9 The optimum of Oligopoly

During previous lectures we have investigated two important forms of market structure: pure competition, where there are typically many small competitors, and pure monopoly, where there is only one large firm in the market. However, much of the world lies between these two extremes. Often there are a number of competitors in the market, but not so many as to regard each of them as having a negligible effect on price. This is the situation known as **oligopoly**. Behavior of firms in case of oligopoly can be either non-cooperative (individual profit maximization) or cooperative (cartel). It is unreasonable to expect one grand model since many different behavior patterns can be observed in the real world. What we want is a guide to some of the possible patterns of behavior and some indication of what factors might be important in deciding when the various models are applicable.

9.1 Bertrand model of oligopoly

Bertrand oligopoly is an example of non-cooperative competition in prices. In Bertrand model firm chooses own price while taking the price of competition as given.

First let's start with a single firm on a market - Arkus. Arkus is facing an inverse demand function:

$$P = 60 - 0.6 * Q$$

Profit is maximized where marginal revenue equals marginal cost (equal to zero for simplicity). In our example the revenue is:

$$R = P * Q = (60 - 0.6 * Q)Q = 60Q - 0.6 * Q^2$$

and hence marginal revenue is:

$$MR = R' = 60 - 1.2 * Q$$

The profit maximizing (monopolistic) quantity is where marginal revenue equals zero which holds for quantity equal to 50 and corresponding profit maximizing price is equal to 30.

Then the second firm, Bona, comes to the market and chooses the price slightly lower in order to get the whole market (Bona can afford this because marginal cost is zero so any price is profitable). Arkus has no customers and hence makes zero profit so it's better to undercut Bona's price to get the whole market and make positive profit. This price war leads to both firms charging basically zero price and earning zero profit.

9.2 Cournot model of oligopoly

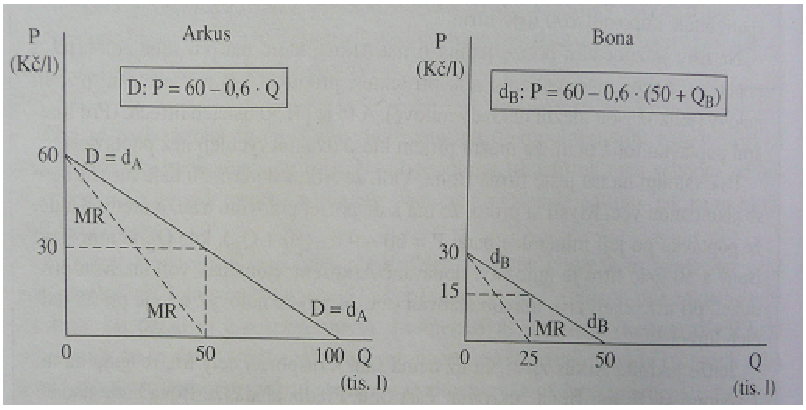
Cournot oligopoly is an example of non-cooperative competition in quantities. In Bertrand competition a firm makes decision about own price while taking the price of competition as given. In Cournot competition a firm chooses own quantity to be produced and takes the quantity produced by a competitor as given. First let start with a single firm on a market - Arkus. Arkus is facing an inverse demand function $P = 60 - 0.6 * Q$. Profit is maximized where marginal revenue equals marginal cost (equal to zero for simplicity) and profit maximizing (monopolistic) quantity is 50. This situation is depicted on the left picture below. Later the second firm, Bona, enters the market. Bona takes Arkus' production of 50 as given and hence the inverse demand function is:

$$P = 60 - 0.6(50 + Q_B)$$

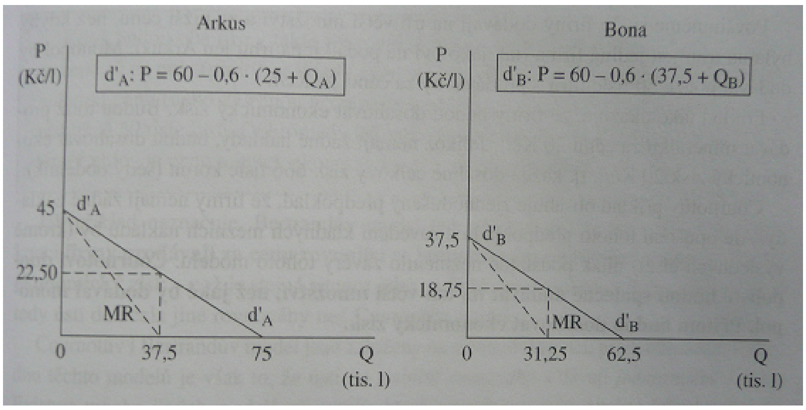
Now the revenues and marginal revenues are:

