be analyzed using the concept of the game theory. For simplicity we assume that only two choices of quantity are possible. Each firm can either choose to produce 25 which in total is the monopoly level of production or, alternatively, each firm can choose Cournot level of production equal to 33.3. This situation can be represented in the table form below. The numbers in the table are profits of company A and B respectively.

A \ B	25 (monopoly quantity)	33.3 (Cournot quantity)
25	750,750	625,833
33.3	833,625	666,666

The Cournot outcome is in the lower right corner where both firms choose the level of output equal to 33.3 and their profit is 666. However, we see that if collusion is possible, the firms would do better to choose the output that maximizes total industry profits and then divide up the profits among themselves. The two firms can decide to sell the monopoly level of production for the monopoly price. This way they maximize their total profit. However, this arrangement is not stable because one firm could increase the output and get temporarily higher profit. That's why we often see that cartel agreements are broken.

Bertrand and Cournot model are both based on very simplified assumptions. The advantage is that they are simple and lead to a single equilibrium. In Bertrand model the prices are essentially equal to marginal cost and firms make zero profits. In Cournot model the prices are much higher and quantity sold is lower.

Note that if we compare oligopoly with monopoly then the level of production in oligopoly is higher and the prices are lower. The price will be higher than marginal cost and hence both firms will have a positive profit (shaded rectangle on the picture above).

9.3 Stackelberg model of oligopoly

When one firm decides about its choices for prices and quantities it may already know the choices made by the other firm. If one firm gets to set its price before the other firm, we call it the **price leader** and the other firm the **price follower**. Similarly, one firm may get to choose its quantity first, in which case it is a **quantity leader** and the other is a **quantity follower**. In the case of quantity leadership, one firm makes a choice before the other firm. This is called the **Stackelberg model**.

Stackelberg competition consists of two stages: First, the leader sets the quantity to be produced. Second, the follower observes the action of the leader and chooses the optimal level of production. So when the leader makes a decision about the quantity to be produced he anticipates followers reaction and hence chooses the quantity accordingly. Notice, that compared to Cournot competition the Stackelberg leader can never be worse off. (Leader can choose the Cournot quantity and then for the follower it is optimal to produce the same amount and we have Cournot outcome. So leader either chooses Cournot quantity or if he chooses a different level of production he must be better off otherwise he wouldn't do that).

To compute Stackelberg outcome we use similar approach as in case of Cournot competition but in this case the firms choose their level of output sequentially rather than simultaneously. In the first stage firm A chooses the level of output and than in the second stage firm B makes the decision. We solve this problem backwards. First we solve the second choice - optimal response of B to A's choice and then we solve for the optimal choice of A.

2^{nd} stage:

$$B: \max_{Q_B} PQ_B = [60 - 0.6(Q_A + Q_B)]Q_B$$

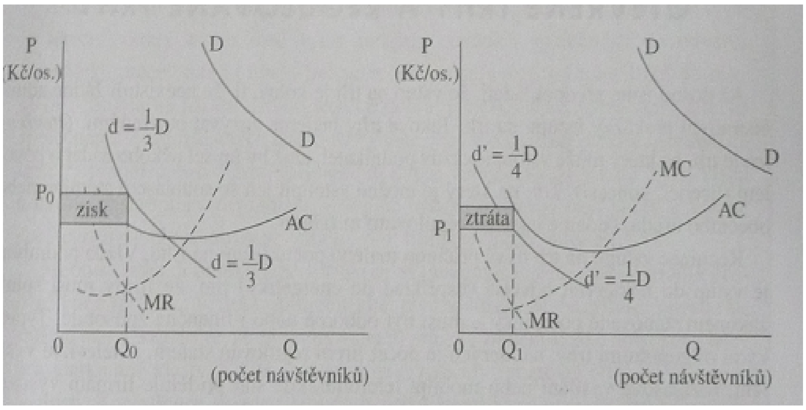
$$FOC: \; 60 - 1.2Q_A - 0.6Q_B = 0 \quad \Rightarrow \quad Q_B = 50 - \frac{Q_A}{2}$$

1^{st} stage:

$$A: \max_{Q_A} PQ_A = [60 - 0.6(Q_A + Q_B)]Q_A \text{ and A knows that } Q_B = 50 - \frac{Q_A}{2}$$

$$\text{Hence: } A: \max_{Q_A} \left[60 - 0.6\left(Q_A + 50 - \frac{Q_A}{2}\right)\right] Q_A = (30 - 0.3Q_A)Q_A$$
$$FOC: \quad 30 - 0.6Q_A = 0 \quad \Rightarrow \quad Q_A = 50$$
$$Q_B = 50 - \frac{Q_A}{2} = 50 - \frac{50}{2} = 25$$

Why do oligopolies exist? The reason is the relationship between the market demand and the optimal size of a firm. Look at the left picture below. If there are three firms on the market, each firm sells one third of market demand at a given price and all three firms make a positive profit. On the right picture we can see the situation when the fourth firm enters the market, each firms sells one fourth of the market demand and all firms make a negative profit till one of them leaves the market.



We have now examined several models of duopoly behavior: quantity leadership (Stackelberg), simultaneous quantity setting (Cournot), simultaneous price setting (Bertrand), and the collusive solution (cartel). How do they compare? In general, collusion results in the smallest industry output and the highest price. Bertrand equilibrium gives us the highest output and the lowest price. The other models give results that are in between these two extremes.

