# 5 Profit maximization, Supply

We already described the technological possibilities now we analyze how the firm chooses the amount to produce so as to maximize its profits. Profits are defined as revenues minus cost. We assume that the firm faces fixed prices (is on a competitive market).

*Example:* A firm is on a competitive market, i.e. takes price of the output as given. Production function is given by $f(x_1, x_2) = x_1^{1/4} x_2^{1/4}$, prices of inputs are $w_1 = 4$, $w_2 = 4$ and price of output is p = 1.

- **Profit-maximization approach:** We maximize profit (revenues minus costs) of the firm.

$$\max_{\{x_1,x_2\}} py - w_1 x_1 - w_2 x_2 \quad \rightarrow \quad \max_{\{x_1,x_2\}} 1x_1^{1/4} x_2^{1/4} - 4x_1 - 4x_2$$

$$FOC[x_1] : \frac{x_2}{4(x_1 x_2)^{3/4}} - 4 = 0$$

$$FOC[x_2] : \frac{x_1}{4(x_1 x_2)^{3/4}} - 4 = 0$$

  Solving these two equations with two unknowns gives:

$$x_1 = x_2 = \frac{1}{256}$$

- **Cost-minimization approach:** Consists of two stages: First, we find minimum cost for producing any given level of output $y$. Second, we find optimal value of output $y$.

  First stage: find minimum cost for arbitrary level of output $y$:

$$\min_{\{x_1,x_2\}} w_1 x_1 + w_2 x_2 \quad \rightarrow \quad \min_{\{x_1,x_2\}} 4x_1 + 4x_2$$

$$\text{such that } \; x_1^{1/4} x_2^{1/4} = y \quad \Rightarrow \quad x_2 = \frac{y^4}{x_1}$$

$$\min_{x_1} 4x_1 + 4\frac{y^4}{x_1}$$

$$\text{FOC: } \; 4 - 4\frac{y^4}{x_1^2} = 0 \quad \Rightarrow \quad x_1 = y^2 \; \text{ and } \; x_2 = y^2$$

  So in this example, our cost function is:

$$c(y) = 4x_1 + 4x_2 = 4y^2 + 4y^2 = 8y^2$$

  Second stage: find optimal level of output $y$:

$$\max_y py - c(y) \quad \rightarrow \quad \max_y y - 8y^2$$

$$\text{FOC: } \; 1 - 16y = 0 \quad \Rightarrow \quad y = \frac{1}{16}$$

$$x_1 = x_2 = y^2 = \frac{1}{256}$$

**Profit maximization ↔ Cost minimization.** If a firm is maximizing profits and if it chooses to supply some output $y$, then it must be minimizing the cost of producing $y$. If this were not so, then there would be some cheaper way of producing y units of output, which would mean that the firm was not maximizing profits in the first place. This simple observation turns out to be quite useful in examining firm behavior.

## 5.1 Profit maximization in short-run:

In short-run the amount of at least one inputs is fixed. In long-run all inputs can be changed.

$$\max_{x_1} pf(x_1, \overline{x_2}) - w_1 x_1 - w_2 \overline{x_2}$$

where:

- $p$ - price of output
- $f(x_1, \overline{x_2})$ - production function
- $x_1, \overline{x_2}$ - inputs, $x_2$ is in short-run fixed at the level $\overline{x_2}$
- $w_1, w_2$ - prices of inputs $x_1, x_2$

For profit maximizing quantity the first order condition has to hold:

$$p \frac{\partial f(x_1, \overline{x_2})}{\partial x_1} = w_1 \quad \text{or} \quad pMP_1 = w_1$$

In other words, **the value of the marginal product of a factor should equal its price.**

In order to understand this rule, think about the decision to employ a little more of factor 1. As you add a little more of it, $\Delta x_1$, you produce $\Delta y = MP_1 \Delta x_1$ more output that is worth $pMP_1 \Delta x_1$. But this marginal output costs $w_1 \Delta x_1$ to produce. If the value of marginal product exceeds its cost, then profits can be increased by increasing input 1. If the value of marginal product is less than its cost, then profits can be increased by decreasing the level of input 1.