$$R = [60 - 0.6(50 + Q_B)]Q_B = [30 - 0.6Q_B]Q_B, \quad MR = 30 - 1.2Q_B$$

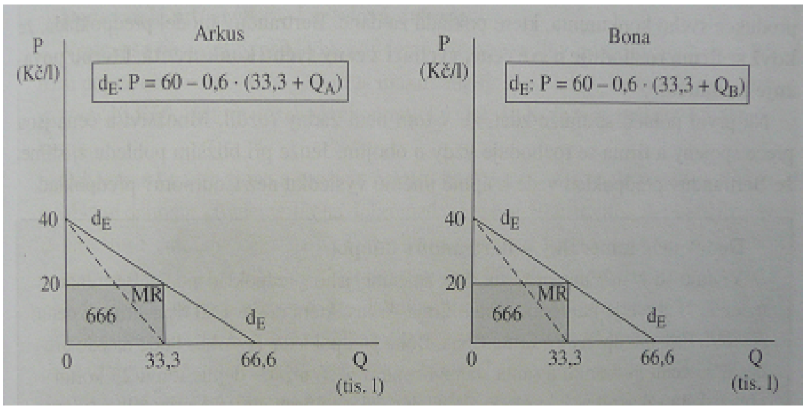
As a result (if we put marginal revenues equal to zero marginal cost) the profit maximizing quantity for Bona is 25. This situation is depicted on the right picture below.



After Bona enters the market and produces 25 the situation changes for Arkus. Now, Arkus takes Bona's production as given and hence the new demand function is $P = 60 - 0.6(25 + Q_A)$ and the new profit maximization quantity will be 37.5. And after this change the new profit maximizing quantity for Bona is 31.25 (look at the picture below).



This behavior will continue till two firms on the market reach equilibrium. Since both firms have zero marginal cost (are the same) the equilibrium will be symmetric, i.e. both firms will produce the same quantity and it will be equal to 33.3. (Note that if Arkus takes Bona's production of 33.3 as given then Arkus' profit maximizing quantity is 33.3 as well and vice versa).



To solve for Cournot outcome we solve maximization problem of the two firms separately:

$$\max_{Q_A} PQ_A = [60 - 0.6(Q_A + Q_B)]Q_A$$
$$\max_{Q_B} PQ_B = [60 - 0.6(Q_A + Q_B)]Q_B$$

FOC (first order conditions) give us:

$$60 - 1.2Q_A - 0.6Q_B = 0$$
$$60 - 1.2Q_B - 0.6Q_A = 0$$

Solving these two equations we get:

$$Q_A = Q_B = 33.\overline{3}$$

This situation can be analyzed using the concept of the game theory. For simplicity we assume that only two choices of quantity are possible. Each firm can either choose to produce 25 which in total is the monopoly level of production or, alternatively, each firm can choose Cournot level of production equal to 33.3. This situation can be represented in the table form below. The numbers in the table are profits of company A and B respectively.

A \ B	25 (monopoly quantity)	33.3 (Cournot quantity)
25	750,750	625,833
33.3	833,625	666,666

The Cournot outcome is in the lower right corner where both firms choose the level of output equal to 33.3 and their profit is 666. However, we see that if collusion is possible, the firms would do better to choose the output that maximizes total industry profits and then divide up the profits among themselves. The two firms can decide to sell the monopoly level of production for the monopoly price. This way they maximize their total profit. However, this arrangement is not stable because one firm could increase the output and get temporarily higher profit. That's why we often see that cartel agreements are broken.

Bertrand and Cournot model are both based on very simplified assumptions. The advantage is that they are simple and lead to a single equilibrium. In Bertrand model the prices are essentially equal to marginal cost and firms make zero profits. In Cournot model the prices are much higher and quantity sold is lower.

Note that if we compare oligopoly with monopoly then the level of production in oligopoly is higher and the prices are lower. The price will be higher than marginal cost and hence both firms will have a positive profit (shaded rectangle on the picture above).

9.3 Stackelberg model of oligopoly

When one firm decides about its choices for prices and quantities it may already know the choices made by the other firm. If one firm gets to set its price before the other firm, we call it the **price leader** and the other firm the **price follower**. Similarly, one firm may get to choose its quantity first, in which case it is a **quantity leader** and the other is a **quantity follower**. In the case of quantity leadership, one firm makes a choice before the other firm. This is called the **Stackelberg model**.

Stackelberg competition consists of two stages: First, the leader sets the quantity to be produced. Second, the follower observes the action of the leader and chooses the optimal level of production. So when the leader makes a decision about the quantity to be produced he anticipates followers reaction and hence chooses the quantity accordingly. Notice, that compared to Cournot competition the Stackelberg leader can never be worse off. (Leader can choose the Cournot quantity and then for the follower it is optimal to produce the same amount and we have Cournot outcome. So leader either chooses Cournot quantity or if he chooses a different level of production he must be better off otherwise he wouldn't do that).

