2   Theory of Demand, Slutsky Equation
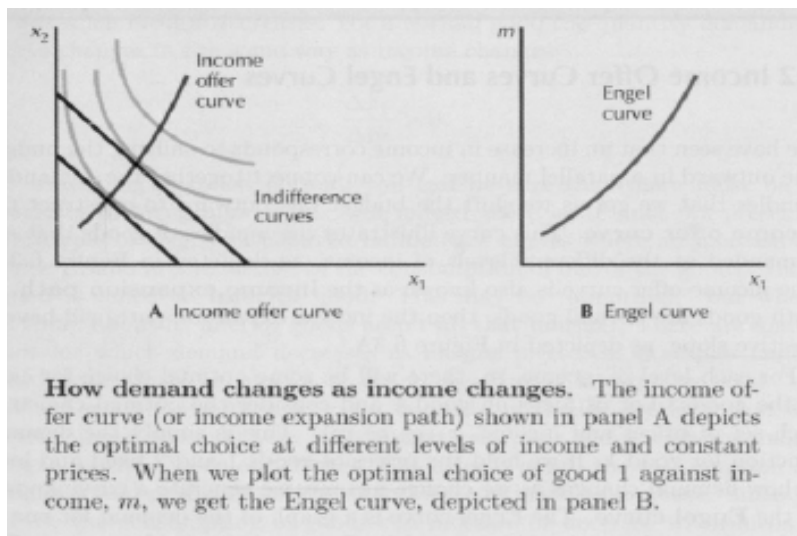
2.1   Theory of Demand

Based on the analysis of consumer's optimal consumption we know that the demand depends on individual preferences, prices, and income. Now we will analyze how the demand changes as prices and income change.

Change in income: (keeping prices constant)

- Normal good: if income increase consumption increase as well

- Inferior good: increase in income causes reduction in consumption (low-quality good)

Set of bundles of goods that are demanded at the different levels of income is called **income offer curve (income expansion path).**

For each level of income, there is some optimal choice for each of the goods. The graph of the demand for one of the goods as a function of income with all prices being held constant is called the **Engel curve**.

How demand changes as income changes. The income of-
fer curve (or income expansion path) shown in panel A depicts
the optimal choice at different levels of income and constant
prices. When we plot the optimal choice of good 1 against in-
come, m, we get the Engel curve, depicted in panel B.

Two goods are perfect substitutes:

If $p_1 < p_2$, consumer spends entire budget on good 1 and therefore buys $x_1 = I/p_1$ units. Income offer curve is a horizontal axis. If income increases, consumption of good 1 increases as well. Engel curve is a linear line, because $I = p_1 x_1$.

Perfect substitutes. The income offer curve (A) and an Engel curve (B) in the case of perfect substitutes.
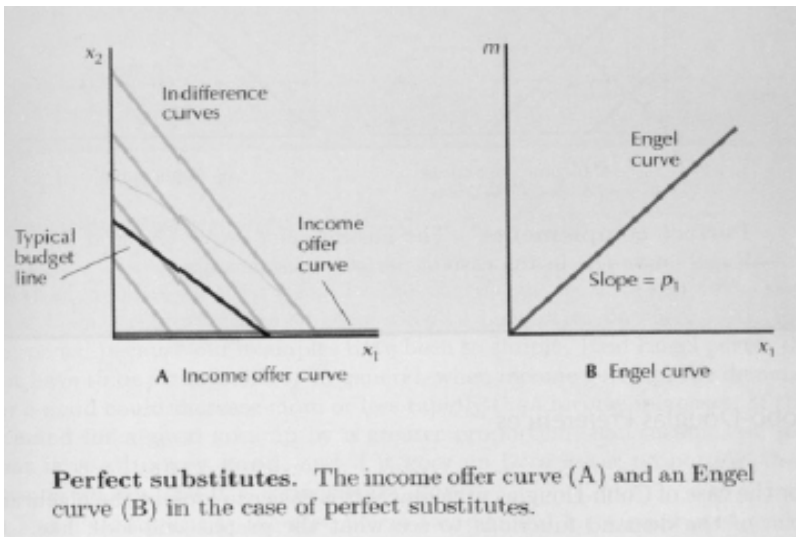
Two goods are perfect complements:

Consumer consumes the same amount of each good, the income curve is the diagonal line through the origin. The demand for good 1 is $x_1 = I/(p_1 + p_2)$ and Engel curve is a straight line with slope $(p_1 + p_2)$.