Summary:

- Output is greater with Cournot duopoly than monopoly, but lower than perfect competition.
- Price is lower with Cournot duopoly than monopoly, but not as low as with perfect competition.
- According to this model the firms have an incentive to form a cartel, effectively turning the Cournot model into a Monopoly. Cartels are usually illegal, so firms might instead tacitly collude using self-imposing strategies to reduce output which, ceteris paribus will raise the price and thus increase profits for all firms involved.

Bertrand versus Cournot

Although both models have similar assumptions, they have very different implications:

- Since the Bertrand model assumes that firms compete on price and not output quantity, it predicts that a duopoly is enough to push prices down to marginal cost level, meaning that a duopoly will result in perfect competition.
- Neither model is necessarily "better." The accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the industry situation.
- If capacity and output can be easily changed, Bertrand is a better model of duopoly competition. If output and capacity are difficult to adjust, then Cournot is generally a better model.
- Under some conditions the Cournot model can be recast as a two stage model, where in the first stage firms choose capacities, and in the second they compete in Bertrand fashion.

Stackelberg versus Cournot

- The Stackelberg and Cournot models are similar because in both, competition is on quantity.
- The first move gives the leader in Stackelberg a crucial advantage.
- There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.
- In Cournot competition, it is the simultaneity of the game (the imperfection of knowledge) that results in neither player (ceteris paribus) being at an advantage.

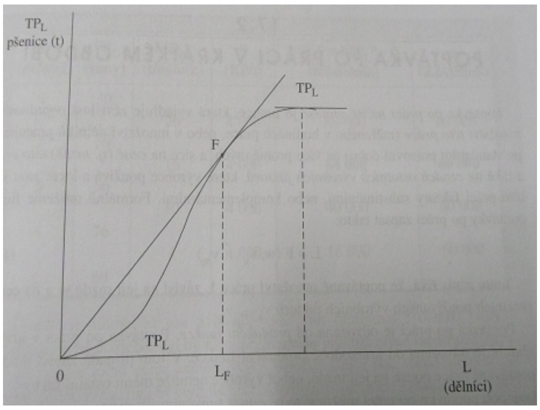
10 D/S of/for Labor

10.1 Demand for Labor

Demand for labor is determined by production function. In short run production function is function of only one variable (labor) remaining factors are fixed (capital). The productivity of a production function can be described by **Average productivity** and **Marginal productivity**. Average product is given by the ratio of total product of labor and number of units of labor used:

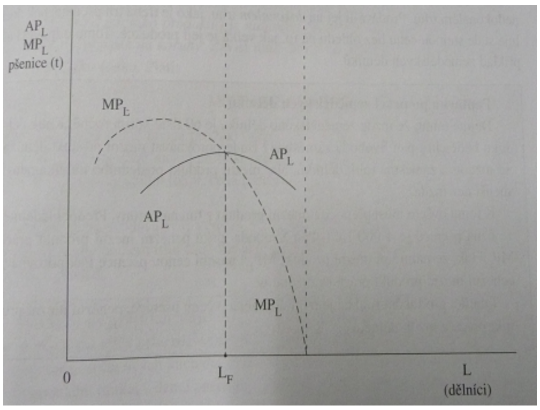
$$AP_L = \frac{TP_L}{L}$$

On the picture below we have a production function of labor. We can find an average product at a given level of labor used L_F by drawing the line between the origin and point F. Average product of labor is then the slope of this line.



Marginal product is a derivative of production function at a given level of labor used. Graphically, marginal product is the slope of a tangent to a production function at a given point.

On the picture below we have depicted both average product and marginal product curve. We see that they intercept at the the maximum of average product curve. AP curve has to be increasing while MP lies above AP and it has to decrease if MP is below AP. (Note that this situation is similar to marginal cost curve intercepting average cost curve at its minimum.)



Demand for labor depends on the price of labor, price of output and production function. In optimum a firm employs so many units of labor (number of workers) so that the value of marginal product of labor equals the wage. Look at the following example:

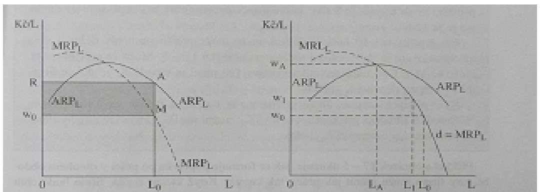
L (dělníci)	TP_L (tuny)	MP_L^n (t/dělník)	P (Kč/t)	$MP_L^P = MP_L^n \cdot P$ (Kč/dělník)	w (Kč/dělník)
1	30				
		20	4 000	80 000	60 000
2	50				
		16	4 000	**64 000**	**60 000**
3	66				
		10	4 000	40 000	60 000
4	76				
		4	4 000	16 000	60 000
5	80				

From the data in the table it follows that this firm should hire three employees. It is so because hiring third employee will increase the profit by 64 000 and the wage of the worker is lower - 60 000. If the firm hires one more worker the fourth worker would bring profit of 40 000 but his wage would have to be higher - 60 000. So profit maximizing firm would hire three workers. If wage decreases below 40 000 it would be optimal to hire four workers. If the wage increases above 64 000 it would be better to hire only two workers.

