11 Edgeworth Diagram

Up until now we have generally considered the market for a single good in isolation. We have viewed the demand and supply functions for a good as depending on it's price alone, disregarding the prices of other goods. But in general the prices of other goods will affect people's demands and supplies for a particular good. Certainly the prices of substitutes and complements for a good will influence the demand for it, and, more subtly, the prices of goods that people sell will affect the amount of income they have and thereby influence much of other goods they will be able to buy.

Up until now we have been ignoring the effect of these other prices on the market equilibrium. When we discussed the equilibrium conditions in a particular market, we only looked at one part of the problem: how demand and supply were affected by the price of the particular good we were examining. This is called **partial equilibrium** analysis.

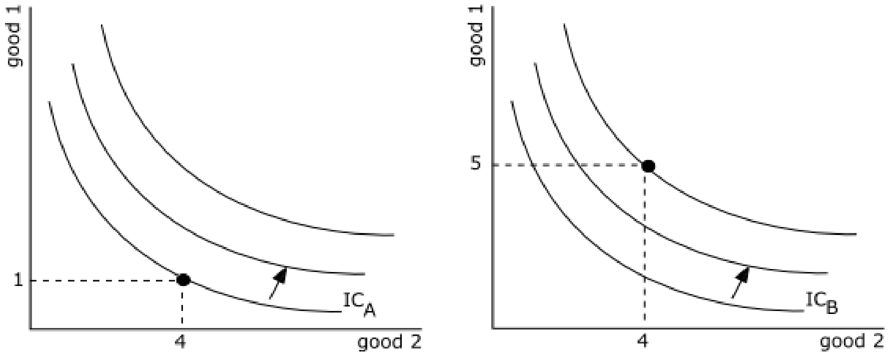
In this lecture we start with so called **general equilibrium** analysis - how demand and supply conditions interact in several markets to determine prices of many goods.

We will analyze a simple example with two people in the economy: A and B and two goods: 1 and 2. We denote A's consumption bundle (x_1^A, x_2^A), where x_1^A is A's consumption of the first good and x_2^A is a consumption of the second good. Similarly, (x_1^B, x_2^B) represents consumption bundle of consumer B. Furthermore, we denote $\omega_A = (\omega_1^A, \omega_2^A)$ an initial endowment bundle of consumer A and we denote initial endowment bundle of consumer B as $\omega_B = (\omega_1^B, \omega_2^B)$.

Example: Consider the following story from the Second World War. There are two prisoners of war in a German camp: British (consumer A) and French (consumer B). Both of them have a right to get some weekly amount of tea (good 1) and coffee (good 2). British prisoner has the endowment $\omega_A = (1, 4)$ and French prisoner, being privileged, has the endowment $\omega_B = (5, 4)$. The prisoners' preferences are given by the following utility functions:

$$u^A(x_1^A, x_2^A) = 2 \ln x_1^A + x_2^A$$
$$u^B(x_1^B, x_2^B) = 4 \ln x_1^B + x_2^B$$

The two prisoners are totally separated and the direct exchange is not possible. Their situation is depicted on the picture below.

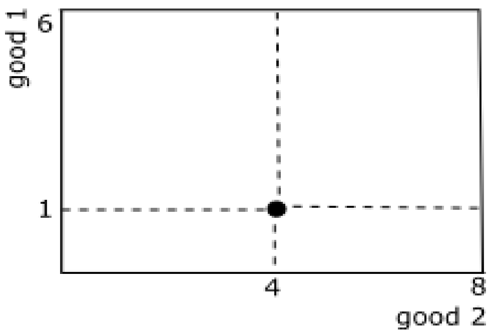


If there is no trade both consumers will consume their endowments. The question is whether both consumers can be better off in presence of trade. To analyze the case where consumers can trader two goods between themselves we use a convenient graphical tool called the Edgeworth box. The Edgeworth box provides a powerful way of graphically studying exchange and the role of markets.