Perfect complements. The income offer curve (A) and an Engel curve (B) in the case of perfect complements.
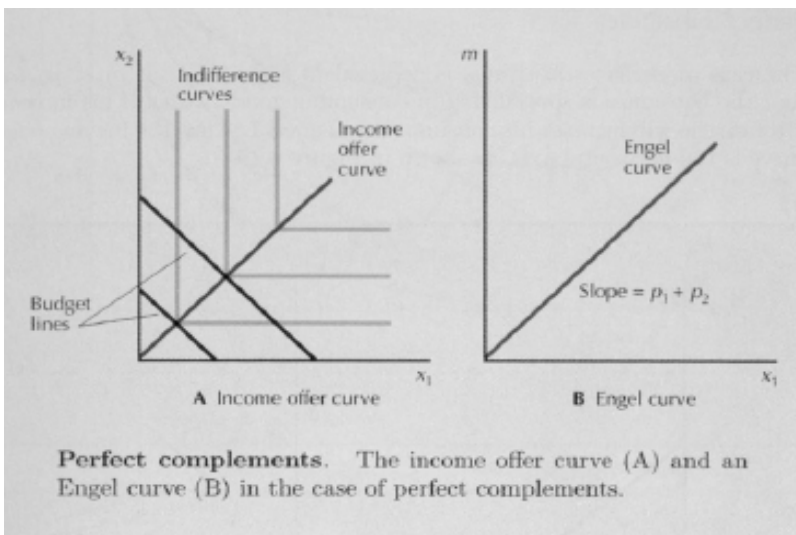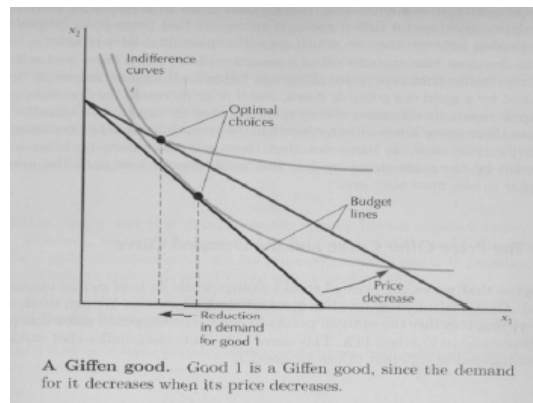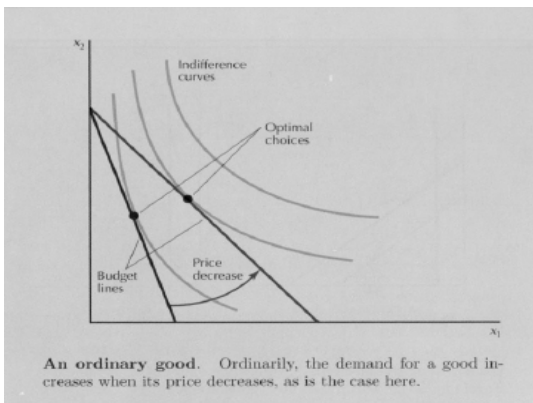
Cobb-Douglas utility function: (not covered on the lecture but useful example) $u(x_1, x_2) = x_1^a x_2^{(1-a)}$. The demand for good 1 is $x_1 = aI/p_1$ and the demand for good 2 is $x_2 = (1-a)I/p_2$. For given prices the demand for both goods is a linear function of income. Doubling income causes double the consumption of both goods. Hence the income offer curve and Engel curve is a straight line.