In other words, the firm chooses to hire so many works that the revenue from marginal product of labor equals wage:

$$MRP_L = MP_L \times MR = w$$

This situation is illustrated on the picture below. If the wage is w_0 that the optimal choice for a firm is to employ L_0 workers. The revenue is a shaded rectangle.

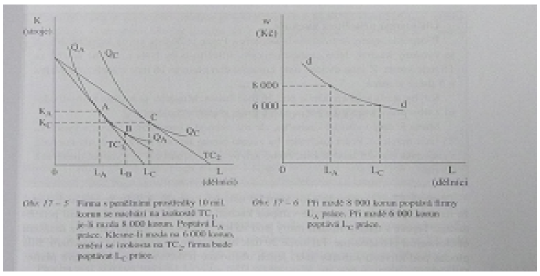


Obr. 17 – 3 Je-li mzda w_0, bude firma poptávat L_0 práce. Bílý obdélník Ow_0ML_0 měří objem mezd a šedý obdélník w_0RAM měří celkový výnos z kapitálu.

Obr. 17 – 4 Křivka poptávky firmy po práci bude shodná s částí křivky MRP_L, která začíná v bodě $MRP_L = ARP_L$.

If the wage increases the optimal level of labor decreases as the worker are more expensive to hire. The demand for labor is actually identical with MRP_L curve. But only up to the point where wage become to high. If the wage is higher than w_A firm will stop hiring any workers because the wage is higher than average revenue from a unit of labor which means that the firm is losing money. In this case it's better to stop production.

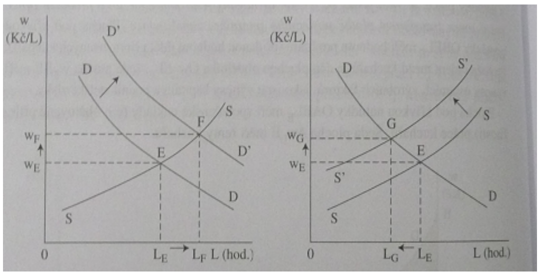
Up to now we analyzed short-run demand for labor. In short-run all factors of production apart from labor are fixed. Now we will analyze the demand for labor in long-run. In long-run all factors of production can be changed. The intuition is the same - if wage increases a firm will hire less workers. The difference between short- and long-run is the motives for this change. In short run decreasing demand for labor is caused be decreasing revenue from marginal product of labor. In long-run decreasing demand for labor is caused by substitution and production effect (similar to substitution and income effect in consumer optimization problem). This situation is depicted on the picture below.



10.2 Supply of Labor

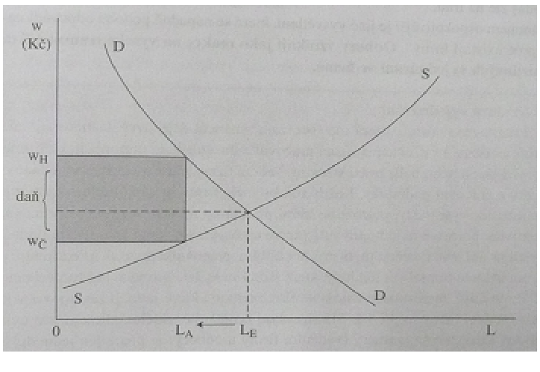
Supply of labor was discussed previously. We analyzed individual behavior when choosing between the labor (or complementary leisure) and consumption. The market supply curve is sum of individual choices and this curve is increasing.

Market equilibrium minimum wage and taxes: The equilibrium wage on the market is determined by the point where the supply and demand for labor intercept. Changes in equilibrium can be caused by changes in demand or supply. If the demand for production increases the demand for labor increases as well and as a result the equilibrium wage increases as well. Look at the left hand side of the picture.

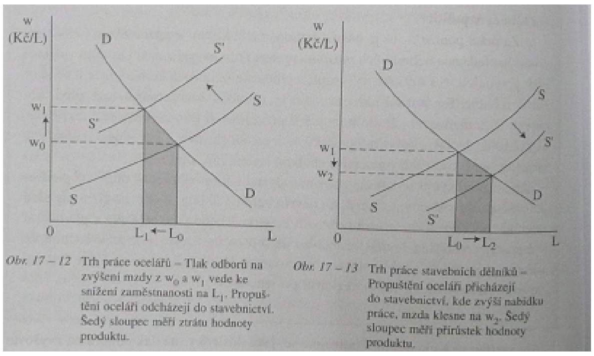


If the wages in neighbor countries increase significantly and it is simple to work abroad or if the wages in similar industries increase the supply of labor curve moves upward and as a result the equilibrium wage increases. see the right hand side picture above.

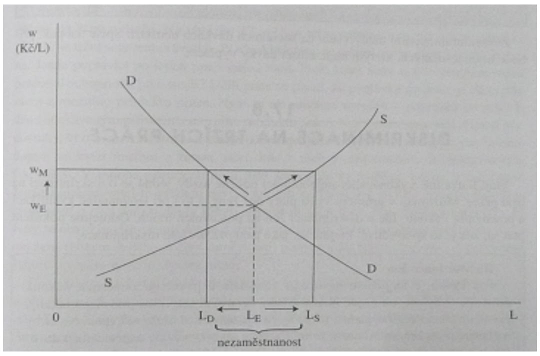
Now we will illustrate the effect of **income tax** on equilibrium wage and number of workers hired. In the Czech Republic the income tax is 15 - 32%. The situation is depicted on the picture below. With no taxes the equilibrium employment rate is L_E. After the income tax is introduced the equilibrium employment decreases to L_A.