Now we analyze profit maximization problem graphically. Profit of the firm is given by:

$$\pi = py - w_1 x_1 - w_2 \overline{x_2}$$

Rearranging terms we get:

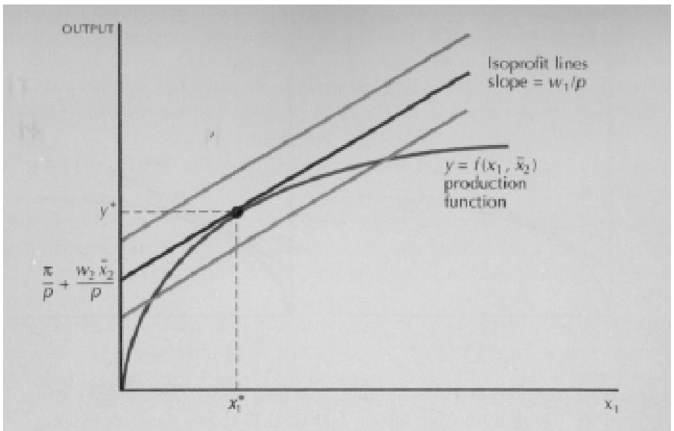
$$y = \frac{\pi}{p} + \frac{w_2}{p} \overline{x_2} + \frac{w_1}{p} x_1$$

The last equation describes **isoprofit lines** - all combinations of inputs and outputs that give a constant level of profit, $\pi$.

As $\pi$ varies we get a set of parallel straight lines each with a slope of $w_1/p$ and each having a vertical intercept of $\pi/p + w_2\overline{x_2}/p$, which measures the profits plus the fixed costs of the firm.

The profit-maximization problem is then to find the point on the production function that has the highest associated isoprofit line. Such a point is illustrated on the following picture. It's characterized by a tangency point - the slope of production function ($MP_1$) equals the slope of the isoprofit line ($w_1/p$). Hence,

$$MP_1 = \frac{w_1}{p}$$




**Profit maximization.** The firm chooses the input and output combination that lies on the highest isoprofit line. In this case the profit-maximizing point is $(x_1^*, y^*)$.

## 5.2 Profit maximization in long-run:

the level of all inputs can be chosen.

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

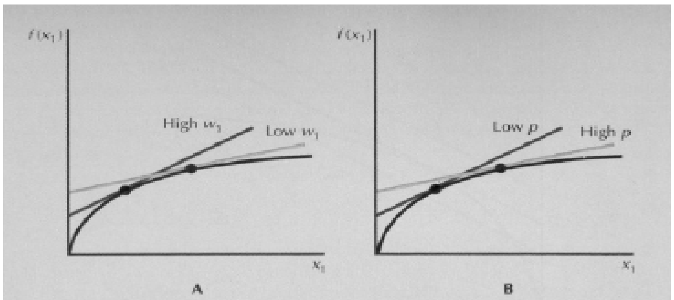
In this case, optimality conditions are:

$$pMP_1(x_1^*, x_2^*) = w_1$$
$$pMP_2(x_1^*, x_2^*) = w_2$$

The last two equations give solution to the profit-optimization problem - expressions for $x_1$ and $x_2$ - **factor demand curves**. These curves measure the relationship between the price of a factor and the profit-maximizing choice of the factor. **The inverse factor demand curve** measures the same relationship but from a different point of view. It measures what the factor prices must be for some given quantity of inputs to be demanded.

**Comparative statics:** we analyze how a firm's choice of inputs and outputs varies as the prices of inputs and outputs vary. How does the optimal choice of factor 1 vary as we vary its factor price $w_1$? Increasing $w_1$ will make the isoprofit line steeper. When the isoprofit line is steeper, the tangency must occur further to the left. Thus the optimal level of factor 1 must decrease. This simply means that as the price of factor 1 increases, the demand for factor 1 must decrease: factor demand curves must slope downward.

Similarly, if the output price decreases the isoprofit line must become steeper and the profit-maximizing choice of factor 1 will decrease.




**Comparative statics.** Panel A shows that increasing $w_1$ will reduce the demand for factor 1. Panel B shows that increasing the price of output will increase the demand for factor 1 and therefore increase the supply of output.