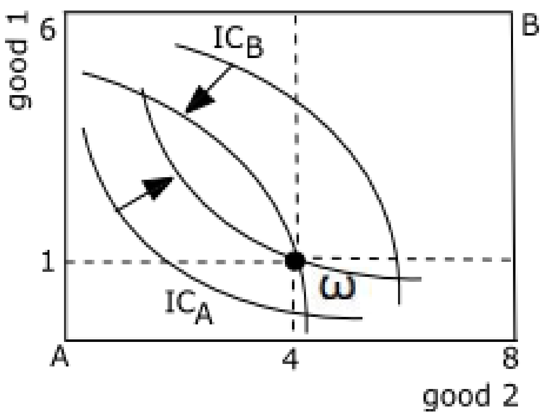
Edgeworth box, Pareto efficient (Pareto optimal) allocations:

The height of the Edgeworth box measures the total amount of good 1 in the economy (in our example 6 units) and the width measures the total amount of good 2 (in our example 8 units). Person A's consumption choices are measured from the lower left-hand corner while person B's choices are measured from the upper right.

Any point in the Edgeworth box indicates a particular distribution of the two goods among the two individuals. Any point in the box describes a possible combination of two goods that consumer A can hold. At the same time this point also indicates the amount of each good that B can hold. If there are 6 units of good 1 and 8 units of good 2 in the economy and if A holds e.g. (3,2) then B must be holding (3,6).

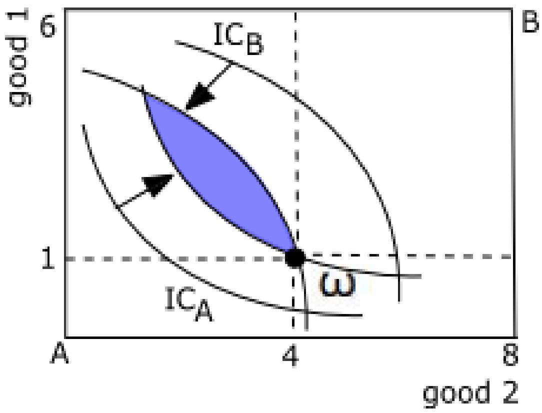


We can depict A's indifference curves in the usual manner, but B's indifference curves take a somewhat different form. To construct them we take a standard diagram for B's indifference curves and turn it upside down. This gives us B's indifference curves on the diagram. If we start at A's origin in the lower left-hand corner and move up and to the right, we will be moving to allocations that are more preferred by A. As we move down and to the left we will be moving do allocations that are more preferred by B.



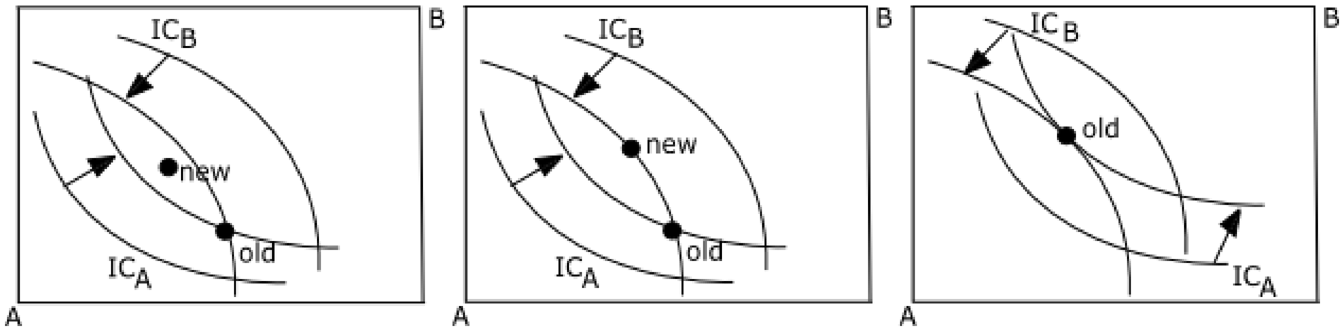
Now that we have both sets of preferences and endowments depicted can begin to analyze the question of how trade takes place. We start at the original endowment of goods, denoted by the point ω. Consider the indifference curves of A and B that pass through their allocation. The region where A is better off than at his endowment consist of all the bundles above her indifference curve through ω. The region where B is better off than at his endowment consists of all the allocations that are above-from his point of view-his indifference curve through ω. (This is below his indifference curve from our point of view).

Where is the region of the box where A and B are both made better off? Clearly it is in the intersection of these two regions. Presumably in the course of their negotiations two people involved will find some mutually advantageous trade (some trade that will move them to some point inside the blue lens-shaped area.)

