3 Risk, Uncertainty

So far we assumed that people's choices are not uncertain - once they decide how to spend their income, they get what they want. However, very often this is not true and people makes choices that involve incomplete information (unknown quality of used car, future value of investment,...) These events have not certain outcome but *expected outcome (expected value.)*

Example: Suppose that a consumer currently has \$10 of wealth and is contemplating a gamble that gives him a 50 percent probability of winning \$5 and a 50 percent probability of losing \$5. His wealth will therefore be random: he has a 50 percent probability of ending up with \$5 and a 50 percent probability of ending up with \$15. The *expected value* of his wealth is:

$$\frac{1}{2} * \$5 + \frac{1}{2} * \$15 = \$10$$

Generally,

$$R_E = \sum \pi_i R_i,$$

where R_i are possible outcomes and π_i are their probabilities.

Fair gamble (game)

A friend of yours offers you to play the following game: "Bet 100CZK and we will flip the coin. If we get a Head you will get additional 50CZK if we get a Tail you will lose 100CZK that you bet." Will you accept this kind of game? The **expected profit** of this game is

$$0.5 \cdot 50 + 0.5 \cdot (-100) = -25$$

The expected profit is negative and therefore you should refuse this game.

Instead, you offer your friend a following game: "Bet 110CZK and we will flip the coin. If we get a Tail you will get additional 90CZK if we get a Head you will lose 110CZK that you bet." Will your friend accept? His expected profit is

$$0.5 \cdot 90 + 0.5 \cdot (-110) = -10$$

The expected profit is negative and therefore your friend should refuse this game.

The only possible game that you both will agree to play is as follows: "Bet 100CZK and we will flip the coin. If we get a Head you will get additional 100CZK if we get a Tail you will lose 100CZK that you bet." In this game your expected profit as well as the expected profit of your friend is 0CZK and we refer to it as a **fair game**.

Small and large bets

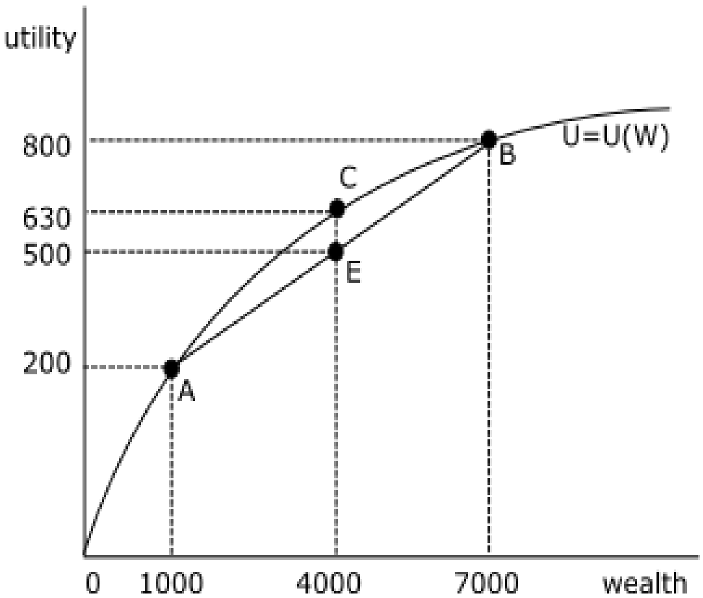
Let's assume two fair games. First: Bet 10CZK and in case of Head you get additional 10CZK; in case of Tail you lose the 10CZK that you bet. Second: Bet 1000CZK and in case of Head you get additional 1000CZK; in case of Tail you lose the 1000CZK that you bet.

Both these games are fair games. In both cases the expected profit is 0CZK. Which one would you chose? Most of the people would go for the first game. In fact, many people would not want to play the fair games at all. This is because so far we only looked at expected value of the games above. But individuals look at **expected utility** rather than expected value. In the next part we will analyze the concept of expected utility.

Assume the following example: Mat has a wealth of 4000CZK. He is trying to decide whether to bet 3000CZK on betting on a flip of coin. If he bets and wins his wealth will be 7000CZK, if he loses he will end up having 1000CZK. Mat is deciding whether to bet or not. Not betting is a safe option with the wealth 4000CZK. Betting is risky with zero expected profit and hence Mat's expected value is also 4000CZK. So the expected values are the same but there is a difference in expected utilities. On the picture below we can see Mat's concave utility function. The expected utility of not playing the game is 630. Expected utility of participating in a bet is