Now we analyze the effect of **unions**. Imagine that unions negotiate the wage increase in a particular industry from w_0 to w_1. Firms are forced to pay more to their workers and as a result the equilibrium employment decreases (left picture below). Those workers who no longer have a job in a given industry will go to other industries which means that the supply curve shifts to the right and as a result the equilibrium wage in the second industry goes down (right picture).



The **minimum wage** also has an impact on the level of employment. If the minimum wage is introduced and it is higher than the equilibrium wage than as a result supply of labor is higher than demand and the unemployment is present.



10.3 D/S of/for Capital

Similarly to decision about the demand for labor a firm will use the level of capital such that the revenue from marginal product of capital equals its price.

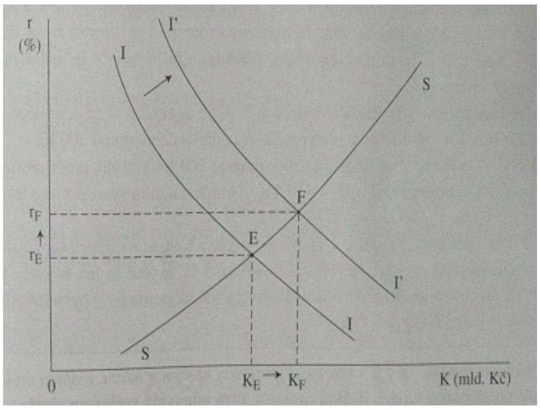
$$MRP_K = v$$

where $v = R + D$, R is the forgone interest and D is depreciation or

$$v = rP + dP$$

where P is the price of the capital (machine), r is interest rate, and d is depreciation rate.for labor.

As the interest rate increases the demand function for capital decreases and supply increases. The equilibrium interest rate is given by their intercept.



The picture above illustrates the situation when the investment environment improves, i.e. there are better investment opportunities. This means that firms want to rent more capital and as a result the the demand curve shifts upwards. The new equilibrium interest rate is higher.

Nominal vs Real interest rate: we have to distinguish between nominal interest rate which measures change in monetary value of capital and real interest rate which measure change in real value of capital.

In other words, the nominal interest rate is the amount, in money terms, of interest payable. For example, suppose a household deposits \$100 with a bank for 1 year and they receive interest of \$10. At the end of the year their balance is \$110. In this case, the nominal interest rate is 10% per annum.

The real interest rate, which measures the purchasing power of interest receipts, is calculated by adjusting the nominal rate charged to take inflation into account. If inflation in the economy has been 10% in the year, then the \$110 in the account at the end of the year buys the same amount as the \$100 did a year ago. The real interest rate, in this case, is zero.

The relationship between nominal and real interest rate is:

$$r_N = r_R + i_e$$

where r_N is nominal interest rate, r_R is real interest rate, and i_e is expected inflation.

Choice of Investment Projects: there are usually several investment possibilities available. How do we decide where to invest free capital? We choose the project with the highest value. The value of project is the present value of all future returns where future returns are discounted to the present date.

Net present value, NPV, is an indicator of how much value an investment or project adds to the value of the firm. In financial theory, if there is a choice between two mutually exclusive alternatives, the one yielding the higher NPV should be selected.

Example: A corporation must decide whether to introduce a new product line. The new product will have startup costs, operational costs, and incoming cash flows over six years. This project will have an immediate (t=0) cash outflow of \$100,000 (which might include machinery, and employee training costs). Other cash outflows for years 1-6 are expected to be \$5,000 per year. Cash inflows are expected to be \$30,000 per year for years 1-6. All cash flows are after-tax, and there are no cash flows expected after year 6. The required rate of return is 10%. The present value (PV) can be calculated for each year:

Year	Cashflow	Present Value
T=0	$\frac{-100,000}{(1+0.10)^0}$	-$100,000
T=1	$\frac{30,000-5,000}{(1+0.10)^1}$	$22,727
T=2	$\frac{30,000-5,000}{(1+0.10)^2}$	$20,661
T=3	$\frac{30,000-5,000}{(1+0.10)^3}$	$18,783
T=4	$\frac{30,000-5,000}{(1+0.10)^4}$	$17,075
T=5	$\frac{30,000-5,000}{(1+0.10)^5}$	$15,523
T=6	$\frac{30,000-5,000}{(1+0.10)^6}$	$14,112

The sum of all these present values is the net present value, which equals $8,881.52. Since the NPV is greater than zero, it would be better to invest in the project than to do nothing, and the corporation should invest in this project if there is no alternative with a higher NPV.

Return vs Risk Investment opportunities bring different returns at different level of risk. More risky opportunities yield higher return.

Example: An individual can invest into stocks or bonds. Stocks are supposed to bring 20% return but the investment is risky and this happens only with 50% probability. On the other hand investment into bonds is safe and yields 8% return with 95% probability. The expected return is as follows:

$$ER_B = 0.08 \times 0.95 = 0.076, \quad \text{i.e. } 7.6\%$$
$$ER_S = 0.2 \times 0.5 = 0.1, \quad \text{i.e. } 10\%$$

We see that expected return of investment into stocks is higher. However, this investment is risky and some investors would prefer bonds to stocks.