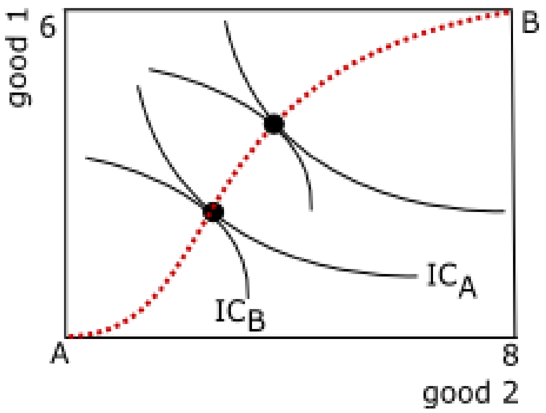


The picture above suggests that the original endowment point is not optimal in some sense. This is because two consumers can be both better off if they exchange the goods in such way that the new allocation lies in the shaded area on the picture above. This brings us to the concept of Pareto optimality. Given a set of alternative allocations of, say, goods or income for a set of individuals, a change from one allocation to another that can make at least one individual better off without making any other individual worse off is called a **Pareto improvement**. An allocation is **Pareto efficient** or **Pareto optimal** when no further Pareto improvements can be made. In other words, we call an allocation **Pareto optimal** when no change can make one consumer better off without making the other worse off.

Let's analyze Pareto optimality on the pictures below. On the left hand picture moving from *old* allocation to *new* makes both consumers better off and therefore the *old* allocation is not Pareto optimal. On the middle picture moving from *old* allocation to *new* makes consumer A better off without making B worse off (consumer B stays on the same indifference curve) and therefore the *old* allocation is not Pareto optimal. On the right hand picture moving from *old* allocation in any direction will make one of the consumers better off but inevitably one of the consumers worse off. There is no room for mutual improvement and hence the *old* allocation is Pareto optimal.



Three pictures above illustrate the fact that the distribution is Pareto optimal if and only if indifference curves are tangent at that point. At a Pareto efficient allocation, each person is on his highest possible indifference curve, given the indifference curve of the other person. Notice that at a Pareto efficient allocation, the marginal rate of substitution is the same for all consumers. The curve connecting such points is known as **contract curve** and is depicted on the picture below as a red dotted line.



What happens if the marginal rate of substitution is not the same for two consumers? Let's analyze the situation in our example at the point of endowment:

$$MRS^A = \frac{MU^A_{x_1}}{MU^A_{x_2}}|(1,4) = \frac{2}{x^A_1}|(1,4) = 2$$
$$MRS^B = \frac{MU^B_{x_1}}{MU^B_{x_2}}|(5,4) = \frac{4}{x^B_1}|(5,4) = \frac{4}{5}$$

The interpretation of these values of marginal rate of substitution is as follows:

Consumer A is willing to get 0.5 units of good 1 in exchange for giving up 1 unit of good 2.

Consumer B is willing to give up 5/4 units of good 1 in order to get 1 unit of good.

Consumer A asks less than consumer B is willing to pay and hence there is room for mutually beneficial trade.

