## 9 D/S of/for Labor

### 9.1 Demand for Labor

Demand for labor depends on the price of labor, price of output and production function. In optimum a firm employs so many units of labor (number of workers) so that the value of marginal product of labor equals the wage. Look at the following example:

| L | TP | MP | P (MR) | VMP | w (MC) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 30 | 20 | 4000 | 8000 | 60000 |
| 2 | 50 | 16 | 4000 | 64000 | 60000 |
| 3 | 66 | 16 | 4000 |  |  |
| 4 | 76 | 10 | 4000 | 40000 | 60000 |
| 5 | 84 | 4 | 4000 | 16000 | 60000 |

From the data in the table it follows that this firm should hire three employees. It is so because hiring third employee will increase the profit by 64000 and the wage of the worker is lower - 60 000. If the firm hires one more worker the fourth worker would bring profit of 40000 but his wage would have to be higher -60000 . So profit maximizing firm would hire three workers. If wage decreases below 40000 it would be optimal to hire four workers. If the wage increases above 64 000 it would be better to hire only two workers.
In other words, the firm chooses to hire so many workers that the value of marginal product of labor equals wage:

$$
V M P_{L}=M P_{L} \times P=w
$$

This situation is illustrated on the picture below. If the wage is $w_{0}$ that the optimal choice for a firm is to employ $L_{0}$ workers. The revenue is a shaded rectangle on the left picture below.



If the wage increases the optimal level of labor decreases as the worker are more expensive to hire. The demand for labor is actually identical with $V M P_{L}$ curve. But only up to the point where wage becomes to high. If the wage is too high firm will stop hiring any workers because the wage is higher than average revenue from a unit of labor which means that the firm is losing money. In this case it's better to stop production. Look at the right picture above. This is the case if wage is higher than maximal level of $V A P_{L}$, i.e. the point where $V M P_{L}$ crosses $V A P_{L}$.

Up to now we analyzed short-run demand for labor. In short-run all factors of production apart from labor are fixed. Now we will analyze the demand for labor in long-run. In long-run all factors of production can be changed. The intuition is the same - if wage increases a firm will hire less workers. The difference between short- and long-run is the motives for this change. In short run decreasing demand for labor is caused be decreasing revenue from marginal product of labor. In long-run decreasing demand for labor is caused by substitution and production effect (similar to substitution and income effect in consumer optimization problem). This situation is depicted on the picture below.


### 9.2 Supply of Labor

## Individual supply of labor is the decision between leisure and consumption:

We discussed the choice between two goods. Similarly, we can illustrate the choice between leisure and consumption. Leisure can be considered as a normal good. Loosely speaking the price of leisure is forgone earnings, i.e. money that consumer could earn if he spent time working instead of enjoying leisure time. The optimal choice of consumption and leisure is a combination where the indifference curve touches the budget line. This choice depends on the wage. If wage increases we can observe substitution effect (leisure becomes more expensive relative to consumption and should be substituted by working time and hence higher consumption) and income effect (we assume that both "consumption" and "leisure" are normal goods $\Rightarrow$ higher income leads to higher consumption of both goods and hence more leisure). The new optimal choice depends on which of the two effects is stronger:


Market equilibrium minimum wage and taxes: The equilibrium wage on the market is determined by the point where the supply and demand for labor intercept. Changes in equilibrium can be caused by changes in demand or supply. If the demand for production increases the demand for labor increases as well and as a result the equilibrium wage increases as well. Look at the left hand side of the picture.


If the wages in neighbor countries increase significantly and it is simple to work abroad or if the wages in similar industries increase the supply of labor curve moves upward and as a result the equilibrium wage increases. see the right hand side picture above.

Now we will illustrate the effect of income tax on equilibrium wage and number of workers hired. In the Czech Republic the income tax is $15-32 \%$. The situation is depicted on the picture below. With no taxes the equilibrium employment rate is $L_{E}$. After the income tax is introduced the equilibrium employment decreases to $L_{A}$.


Now we analyze the effect of unions. Imagine that unions negotiate the wage increase in a particular industry from $w_{0}$ to $w_{1}$. Firms are forced to pay more to their workers and as a result the equilibrium employment decreases (left picture below). Those workers who no longer have a job in a given industry will go to other industries which means that the supply curve shifts to the right and as a result the equilibrium wage in the second industry goes down (right picture).


The minimum wage also has an impact on the level of employment. If the minimum wage is introduced and it is higher than the equilibrium wage than as a result supply of labor is higher than demand and the unemployment is present.


### 9.3 D/S of/for Capital

Similarly to decision about the demand for labor a firm will use the level of capital such that the revenue from marginal product of capital equals its price.

$$
V M P_{K}=v
$$

where $v=R+D, R$ is the forgone interest and $D$ is depreciation or

$$
v=r P+d P
$$

where $P$ is the price of the capital (machine), $r$ is interest rate, and $d$ is depreciation rate.for labor.

As the interest rate increases the demand function for capital decreases and supply increases. The equilibrium interest rate is given by their intercept.


The picture above illustrates the situation when the investment environment improves, i.e. there are better investment opportunities. This means that firms want to rent more capital and as a result the the demand curve shifts upwards. The new equilibrium interest rate is higher.

Nominal vs Real interest rate: we have to distinguish between nominal interest rate which measures change in monetary value of capital and real interest rate which measure change in real value of capital.