Cobb-Douglas. An income offer curve (A) and an Engel curve (B) for Cobb-Douglas utility.
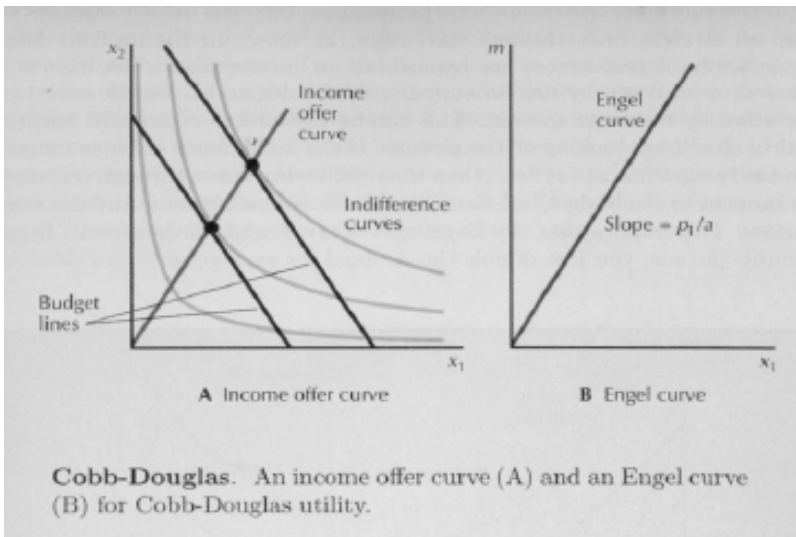
*Change in one price:* (keeping the other price and income constant)

- Ordinary good: decrease in price causes increase in consumption of a given good

- Giffen good: decrease in price results in reduction in its demand

An ordinary good. Ordinarily, the demand for a good increases when its price decreases, as is the case here.

A Giffen good. Good 1 is a Giffen good, since the demand for it decreases when its price decreases.

2.2 Slutsky Equation

When the price of a good changes, there are two sorts of effects: the rate at which you can exchange on good for another changes, an the total purchasing power of your income is altered. The change in demand due to the change in the rate of exchange between the two goods is called **substitution effect**. The change in demand due to having more purchasing power is called the **income effect**.

Note: There are two approaches to decompose total effect into substitution and income effect. Below we use red color for Slutsky's approach (pivot budget line goes through the original consumption bundle). Alternative approach is Hick's approach (pivot budget line is such that the consumer has the same level of utility as in the original point of consumption) for which we use green color. These approaches are equivalent.
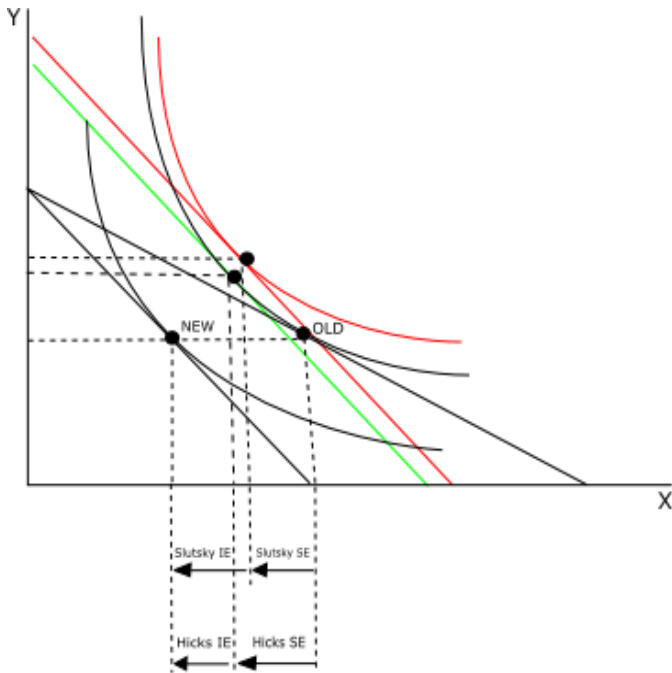
Slutsky's equation:

$$x_1(p_1', I) - x_1(p_1, I) = [x_1(p_1', I') - x_1(p_1, I)] + [x_1(p_1', I) - x_1(p_1', I')]$$

Change in demand = Substitution effect + Income effect


Substitution effect:

Is always negative. If price of good 1 decreases, new optimal choice must involve consuming at least as much of good 1 as originally. (The original bundle is preferred to all the bundles on pivot budget line to the right from original point of consumption - these were affordable but consumer did not choose them).