To compute Stackelberg outcome we use similar approach as in case of Cournot competition but in this case the firms choose their level of output sequentially rather than simultaneously. In the first stage firm A chooses the level of output and than in the second stage firm B makes the decision. We solve this problem backwards. First we solve the second choice - optimal response of B to A's choice and then we solve for the optimal choice of A.

2^{nd} stage:

$$B: \max_{Q_B} PQ_B = [60 - 0.6(Q_A + Q_B)]Q_B$$

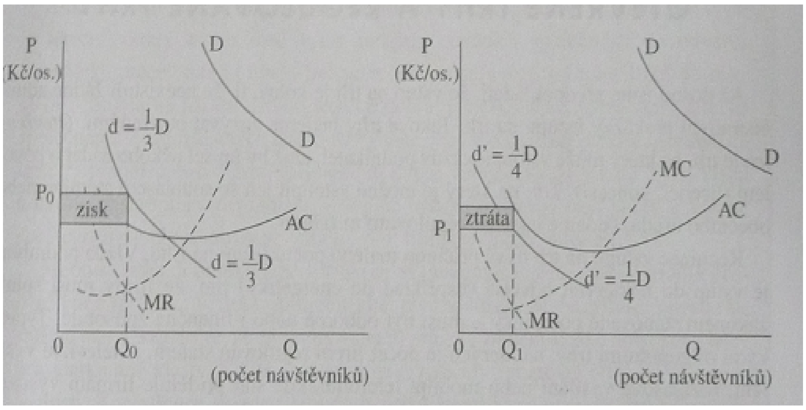
$$FOC: \; 60 - 1.2Q_A - 0.6Q_B = 0 \quad \Rightarrow \quad Q_B = 50 - \frac{Q_A}{2}$$

1^{st} stage:

$$A: \max_{Q_A} PQ_A = [60 - 0.6(Q_A + Q_B)]Q_A \text{ and A knows that } Q_B = 50 - \frac{Q_A}{2}$$

$$\text{Hence: } A: \max_{Q_A} \left[60 - 0.6\left(Q_A + 50 - \frac{Q_A}{2}\right)\right] Q_A = (30 - 0.3Q_A)Q_A$$
$$FOC: \quad 30 - 0.6Q_A = 0 \quad \Rightarrow \quad Q_A = 50$$
$$Q_B = 50 - \frac{Q_A}{2} = 50 - \frac{50}{2} = 25$$

Why do oligopolies exist? The reason is the relationship between the market demand and the optimal size of a firm. Look at the left picture below. If there are three firms on the market, each firm sells one third of market demand at a given price and all three firms make a positive profit. On the right picture we can see the situation when the fourth firm enters the market, each firms sells one fourth of the market demand and all firms make a negative profit till one of them leaves the market.



We have now examined several models of duopoly behavior: quantity leadership (Stackelberg), simultaneous quantity setting (Cournot), simultaneous price setting (Bertrand), and the collusive solution (cartel). How do they compare? In general, collusion results in the smallest industry output and the highest price. Bertrand equilibrium gives us the highest output and the lowest price. The other models give results that are in between these two extremes.

Summary:

- Output is greater with Cournot duopoly than monopoly, but lower than perfect competition.
- Price is lower with Cournot duopoly than monopoly, but not as low as with perfect competition.
- According to this model the firms have an incentive to form a cartel, effectively turning the Cournot model into a Monopoly. Cartels are usually illegal, so firms might instead tacitly collude using self-imposing strategies to reduce output which, ceteris paribus will raise the price and thus increase profits for all firms involved.

Bertrand versus Cournot

Although both models have similar assumptions, they have very different implications:

- Since the Bertrand model assumes that firms compete on price and not output quantity, it predicts that a duopoly is enough to push prices down to marginal cost level, meaning that a duopoly will result in perfect competition.
- Neither model is necessarily "better." The accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the industry situation.
- If capacity and output can be easily changed, Bertrand is a better model of duopoly competition. If output and capacity are difficult to adjust, then Cournot is generally a better model.
- Under some conditions the Cournot model can be recast as a two stage model, where in the first stage firms choose capacities, and in the second they compete in Bertrand fashion.

Stackelberg versus Cournot

- The Stackelberg and Cournot models are similar because in both, competition is on quantity.
- The first move gives the leader in Stackelberg a crucial advantage.
- There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.
- In Cournot competition, it is the simultaneity of the game (the imperfection of knowledge) that results in neither player (ceteris paribus) being at an advantage.