1 Budget Set, Preferences, Consumer's Optimum

People choose the best things they can afford.

Every consumer makes a decision about how much to consume of each good. Without any sort of restrictions, we would want to eat as much as we could of everything. However, economics is about choice under scarcity. In particular, each of us has limited money to buy all the things that we like. Hence, we're going to introduce the budget set constraint.

1.1 The budget set

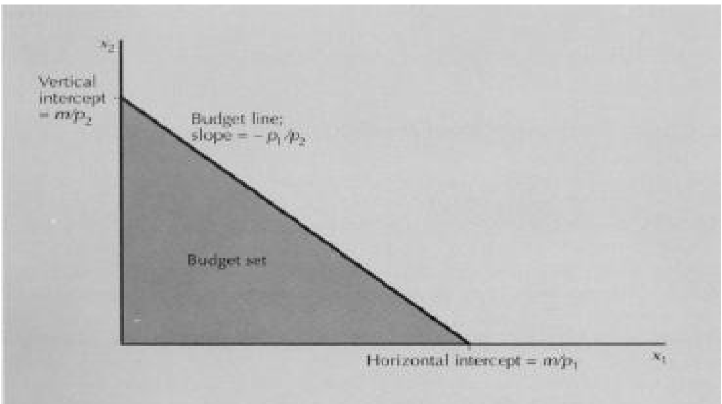
People choose things they can afford.

In general, we'll take prices and the consumer's income as given. The budget set consists of all bundles of goods that the consumer can afford at given prices and income. We can write down in an equation the amount of each good that I can consume.

Assume two goods X and Y, their prices P_X and P_Y and income I. Then the budget set is given by:

$$P_X X + P_Y Y \leq I \quad \Rightarrow \quad \text{Budget line is given by: } Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$$

The absolute easiest way to draw a budget set is to start by figuring out how much of each good I could buy if I spent all my money on it. To find this number, all you need to do is divide your income by the price of each good. You can then put these points on the graph and connect them with a line. That line is the budget set. (Why is the line straight?)



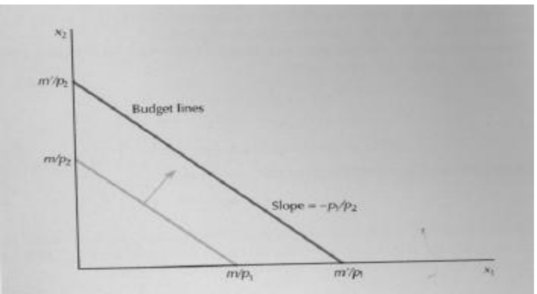
The budget set. The budget set consists of all bundles that are affordable at the given prices and income.

The formula tells us how many units of good 2 the consumer needs to consume in order to just satisfy the budget constraint if she is consuming X units of good 1. The slope of the budget line measures the rate at which the market is willing to substitute good 1 for good 2, or opportunity cost of consuming good 1.

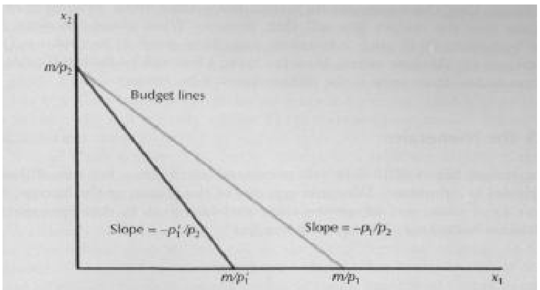
Changes in budget line:

When prices and incomes change, the set of goods that a consumer can afford changes as well. How?

- **Change in income:** Increase in income result in parallel shift outward of the budget line. The intercepts change, the slope remains the same.
- **Change in price:** Increase in price of good 1 result in shift of the horizontal intercept of the budget line inward.
- **Change in both prices and income by the same factor:** Budget set remains the same.



Increasing income. An increase in income causes a parallel shift outward of the budget line.



Increasing price. If good 1 becomes more expensive, the budget line becomes steeper.

Numeraire: Setting one of the prices equal to 1 and adjusting the other price appropriately does not change the budget set at all. If we set price of one good to 1 we refer to this good a numeraire and to this price as numeraire price. It is convenient to do that because there will be one variable less.

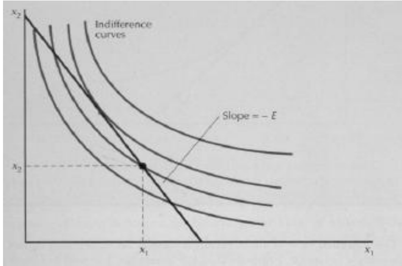
1.2 Preferences

People choose the best things.

We suppose that preferences are:

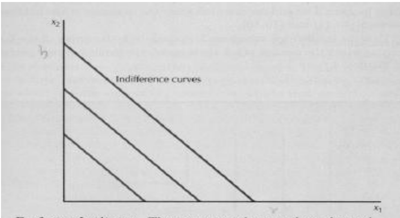
- **complete**, i.e. we suppose that given any two consumption bundles (x_1, x_2) and (y_1, y_2), the consumer can rank them as to their desirability. That is, the consumer can determine that one of the bundles is strictly better than the other, or decide that she is indifferent between the two
- **reflexive**, i.e. any bundle is at least as good as itself: $(x_1, x_2) \succeq (x_1, x_2)$
- **transitive**, i.e. if $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$

Indifference curve: is a graphical representation of preferences; it is a set of bundles for which the consumer is just indifferent. Start in some bundle. Now think about giving a little bit less of good 1 to the consumer. How much more of good 2 do you have to give him in order to keep his utility level?