11 Edgeworth Diagram

Up until now we have generally considered the market for a single good in isolation. We have viewed the demand and supply functions for a good as depending on it's price alone, disregarding the prices of other goods. But in general the prices of other goods will affect people's demands and supplies for a particular good. Certainly the prices of substitutes and complements for a good will influence the demand for it, and, more subtly, the prices of goods that people sell will affect the amount of income they have and thereby influence much of other goods they will be able to buy.

Up until now we have been ignoring the effect of these other prices on the market equilibrium. When we discussed the equilibrium conditions in a particular market, we only looked at one part of the problem: how demand and supply were affected by the price of the particular good we were examining. This is called **partial equilibrium** analysis.

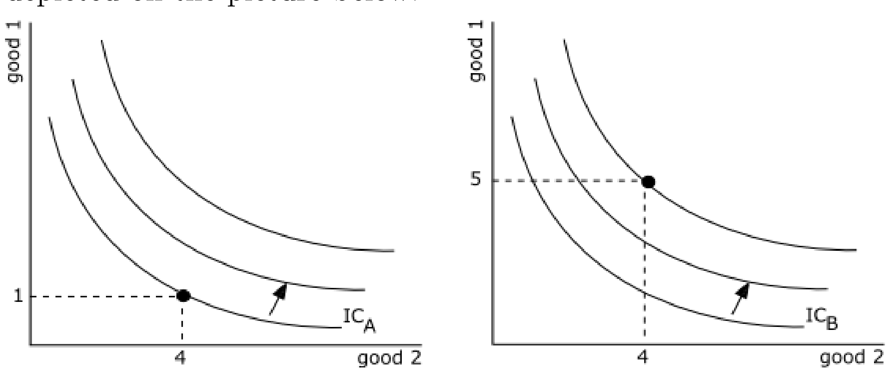
In this lecture we start with so called **general equilibrium** analysis - how demand and supply conditions interact in several markets to determine prices of many goods.

We will analyze a simple example with two people in the economy: A and B and two goods: 1 and 2. We denote A's consumption bundle (x_1^A, x_2^A), where x_1^A is A's consumption of the first good and x_2^A is a consumption of the second good. Similarly, (x_1^B, x_2^B) represents consumption bundle of consumer B. Furthermore, we denote $\omega_A = (\omega_1^A, \omega_2^A)$ an initial endowment bundle of consumer A and we denote initial endowment bundle of consumer B as $\omega_B = (\omega_1^B, \omega_2^B)$.

Example: Consider the following story from the Second World War. There are two prisoners of war in a German camp: British (consumer A) and French (consumer B). Both of them have a right to get some weekly amount of tea (good 1) and coffee (good 2). British prisoner has the endowment $\omega_A = (1, 4)$ and French prisoner, being privileged, has the endowment $\omega_B = (5, 4)$. The prisoners' preferences are given by the following utility functions:

$$u^A(x_1^A, x_2^A) = 2\ln x_1^A + x_2^A$$
$$u^B(x_1^B, x_2^B) = 4\ln x_1^B + x_2^B$$

The two prisoners are totally separated and the direct exchange is not possible. Their situation is depicted on the picture below.

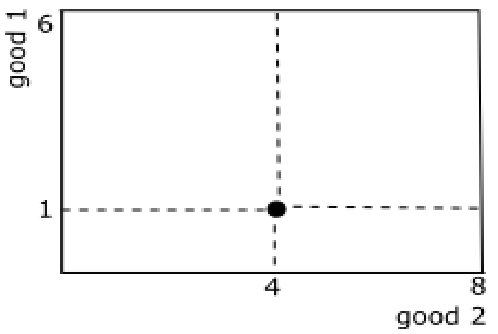


If there is no trade both consumers will consume their endowments. The question is whether both consumers can be better off in presence of trade. To analyze the case where consumers can trader two goods between themselves we use a convenient graphical tool called the Edgeworth box. The Edgeworth box provides a powerful way of graphically studying exchange and the role of markets.

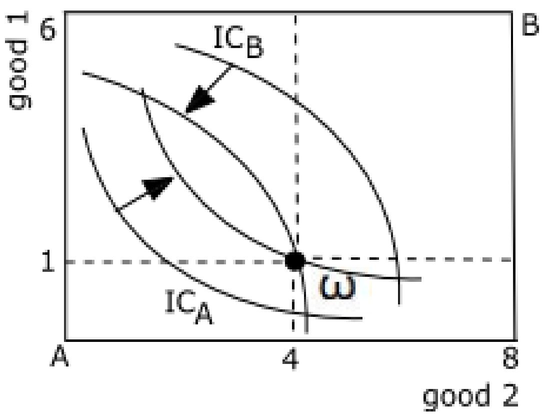
Edgeworth box, Pareto efficient (Pareto optimal) allocations:

The height of the Edgeworth box measures the total amount of good 1 in the economy (in our example 6 units) and the width measures the total amount of good 2 (in our example 8 units). Person A's consumption choices are measured from the lower left-hand corner while person B's choices are measured from the upper right.