In other words, the nominal interest rate is the amount, in money terms, of interest payable. For example, suppose a household deposits $\$ 100$ with a bank for 1 year and they receive interest of $\$ 10$. At the end of the year their balance is $\$ 110$. In this case, the nominal interest rate is $10 \%$ per annum.

The real interest rate, which measures the purchasing power of interest receipts, is calculated by adjusting the nominal rate charged to take inflation into account. If inflation in the economy has been $10 \%$ in the year, then the $\$ 110$ in the account at the end of the year buys the same amount as the $\$ 100$ did a year ago. The real interest rate, in this case, is zero.

The relationship between nominal and real interest rate is:

$$
r_{N}=r_{R}+i_{e}
$$

where $r_{N}$ is nominal interest rate, $r_{R}$ is real interest rate, and $i_{e}$ is expected inflation.

Choice of Investment Projects: there are usually several investment possibilities available. How do we decide where to invest free capital? We choose the project with the highest value. The value of project is the present value of all future returns where future returns are discounted to the present date.

Net present value, NPV, is an indicator of how much value an investment or project adds to the value of the firm. In financial theory, if there is a choice between two mutually exclusive alternatives, the one yielding the higher NPV should be selected.

Example: We consider two investment projects. Amount of money necessary to be invested and returns are summarized below and the interest rate is $10 \%$ :

|  | $1^{\text {st }}$ year | $2^{\text {nd }}$ year | $3^{\text {rd }}$ year |
| :---: | :---: | :---: | :---: |
| A | -100000 | 80000 | 50000 |
| B | -110000 | 60000 | 90000 |

$$
\begin{aligned}
& N P V_{A}=-100000+\frac{80000}{1+0.1}+\frac{50000}{(1+0.1)^{2}}=14050 \\
& N P V_{B}=-110000+\frac{60000}{1+0.1}+\frac{90000}{(1+0.1)^{2}}=18925
\end{aligned}
$$

Since the net present value of project $B$ is higher we should choose project $B$.

Return vs Risk Investment opportunities bring different returns at different level of risk. More risky opportunities yield higher return.

Example: An individual can invest into stocks or bonds. Stocks are supposed to bring $20 \%$ return but the investment is risky and this happens only with $50 \%$ probability. On the other hand investment into bonds is safe and yields $8 \%$ return with $95 \%$ probability. The expected return is as follows:

$$
\begin{aligned}
& E R_{B}=0.08 \times 0.95=0.076, \text { i.e. } 7.6 \% \\
& E R_{S}=0.2 \times 0.5=0.1, \quad \text { i.e. } 10 \%
\end{aligned}
$$

We see that expected return of investment into stocks is higher. However, this investment is risky and some investors would prefer bonds to stocks.

## Intertemporal choice (choice between current and future consumption):

So far we only studied static choices, but life is full of intertemporal choices (should I study for my test today or tomorrow; should I save or should I consume now, school, cigarettes, alcohol). When modeling intertemporal choice, economists treat one physical good consumed at two different times as two different goods.
We consider in this lecture the optimal allocation decision through time. In particular, we examine the optimal allocation of income to consumption through time. This is important as people often receive the income through time in a way that does not correspond to their preferred consumption stream through time. The individual needs to rearrange his or her income stream. This is achieved by borrowing and saving - through the use of capital markets. We assume here perfect capital markets, by which is meant that the individual can borrow and save as much as he or she wants at a constant and given rate of interest, which we shall denote by $r$. If the rate of interest is $10 \%$ then $\mathrm{r}=.1$; if the rate of interest is $20 \%$ then $\mathrm{r}=.2$; and so on. To keep our analysis simple we assume a two period world. Each individual gets current income $I_{a}$ and expected future income $I_{b}$ and we denote current consumption $C_{a}$ and future consumption $C_{b}$. (For simplicity we assume a single consumption good and assume a price of 1 in both periods.) It may be the case that the individual is happy to currently consume his or her income $I_{a}$ and to consume his or her income $I_{b}$ in future. However, the individual may prefer to rearrange his or her consumption by borrowing or lending. If $r$ is zero the possibilities are obvious: the maximum he or she could consume now
is $I_{a}+I_{b}$ with zero consumption in future and the maximum he or she could consume in future is the same. More generally the choice of $C_{a}$ and $C_{b}$ must satisfy the budget constraint:

$$
C_{a}+C_{b}=I_{a}+I_{b}
$$

If there is a positive rate of interest things are a little more complicated. If the individual wanted to consume nothing now then he or she could save the income $I_{a}$, investing it at the rate of interest $r$, earning interest $r I_{a}$ and thus in future having

$$
I_{a}(1+r)+I_{b}
$$

Alternatively if he or she wanted to consume nothing in future, then now he or she could spend his current income plus what he or she could borrow on the strength of being able to pay back in future.

$$
I_{a}+I_{b} /(1+r)
$$



Generally, the future value of the consumption stream must equal the future value of the income stream:

$$
C_{a}(1+r)+C_{b}=I_{a}(1+r)+I_{b}
$$

Alternatively, the present value of the consumption stream must equal the present value of the income stream:

$$
C_{a}+C_{b} /(1+r)=I_{a}+I_{b} /(1+r)
$$

Change of interest rate $r$ : changes the slope of budget constraint. Substitution effect (current consumption is more expensive) $\Rightarrow$ higher savings. Income effect (increase in future income) $\Rightarrow$ higher current consumption. The total effect depends on the shape of indifference curves, i.e. on whether substitution effect is stronger than income effect or vice versa.