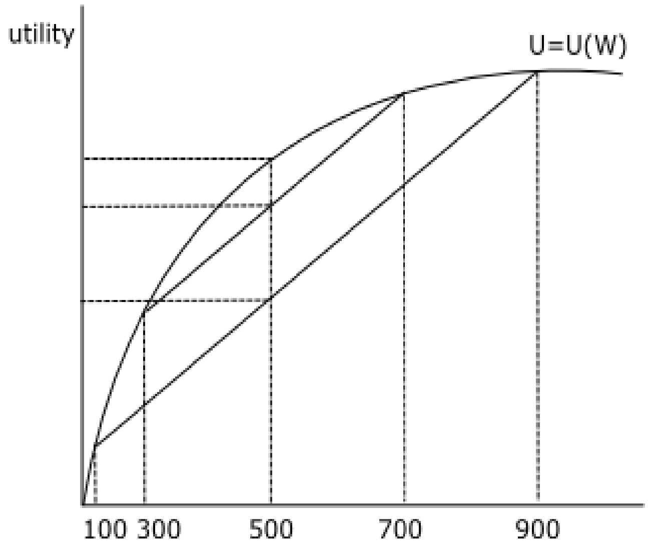
$$0.5 \cdot 200 + 0.5 \cdot 800 = 500$$

The expected utility of not betting the money is higher and therefore Mat decides not to participate in this game.



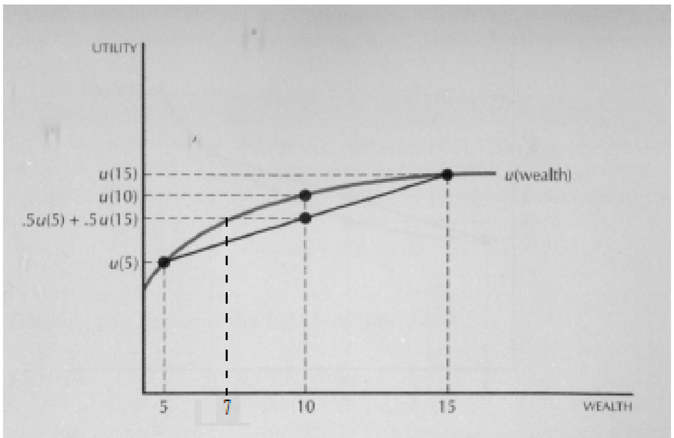
When there are several alternatives we could think that the individual should choose the alternative with the highest expected value. Is this always the case? Imagine a lottery where with 50% probability you win \$90 and with 50% probability you win \$110. The expected value is \$100. Now imagine a lottery where with 99% probability you win \$1 and with 1% probability you win \$10001. The expected value is \$101. So you should choose the second lottery. However, most of people would prefer the first lottery to the second one. The reason is that people do not compare expected values, they compare expected utilities and in general people are risk averse and the second lottery is much riskier than the first one.

Now, let's get back to preferring small stakes to large - look at the picture below. We illustrated two games (lotteries) with the same expected utility (500). But sage lottery involves betting winning/losing 200 and the risky lottery involves winning/losing 400. While the expected value is the same in both cased, the expected utility is higher in safe lottery than in risky lottery.



In the first example at the beginning of this lecture the *expected utility* is:

$$\frac{1}{2}u(\$5) + \frac{1}{2}u(\$15)$$



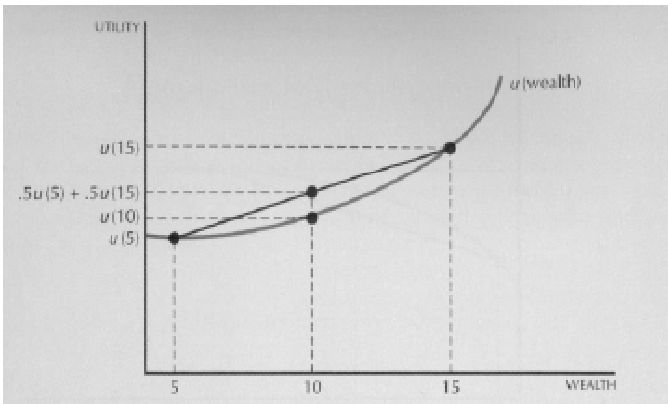
Risk aversion. For a risk-averse consumer the utility of the expected value of wealth, $u(10)$, is greater than the expected utility of wealth, $.5u(5) + .5u(15)$.

Note that in this diagram the expected utility of wealth is less than the utility of the expected wealth:

$$u\left(\frac{1}{2} * \$5 + \frac{1}{2} * \$15\right) = u(\$10) > \frac{1}{2}u(\$5) + \frac{1}{2}u(\$15)$$

In this case, consumer is **risk averse** since expected value is preferred to gamble.