Any point in the Edgeworth box indicates a particular distribution of the two goods among the two individuals. Any point in the box describes a possible combination of two goods that consumer A can hold. At the same time this point also indicates the amount of each good that B can hold. If there are 6 units of good 1 and 8 units of good 2 in the economy and if A holds e.g. (3,2) then B must be holding (3,6).

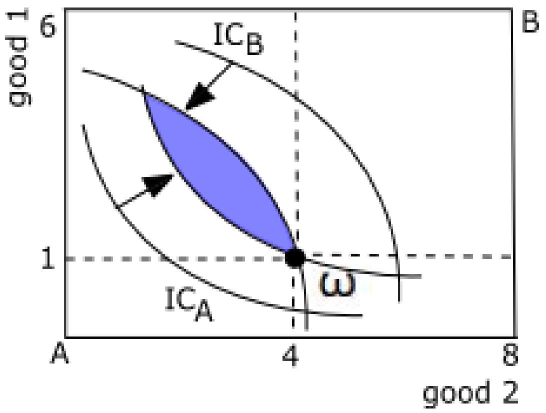


We can depict A's indifference curves in the usual manner, but B's indifference curves take a somewhat different form. To construct them we take a standard diagram for B's indifference curves and turn it upside down. This gives us B's indifference curves on the diagram. If we start at A's origin in the lower left-hand corner and move up and to the right, we will be moving to allocations that are more preferred by A. As we move down and to the left we will be moving do allocations that are more preferred by B.



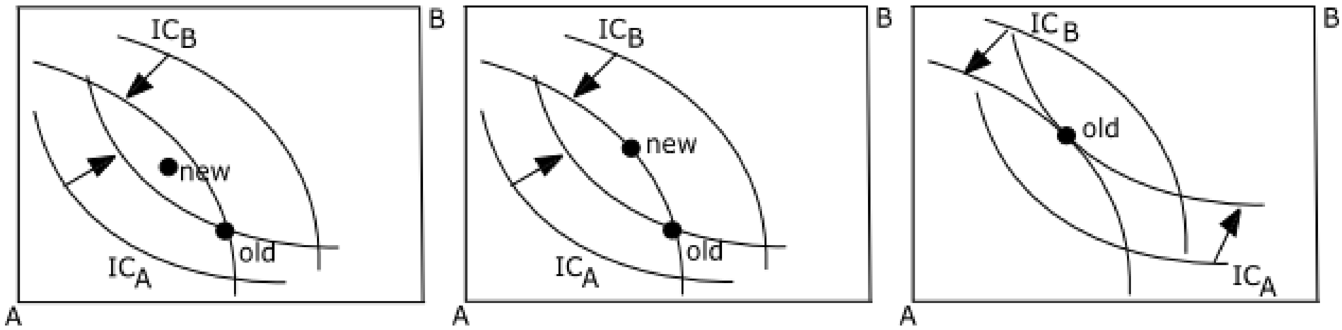
Now that we have both sets of preferences and endowments depicted can begin to analyze the question of how trade takes place. We start at the original endowment of goods, denoted by the point ω. Consider the indifference curves of A and B that pass through their allocation. The region where A is better off than at his endowment consist of all the bundles above her indifference curve through ω. The region where B is better off than at his endowment consists of all the allocations that are above-from his point of view-his indifference curve through ω. (This is below his indifference curve from our point of view).

Where is the region of the box where A and B are both made better off? Clearly it is in the intersection of these two regions. Presumably in the course of their negotiations two people involved will find some mutually advantageous trade (some trade that will move them to some point inside the blue lens-shaped area.)

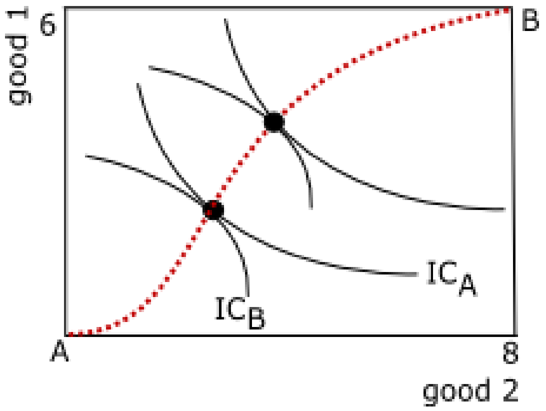


The picture above suggests that the original endowment point is not optimal in some sense. This is because two consumers can be both better off if they exchange the goods in such way that the new allocation lies in the shaded area on the picture above. This brings us to the concept of Pareto optimality. Given a set of alternative allocations of, say, goods or income for a set of individuals, a change from one allocation to another that can make at least one individual better off without making any other individual worse off is called a **Pareto improvement**. An allocation is **Pareto efficient** or **Pareto optimal** when no further Pareto improvements can be made. In other words, we call an allocation **Pareto optimal** when no change can make one consumer better off without making the other worse off.

Let's analyze Pareto optimality on the pictures below. On the left hand picture moving from *old* allocation to *new* makes both consumers better off and therefore the *old* allocation is not Pareto optimal. On the middle picture moving from *old* allocation to *new* makes consumer A better off without making B worse off (consumer B stays on the same indifference curve) and therefore the *old* allocation is not Pareto optimal. On the right hand picture moving from *old* allocation in any direction will make one of the consumers better off but inevitably one of the consumers worse off. There is no room for mutual improvement and hence the *old* allocation is Pareto optimal.