Finally, we can ask what will happen if the price of factor 2 changes? Because this is a short-run analysis, changing the price of factor 2 will not change the firm's choice of factor 2-in the short run, the level of factor 2 is fixed at $\overline{x_2}$. Changing the price of factor 2 has no effect on the slope of the isoprofit line. Thus the optimal choice of factor 1 will not change, nor will the supply of output.

**Profit maximization and returns to scale:** There is an important relationship between competitive profit maximization and returns to scale. Suppose that the firm's production function exhibits constant returns to scale and that it is making positive profits in equilibrium. Then consider what would happen if it doubled the level of its input usage. According to the constant returns to scale hypothesis, it would double its output level and its profits would also double. But this contradicts the assumption that its original choice was profit maximizing. This argument shows that the only reasonable long-run level of profits for a competitive firm that has constant returns to scale at all levels of output is a zero level of profits. (Of course if a firm has negative profits in the long run, it should go out of business.)

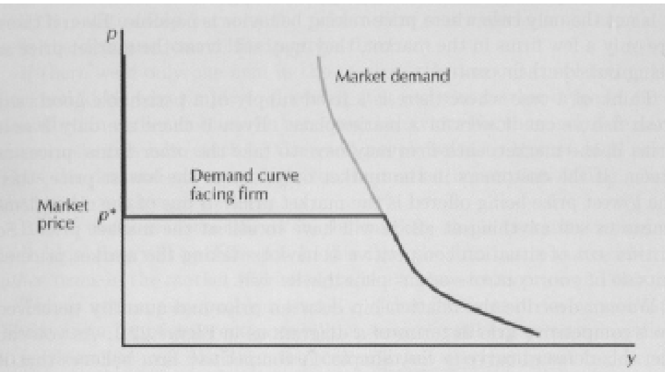
Firms exist to maximize profits so how can it be that they can only get zero profits in the long run? Think about what would happen to a firm that did try to expand indefinitely. Three things might occur.

- First, the firm could get so large that it could not really operate effectively. This is just saying that the firm really doesn't have constant returns to scale at all levels of output. Eventually, due to coordination problems, it might enter a region of decreasing returns to scale.
- Second, the firm might get so large that it would totally dominate the market for its product. In this case there is no reason for it to behave competitively-to take the price of output as given. Instead, it would make sense for such a firm to try to use its size to influence the market price. The model of competitive profit maximization would no longer be a sensible way for the firm to behave, since it would effectively have no competitors. We'll investigate more appropriate models of firm behavior in this situation when we discuss monopoly.
- Third, if one firm can make positive profits with a constant returns to scale technology, so can any other firm with access to the same technology. If one firm wants to expand its output, so would other firms. But if all firms expand their outputs, this will certainly push down

## 5.3 Firm supply

If the firm could choose quantity of production and the price freely it would choose arbitrarily large price and quantity. However, firm faces technological (production function) and market constraints (firm can set whatever price it wants, but it can only sell as much as people are willing to buy). We call the relationship between the price a firm sets and the amount that it sells the **demand curve** facing the firm. In this lecture we analyze a simple market environment - **pure competition**. In pure competition the price of a good is independent of firm's behavior. Prices are given and firms are **price takers** and only choose the level of quantity. This is plausible assumption if we imagine market with a very large number of small firms (wheat farmers in the USA, hot dog sellers on Vaclavske namesti, ...).

A competitive firm faces the following demand. If the price charged is higher than the market price the firm sells nothing. It the firm sells for the market price it can sell whatever amount it wants and if the price is lower than the market price it will get the entire market.




**The demand curve facing a competitive firm.** The firm's demand is horizontal at the market price. At higher prices, the firm sells nothing, and below the market price it faces the entire market demand curve.

**Supply decision of a competitive firm:**

Let us use the facts we have discovered about cost curves to figure out the supply curve of a competitive firm. Thus the maximization problem facing a competitive firm is

$$\max_y \{\text{revenues - costs}\} = \max_y py - c(y)$$

A profit maximizing firm chooses a level of output such that marginal revenue (extra revenue gained by one more unit of output) equals marginal (extra) cost. If this condition did not hold, the firm could always increase its profits by changing its level of output.

