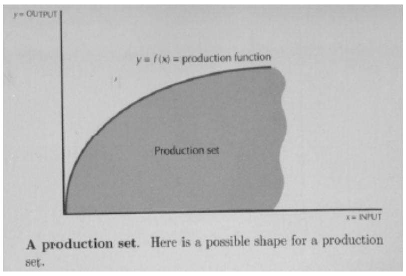
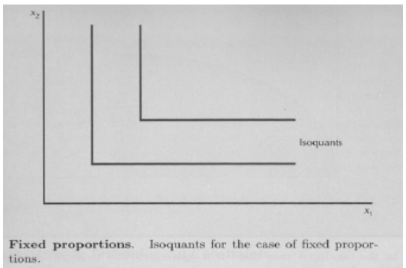
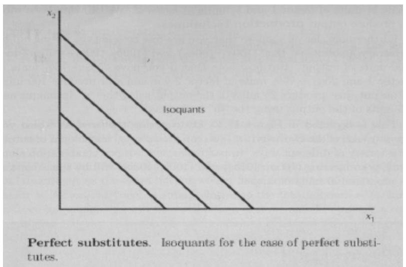
4 Optimum of the firm, Cost minimization

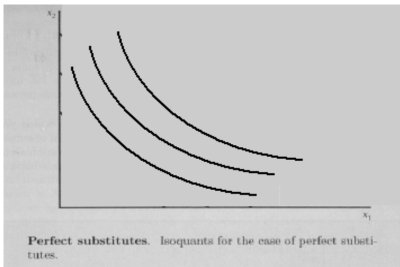
Firm's production is constrained by technology. Technology can by described by production set, or production function. In case of only one input (only one factor of production) we can illustrate production set and production function in the following way.



Usually, we assume that a firm uses two inputs. In this case we use **isoquants** to depict production relations. An isoquant is the set of all possible combinations of inputs 1 and 2 that are just sufficient to produce a given amount of output.







$f\{x_1, x_2\} = \min\{x_1, x_2\}$ $\quad$ $f\{x_1, x_2\} = x_1 + x_2$ $\quad$ $f\{x_1, x_2\} = Ax_1^a x_2^b$

Properties of technology: we assume that the production function is:

- monotonic: if we increase the amount of at least one of the inputs, we will get at least as much output as originally (free disposal)
- convex: if we can produce output y in two different ways - using (x_1, x_2) or (z_1, z_2) units of inputs, then their weighted average will produce at least y units of output

Let's assume that we are operating at some point (x_1, x_2) and that we consider to use a little bit more of factor 1 while keeping factor 2 fixed at level x_2. The additional output that we get is called **marginal product of factor 1**. We typically expect that the marginal product of a factor will diminish as we get more and more of that factor. This is called the law of diminishing marginal product.

Now again assume that we are operating at some point (x_1, x_2), and that we consider giving up a little bit of factor 1 and using just enough more of factor 2 to produce the same amount of output y. How much extra of factor 2, Δx_2, do we need if we are going to give up a little bit of factor 1, Δx_1? This is just the slope of the isoquant and we refer to if as the **technical rate of substitution**

$$TRS_{(x_1,x_2)} = \frac{\Delta x_1}{\Delta x_2} = -\frac{MP_1(x_1,x_2)}{MP_2(x_1,x_2)}$$

Returns to scale: What happens if instead of increasing the amount of 1 input we increase the amount of all the inputs in the same proportions? If, for example, we use twice as much of each input, how much output will we get? The most likely outcome is that we will get twice as much output (a firm should be able to replicate what it is doing right now). This is called **constant returns to scale**. Mathematically:

$$f(2x_1, 2x_2) = 2f(x_1, x_2)$$

Or generally,

$$f(tx_1, tx_2) = tf(x_1, x_2)$$

Note: it is possible to have constant returns to scale and diminishing marginal product at the same time. For example if $f(x_1, x_2) = \sqrt{x_1 x_2}$. Then $f(2x_1, 2x_2) = \sqrt{2x_1 2x_2} = 2\sqrt{x_1 x_2}$. So this function has constant returns to scale. At the same time we observe diminishing marginal product:

$$MP_1 = \frac{\partial f(x_1,x_2)}{\partial x_1} = \frac{\partial \sqrt{(x_1 x_2)}}{\partial x_1} = \frac{x_2}{2\sqrt{(x_1 x_2)}}$$

This is decreasing function in x_1; i.e. if we keep increasing x_1 output will increase as well but slower and slower.

Similarly, we can have **increasing returns to scale**:

$$f(tx_1, tx_2) > tf(x_1, x_2)$$

(Ex: If we double the diameter of an oil pipe, we use twice as much material but are able to transport 4 times as much oil.)

and **decreasing returns to scale**:

$$f(tx_1, tx_2) < tf(x_1, x_2)$$

(Ex: Usually a short run case, where we can not double all the inputs. Otherwise something goes wrong, because it was possible just to replicate what a firm was doing and get constant returns to scale.)