Three pictures above illustrate the fact that the distribution is Pareto optimal if and only if indifference curves are tangent at that point. At a Pareto efficient allocation, each person is on his highest possible indifference curve, given the indifference curve of the other person. Notice that at a Pareto efficient allocation, the marginal rate of substitution is the same for all consumers. The curve connecting such points is known as **contract curve** and is depicted on the picture below as a red dotted line.



What happens if the marginal rate of substitution is not the same for two consumers? Let's analyze the situation in our example at the point of endowment:

$$MRS^A = \frac{MU^A_{x_1}}{MU^A_{x_2}}|(1,4) = \frac{2}{x^A_1}|(1,4) = 2$$
$$MRS^B = \frac{MU^B_{x_1}}{MU^B_{x_2}}|(5,4) = \frac{4}{x^B_1}|(5,4) = \frac{4}{5}$$

The interpretation of these values of marginal rate of substitution is as follows:

Consumer A is willing to get 0.5 units of good 1 in exchange for giving up 1 unit of good 2.

Consumer B is willing to give up 5/4 units of good 1 in order to get 1 unit of good.

Consumer A asks less than consumer B is willing to pay and hence there is room for mutually beneficial trade.

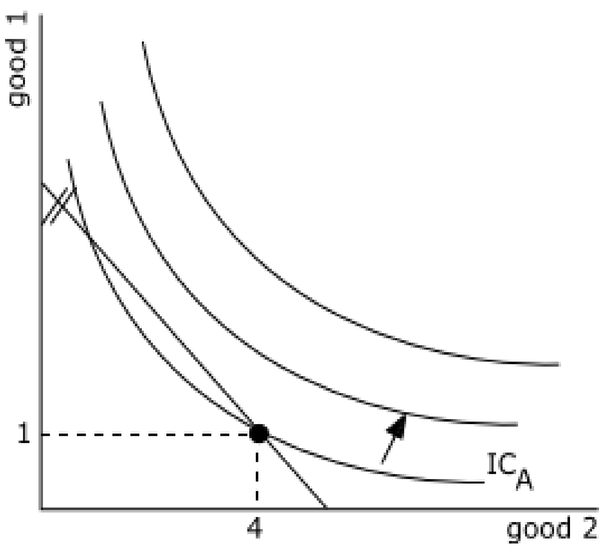
To summarize, a Pareto efficient allocation can be described as an allocation where:

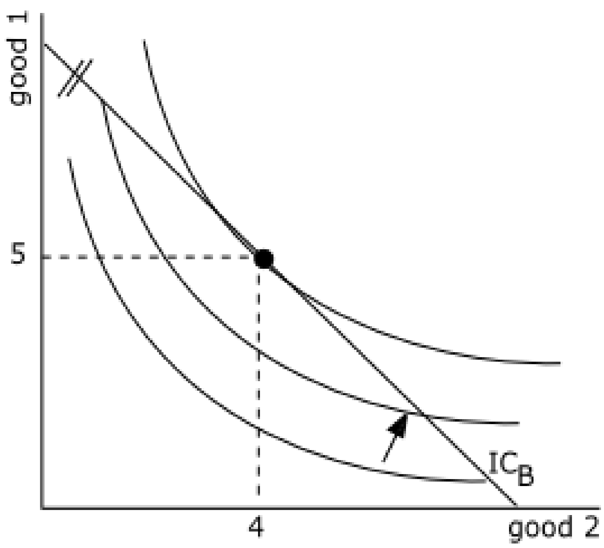
- There is no way to make all the people involved better off; or
- there is no way to make some individual better off without making someone else worse off; or
- all of the gains from trade have been exhausted; or
- there are no mutually advantageous trades to be made, and so on.

General equilibrium:

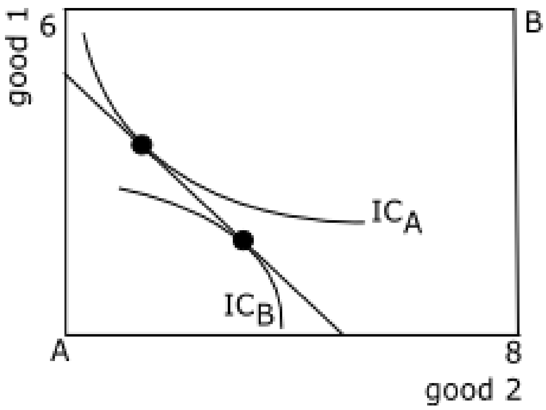
The equilibrium of the trading process described above-the set of Pareto efficient allocations-is very important, but it still leaves a lot of ambiguity about where the agents end up. The reason is that the trading process we have described is very general. Essentially we have only assumed that the two parties will move to some allocation where they are both made better off.

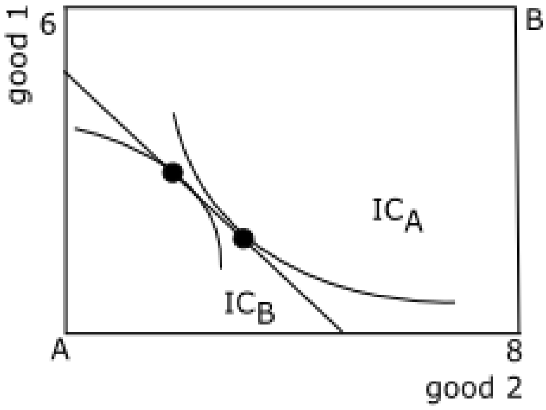
Now assume that p_1 and p_2 are prices of goods 1 and 2. Consumers A and B observe these prices and see how much their endowment is worth at the prices. Consequently consumers decide how much of each good he or she would want to buy at given prices. The situation is depicted on the picture below. Note that the slope of the budget constraint is the same for both consumers but it goes through different endowment points.