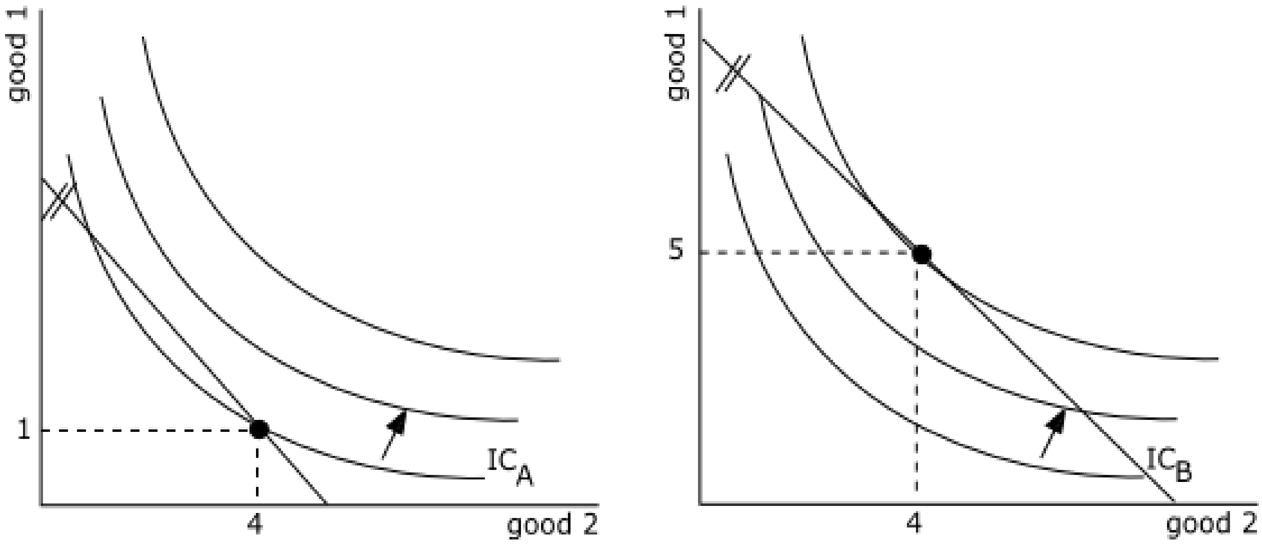
To summarize, a Pareto efficient allocation can be described as an allocation where:

- There is no way to make all the people involved better off; or
- there is no way to make some individual better off without making someone else worse off; or
- all of the gains from trade have been exhausted; or
- there are no mutually advantageous trades to be made, and so on.

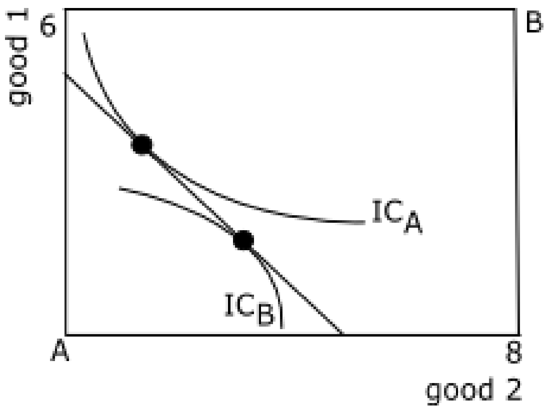
General equilibrium:

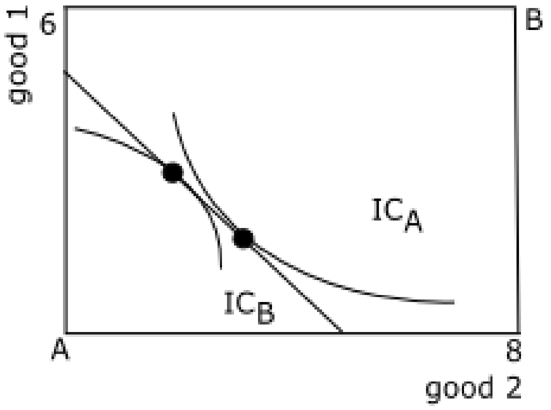
The equilibrium of the trading process described above-the set of Pareto efficient allocations-is very important, but it still leaves a lot of ambiguity about where the agents end up. The reason is that the trading process we have described is very general. Essentially we have only assumed that the two parties will move to some allocation where they are both made better off.

Now assume that p_1 and p_2 are prices of goods 1 and 2. Consumers A and B observe these prices and see how much their endowment is worth at the prices. Consequently consumers decide how much of each good he or she would want to buy at given prices. The situation is depicted on the picture below. Note that the slope of the budget constraint is the same for both consumers but it goes through different endowment points.

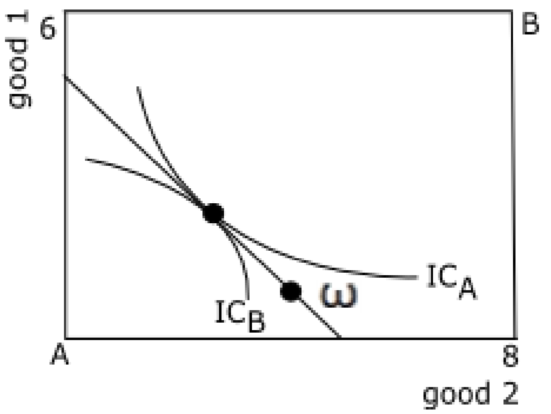


Now we get back to the Edgeworth box. Note that the budget lines of two consumers coincide into a single straight line. In any equilibrium the bundle consumed by consumer A and B has to be such that these two points coincide. See the picture below: on the left hand picture two consumers altogether consume more more units of good 1 than is available in the economy and not all units of good 2. On the right hand picture two consumers consume more units of good 2 than is available in the economy and not all units of good 1.