4.1 Cost minimization

Suppose that we have two factors of production,x_1 and x_2, and their prices w_1 and w_2, and that we want to figure out the cheapest way to produce a given level of output, y. For a given amounts of two factors the cost of production is

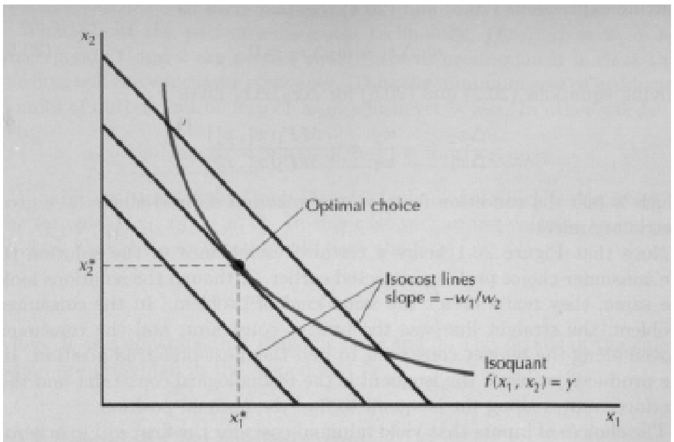
$$w_1x_1 + w_2x_2$$

Suppose that we want to plot all the combinations of inputs x_1 and x_2 that have some given level of cost, C. We can write this as

$$w_1x_1 + w_2x_2 = C \quad \Leftrightarrow \quad x_2 = \frac{C}{w_2} - \frac{w_1}{w_2}x_1$$

This is a straight line with a slope of $-\frac{w_1}{w_2}$ and a vertical intercept of $\frac{C}{w_2}$. As we let C change we get many different **isocost lines** - every point has the same cost and higher isocost lines are associated with higher costs.

So the cost-minimization problem of the firm is to find such point on given isoquant that lies on the lowest isocost line.



Cost minimization. The choice of factors that minimize production costs can be determined by finding the point on the isoquant that has the lowest associated isocost curve.

Note that if isoquant is a smooth curve then the cost-minimizing point will be characterized by a tangency condition: the slope of the isoquant must be equal to the slope of the isocost curve. Or the technical rate of substitution must equal the factor price ratio:

$$-\frac{MP_1}{MP_2} = TRS = -\frac{w_1}{w_2}$$

Mathematically, we denote this solution $c(w_1, w_2, y)$, where this function is called **cost function** and it measures the minimum cost of producing a given level of of output y at given factor prices w_1 and w_2.

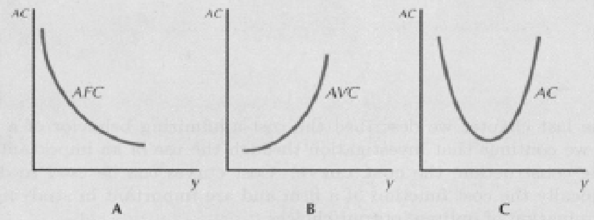
4.2 Cost curves

Types of costs:

- Fixed costs - independent of the level of output
- Variable costs - expenses that change in proportion to the amount of output produced
- Total costs = fixed costs + variable costs

Average costs: costs per unit of output. The average variable cost function measures the variable cost per unit of output, average fixed cost function measures the fixed costs per unit of output.

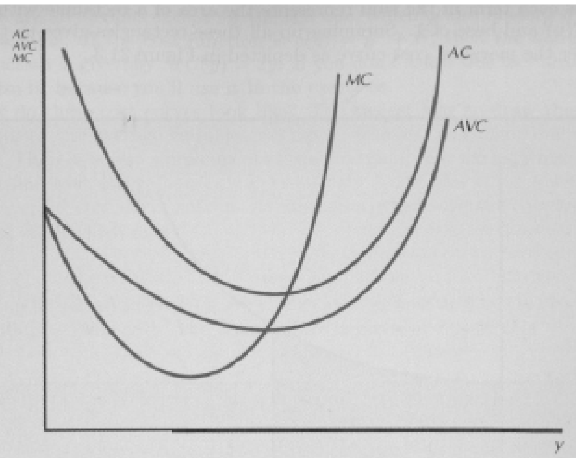
$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} = AVC(y) + AFC(y)$$



Construction of the average cost curve. (A) The average fixed costs decrease as output is increased. (B) The average variable costs eventually increase as output is increased. (C) The combination of these two effects produces a U-shaped average cost curve.

Marginal costs:

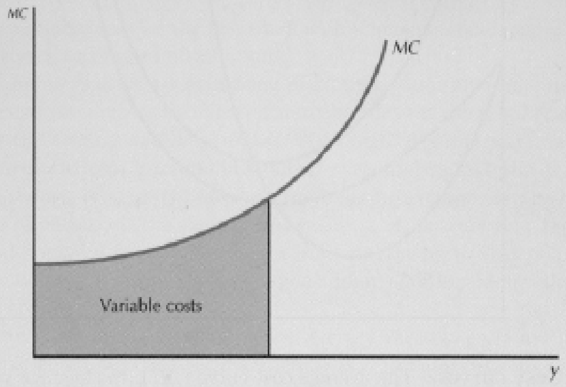
There is one more cost curve of interest: the marginal cost curve. The marginal cost curve measures the change in costs for a given change in output. That is, at any given level of output y, we can ask how costs will change if we change output by some amount Δy. Often we think of Δy as being one unit of output, so that marginal cost indicates the change in our costs if we consider producing one more discrete unit of output.



Cost curves. The average cost curve (AC), the average variable cost curve (AVC), and the marginal cost curve (MC).

Note that MC curve intercept AC and AVC curve in their minimum. If AC curve decreases it must be the case that marginal cost is lower than average cost. If AC increases that marginal cost must by higher than average cost. Thus we know that the marginal cost curve must lie below the average cost curve to the left of its minimum point and above it to the right. This implies that the marginal cost curve must intersect the average variable cost curve at its minimum point.

Also note that the area below the marginal cost curve up to point of output y gives variable cost of production. Marginal cost curve measures the cost of each additional unit of output. If we add up the cost of producing each unit of output we get total variable cost.



Marginal cost and variable costs. The area under the marginal cost curve gives the variable costs.