Income effect:

Can be positive or negative. If the good is a normal good, then the decrease in income will lead to a decrease in demand. If the good is an inferior good, then the decrease in income will lead to an increase in demand. Income effect can be positive or negative.

SE	IE	TE
$p_1 \nearrow \Rightarrow x_1 \searrow$ (neg)	$I \searrow \Rightarrow x_1 \searrow$ (normal good)	$p_1 \nearrow \Rightarrow x_1 \searrow$ (ordinary)
$p_1 \nearrow \Rightarrow x_1 \searrow$ (neg)	$I \searrow \Rightarrow x_1 \nearrow$ (a little) (weak inferior good)	$p_1 \nearrow \Rightarrow x_1 \searrow$ (ordinary)
$p_1 \nearrow \Rightarrow x_1 \searrow$ (neg)	$I \searrow \Rightarrow x_1 \nearrow$ (a lot) (strong inferior good)	$p_1 \nearrow \Rightarrow x_1 \nearrow$ (Giffen)

We have a Giffen good only in case of very inferior goods, in case of only a little inferior good and normal good total effect is always negative. (Every Giffen good in inferior but not every inferior good is also Giffen).
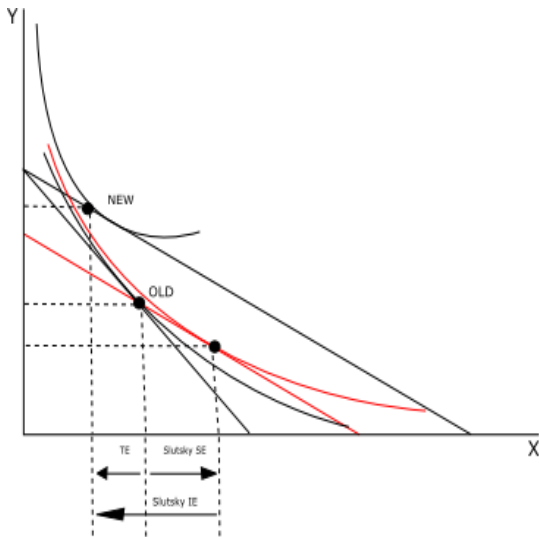
2.3 Elasticity

Demand responds to changes in prices and/or income. Elasticity measures how responsive the demand is.

The price elasticity of demand, ϵ, is defined to be the percent change in quantity divided by the percent change in price:

$$\epsilon = \frac{\Delta q/q}{\Delta p/p}$$

- elastic demand: elasticity is greater than 1 in absolute value (if there are a lot of substitutes)

- inelastic demand: elasticity is less than 1 in absolute value (if there are very few substitutes)

- unit elastic demand: elasticity is equal to 1 in absolute value

Elasticity and revenue: If demand is very responsive to price, very elastic, then an increase in price will reduce demand so much that revenue will fall. If demand is very unresponsive to price, very inelastic, then an increase in price will not change demand very much and overall revenue will increase. If elasticity is equal to -1 and the price increases by 1 percent, the quantity will decrease by 1 percent, so overall revenue does not change.

The income elasticity of demand, ϵ_I, is defined to be the percent change in quantity divided by the percent change in income:

$$\epsilon_I = \frac{\Delta q/q}{\Delta I/I}$$

Income elasticity of demand is positive for normal good and negative for an inferior good.

The cross price elasticity, ϵ_c:

$$\epsilon_c = \frac{\Delta q_x/q_x}{\Delta p_y/p_y}$$

is positive if substitution effect is greater than income effect, negative otherwise.