It could happen that the consumer prefers a a random distribution of wealth to its expected value, in which case we say that the consumer is a **risk lover**.



Risk loving. For a risk-loving consumer the expected utility of wealth, $.5u(5) + .5u(15)$, is greater than the utility of the expected value of wealth, $u(10)$.

The intermediate case is that of a linear utility function. Here the consumer is **risk neutral**: the expected utility of wealth is the utility of its expected value. In this case the consumer doesn't care about the riskiness of his wealth at all-only about its expected value.

3.1 Insurance against risk

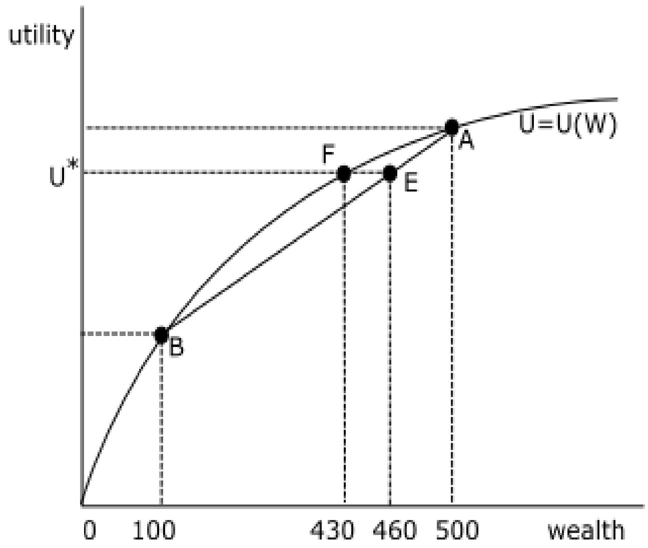
Risk premium: maximum amount of money that an individual is willing to pay in order to avoid the gamble. Risk-averse individual in example above is willing to pay up to \$3 to avoid the gamble, therefore the risk premium is \$3. If someone offers this individual insurance against the uncertainty; i.e. insurance company offers to keep individual's wealth constant and the price of this insurance is at most \$3 the individual buys this insurance.

The expected utility is the same whether the individual gets the insurance or not. Then why is the individual willing to buy it in the first place? Because when there is no insurance, marginal utility in the event of loss is higher than if no loss occurs (because of diminishing marginal utility of risk-averse individual). Hence, the transfer from no-loss to the loss situation must increase total utility.

Example: Tom owns a car of value 400 000 and he also has cash of 100 000. His wealth altogether is therefore 500 000. There is 10% probability that the car gets stolen hence Tom is thinking whether to buy insurance or not. Let's he what is the maximum amount of money that Tom would be willing to pay for the insurance that pays full price of the car (400 000) if it gets stolen. Without any insurance the expected utility is

$$0.1 \cdot 100000 + 0.9 \cdot 500000 = 460000$$

And going through point E the corresponding expected utility is U^*. Tom is will to pay for the insurance up to the price that will give him the same level of utility as in no insurance case. On the picture below wealth of 430 000 gives the level of utility U^*. Therefore Tom will be willing to pay up to 70 000 (computed as 500 000-430 000) for the insurance of his car.



3.2 Diversification

Suppose, for example, that shares of the raincoat company and the sunglasses company currently sell for \$10 a piece. If it is a rainy summer, the raincoat company will be worth \$20 and the sunglasses company will be worth \$5. If it is a sunny summer, the payoffs are reversed: the sunglasses company will be worth \$20 and the raincoat company will be worth \$5. If you invest your entire \$100 in the sunglasses company, you are taking a gamble that has a 50 percent chance of giving you \$200 and a 50 percent chance of giving you \$50. The same magnitude of payoffs results if you invest all your money in the sunglasses company: ill either case you have an expected payoff of \$125. But look what happens if you put half of your money in each. Then, if it is sunny you get \$100 from the sunglasses investment and \$25 hom the raincoat investment. But if it is rainy, you get \$100 from the raincoat investment and \$25 hom the sunglasses investment. Either way, you end up with \$125 for sure. By diversifying your investment in the two companies, you have managed to reduce the overall risk of your investment, while keeping the expected payoff the same.

Diversification was quite easy in this example: the two assets were perfectly negatively correlated-when one went up, the other went down. Pairs of assets like this can be extremely valuable because they call reduce risk so dramatically. But, alas, they are also very hard to find. Most asset values move together: when GM stock is high, so is Ford stock, and so is Goodrich stock. But as long as asset price movements are not perfectly positively correlated, there will be some gains from diversification.