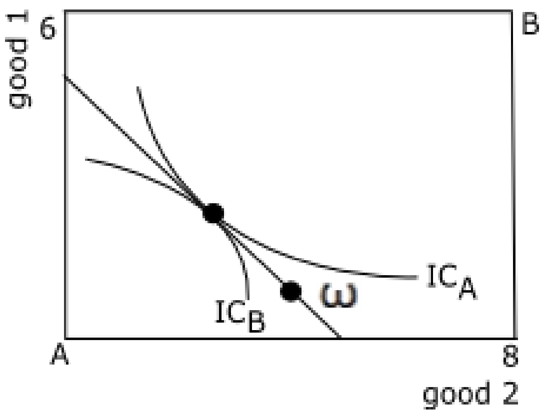


Now we get back to the Edgeworth box. Note that the budget lines of two consumers coincide into a single straight line. In any equilibrium the bundle consumed by consumer A and B has to be such that these two points coincide. See the picture below: on the left hand picture two consumers altogether consume more more units of good 1 than is available in the economy and not all units of good 2. On the right hand picture two consumers consume more units of good 2 than is available in the economy and not all units of good 1.





The analysis up to the point suggests that the market equilibrium has to be some point on a contract curve. In other words an equilibrium has to be Pareto optimal allocation. To reiterate if the equilibrium is not Pareto optimal allocation there is a way to make both consumers better off and hence the original situation can not be the equilibrium. The question is which Pareto optimal allocation in particular will be a result of the market interaction when two consumers can exchange two goods. This depends on initial endowment. In equilibrium the budget line has to go through the endowment point and indifference curves of both consumers have to be tangents to this budget line and also tangents one to another. This leads to a single possible combination of prices p_1 and p_2 that determine the slope of the budget line. This in turn gives us a single possible equilibrium. See the picture below.



After the graphical solution we find the prices algebraically in the following way. First, we solve the utility maximization problem of each consumer separately. In the second step we use so called market-clearing conditions to find the equilibrium prices.

1. We choose good 1 to be a numeraire, therefore $p_1 = 1$ and for simplicity we denote $p_2 = p$.

$$\text{Consumer A:} \quad \max_{\{x_1^A, x_2^A\}} 2\ln x_1^A + x_2^A$$
$$s.t.\ \ p_1 x_1^A + p_2 x_2^A = p_1 + 4p_2 \quad \Rightarrow \quad x_1^A + p x_2^A = 1 + 4p$$
$$\text{Consumer B:} \quad \max_{\{x_1^B, x_2^B\}} 4\ln x_1^B + x_2^B$$
$$s.t.\ \ p_1 x_1^B + p_2 x_2^B = 5p_1 + 4p_2 \quad \Rightarrow \quad x_1^B + p x_2^B = 5 + 4p$$

Now we plug budget constraints into the objective functions and take the first order conditions:

$$\text{Consumer A:} \quad \max_{x_2^A} 2\ln(1+4p-px_2^A)+x_2^A$$

$$\text{FOC:} \frac{2(-p)}{1+4p-px_2^A}+1=0 \Rightarrow x_2^A=\frac{2p+1}{p}$$

$$\text{Consumer B:} \quad \max_{x_2^B} 4\ln(5+4p-px_2^B)+x_2^B$$

$$\text{FOC:} \frac{4(-p)}{5+4p-px_2^B}+1=0 \Rightarrow x_2^B=\frac{5}{p}$$

Therefore the demand functions are $x_2^A=\frac{2p+1}{p}$ and $x_2^B=\frac{5}{p}$. And from budget constraints we get the demands $x_1^A=2p$ and $x_1^B=4p$.

2. Competitive equilibrium consists of equilibrium prices (only price p_2 needs to be determined since we set the price p_1 equal to 1) and allocations $\{x_1^A,x_2^A\},\{x_1^B,x_2^B\}$. In equilibrium both markets (market for good 1 and market for good 2) clear:

$$x_1^A+x_1^B=\omega_1^A+\omega_1^B \quad \Leftrightarrow \quad 2p+4p=6 \quad \Rightarrow \quad p=1$$

Here, we check if market for good 2 clears for price p=1 as well:

$$\frac{2p+1}{p}+\frac{5}{p}=8 \quad \Rightarrow \quad p=1$$

Hence, the competitive equilibrium is:

$$\{x_1^A,x_2^A\}=(2,3);\ \{x_1^B,x_2^B\}=(4,5);\ p_1=1;\ p_2=1.$$

Special forms of utility function: If two goods are substitutes or complement we will not find interior solution to our general equilibrium problem like in the previous example, we will have corner solution. On the left picture below goods are substitutes for both consumers and the set of Pareto optimal allocations is upper and left side of the Edgeworth box. On the right picture below goods are complements for consumer A and substitutes for consumer B. In this case the contract curve is the straight line.