The analysis up to the point suggests that the market equilibrium has to be some point on a contract curve. In other words an equilibrium has to be Pareto optimal allocation. To reiterate if the equilibrium is not Pareto optimal allocation there is a way to make both consumers better off and hence the original situation can not be the equilibrium. The question is which Pareto optimal allocation in particular will be a result of the market interaction when two consumers can exchange two goods. This depends on initial endowment. In equilibrium the budget line has to go through the endowment point and indifference curves of both consumers have to be tangents to this budget line and also tangents one to another. This leads to a single possible combination of prices p_1 and p_2 that determine the slope of the budget line. This in turn gives us a single possible equilibrium. See the picture below.



After the graphical solution we find the prices algebraically in the following way. First, we solve the utility maximization problem of each consumer separately. In the second step we use so called market-clearing conditions to find the equilibrium prices.

1. We choose good 1 to be a numeraire, therefore $p_1 = 1$ and for simplicity we denote $p_2 = p$.

$$\text{Consumer A:} \quad \max_{\{x_1^A, x_2^A\}} 2\ln x_1^A + x_2^A$$
$$s.t. \;\; p_1 x_1^A + p_2 x_2^A = p_1 + 4p_2 \quad \Rightarrow \quad x_1^A + p x_2^A = 1 + 4p$$
$$\text{Consumer B:} \quad \max_{\{x_1^B, x_2^B\}} 4\ln x_1^B + x_2^B$$
$$s.t. \;\; p_1 x_1^B + p_2 x_2^B = 5p_1 + 4p_2 \quad \Rightarrow \quad x_1^B + p x_2^B = 5 + 4p$$

Now we plug budget constraints into the objective functions and take the first order conditions:

$$\text{Consumer A:} \quad \max_{x_2^A} 2\ln(1+4p-px_2^A)+x_2^A$$
$$\text{FOC:} \frac{2(-p)}{1+4p-px_2^A}+1=0 \Rightarrow x_2^A=\frac{2p+1}{p}$$
$$\text{Consumer B:} \quad \max_{x_2^B} 4\ln(5+4p-px_2^B)+x_2^B$$
$$\text{FOC:} \frac{4(-p)}{5+4p-px_2^B}+1=0 \Rightarrow x_2^B=\frac{5}{p}$$

Therefore the demand functions are $x_2^A=\frac{2p+1}{p}$ and $x_2^B=\frac{5}{p}$. And from budget constraints we get the demands $x_1^A=2p$ and $x_1^B=4p$.

2. Competitive equilibrium consists of equilibrium prices (only price p_2 needs to be determined since we set the price p_1 equal to 1) and allocations $\{x_1^A, x_2^A\}, \{x_1^B, x_2^B\}$. In equilibrium both markets (market for good 1 and market for good 2) clear:

$$x_1^A+x_1^B=\omega_1^A+\omega_1^B \quad \Leftrightarrow \quad 2p+4p=6 \quad \Rightarrow \quad p=1$$

Here, we check if market for good 2 clears for price p=1 as well:

$$\frac{2p+1}{p}+\frac{5}{p}=8 \quad \Rightarrow \quad p=1$$

Hence, the competitive equilibrium is:

$$\{x_1^A, x_2^A\}=(2,3);\ \{x_1^B, x_2^B\}=(4,5);\ p_1=1;\ p_2=1.$$

Special forms of utility function: If two goods are substitutes or complement we will not find interior solution to our general equilibrium problem like in the previous example, we will have corner solution. On the left picture below goods are substitutes for both consumers and the set of Pareto optimal allocations is upper and left side of the Edgeworth box. On the right picture below goods are complements for consumer A and substitutes for consumer B. In this case the contract curve is the straight line.