In the case of a competitive firm, marginal revenue is simply the price. To see this, ask how much extra revenue a competitive firm gets when it increases its output by $\Delta y$. We have

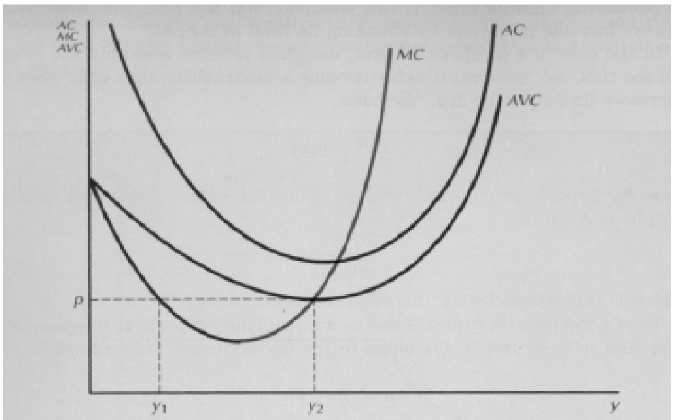
$$\Delta R = p\Delta y$$
$$\frac{\Delta R}{\Delta y} = p$$

Thus a competitive firm will choose a level of output $y$ where the marginal cost that it faces at $y$ is just equal to the market price:

$$p = MC(y)$$

Whatever the level of the market price $p$, the firm will choose a level of output $y$ where $p = MC(y)$. Thus the marginal cost curve of competitive firm is precisely its supply curve. Or put another way, the market price is precisely marginal cost-as long as each firm is producing at its profit-maximizing level.

Graphical representation of the optimality condition that $p = MC$ is depicted on the graph below. Note that this condition holds for two different levels of output $y_1$ and $y_2$. But only $y_2$ is profit maximizing level of output. Marginal cost curve is decreasing on the part where $p = y_1$. The level of output where marginal cost curve is decreasing can never be optimal, because the revenue of producing one more unit of output would be higher than its cost.




**Marginal cost and supply.** Although there are two levels of output where price equals marginal cost, the profit-maximizing quantity supplied can lie only on the upward-sloping part of the marginal cost curve.

**Shut-down condition:** Sometimes a firm can be better off to stop production. This is the case if the price is so low that the it does not even cover the variable cost. Only the portions of the marginal cost curve that lie above the average variable cost curve are possible points on the supply curve. If a point where price equals marginal cost is beneath the average variable cost curve, the firm would optimally choose to produce zero units of output.

Since price equals marginal cost at each point on the supply curve, the market price must be a measure of marginal cost for every firm operating in the industry. A firm that produces a lot of output and a firm that produces only a little output must have the same marginal cost, if they are both maximizing profits. The total cost of production of each firm can be very different, but the marginal cost of production must be the same.

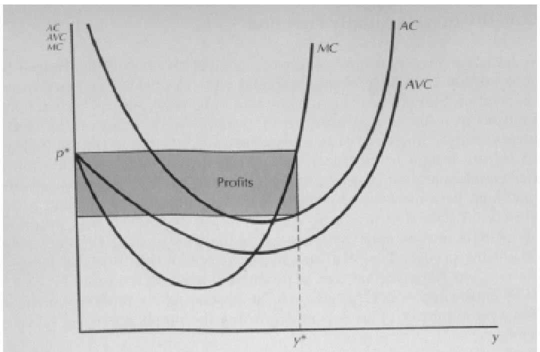
Given the market price we can now compute the optimal operating position for the firm from the condition that $p = MC(y)$. Given the optimal operating position we can compute the profits of the firm. Total revenue is given by

$$TR = p^* y^*$$

and total cost is given by

$$c(y) = y\frac{c(y)}{y} = yAC(y)$$

The profit is given by revenues minus costs and it is the shaded area on the picture below.




**Profits.** Profits are the difference between total revenue and total costs, as shown by the colored rectangle.

If a point where price equals marginal cost is beneath the average variable cost curve, the profit would be negative and the firm would optimally choose to produce zero units of output. If this point is on the average variable cost curve, the profit of the firm is zero.