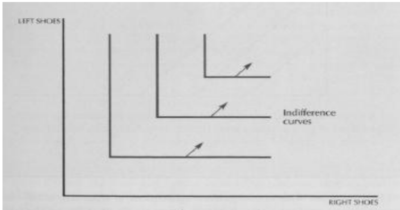


The shape of the indifference curves can be different:

- perfect substitutes: if the consumer is willing to substitute one good for the other at a constant rate (linear utility function $u(x_1, x_2) = x_1 + x_2$)



Perfect substitutes. The consumer only cares about the total number of pencils, not about their colors. Thus the indifference curves are straight lines with a slope of −1.



Perfect complements. The consumer always wants to consume the goods in fixed proportions to each other. Thus the indifference curves are L-shaped.

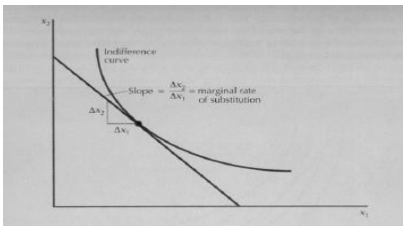
- perfect complements: goods that are always consumed together in fixed proportions (utility function $u(x_1, x_2) = \min\{x_1, x_2\}$)
- (neutrals: a good that the consumer doesn't care about, bads, satiation point)

Well-behaved indifference curves: we assume that more is better (monotonicity of preferences, satiation is not possible, negative slope), indifference curves are convex. (Cobb-Douglas utility function $u(x_1, x_2) = x_1^a x_2^b$)

Indifference curves representing distinct levels of preference cannot cross. If they cross, transitivity and "more is better" is violated.

Marginal rate of substitution (MRS): the slope of indifference curve at a particular point. It is the rate at which the consumer is willing to substitute one good for the other.

Suppose that we take a little of good 1, Δx_1 away from the consumer. Then we give him Δx_2 the amount that is sufficient to put him back on his indifference curve, so that he is just as well off after the substitution of x_2 for x_1 as he was before. We thing of the ratio of $\Delta x_2/\Delta x_1$ as being the rate at which the consumer is willing to substitute good 2 for good 1. If Δx_1 is very small change, marginal change, than $\Delta x_2/\Delta x_1$ measures the marginal rate of substitution of good 2 for good 1. As Δx_1 gets smaller, $\Delta x_2/\Delta x_1$ approaches the slope of the indifference curve.



The marginal rate of substitution (MRS). The marginal rate of substitution measures the slope of the indifference curve.

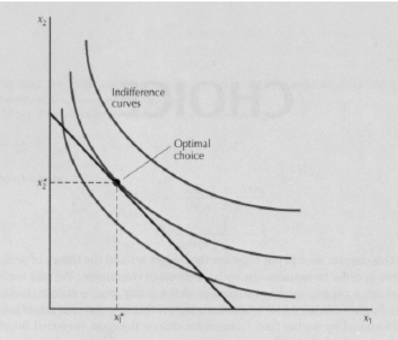
MRS is a negative number for well-behaved preferences, -1 for perfect substitutes, infinite for neutrals, zero or infinity for perfect complements. Moreover, for strictly convex indifference curves, the MRS - the slope - decreases (in absolute value) as we increase x_1. Thus the indifference curves exhibit a diminishing marginal rate of substitution. The more you have of one good, the more willing you are to give some of it up in exchange for the other good.

Marginal utility (MU$_1$): measures change in utility associated with a small change in the amount of good 1. $MU_1 = \Delta U/\Delta x_1$. Similarly, $MU_2 = \Delta U/\Delta x_2$

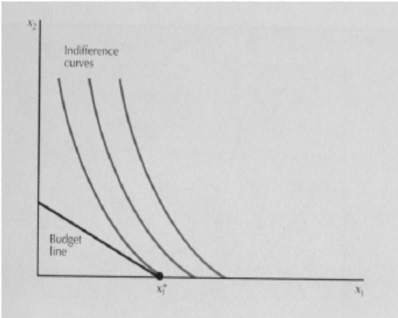
Note that: $MRS = \frac{\Delta x_2}{\Delta x_1} = \frac{MU_1}{MU_2}$

1.3 Consumer's optimum

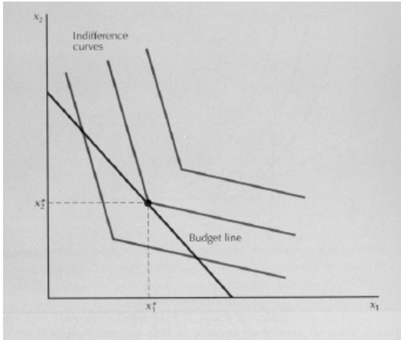
People choose the best things they can afford - People choose the most preferred bundle from their budget set.



Optimal choice. The optimal consumption position is where the indifference curve is tangent to the budget line.



Boundary optimum. The optimal consumption involves consuming zero units of good 2. The indifference curve is not tangent to the budget line.



Kinky tastes. Here is an optimal consumption bundle where the indifference curve doesn't have a tangent.

The optimal choice (x_1^*, x_2^*) is the bundle in which indifference curve is tangent to the budget line (interior optimum). This means that the slope of the indifference curve is equal to the slope of the budget line, i.e. $MRS = \frac{p_X}{p_Y}$. The optimality condition can also be given by:

$$\frac{MU_X}{p_X} = \frac{MU_Y}{p_Y} \quad \text{or} \quad \frac{MU_X}{p_X} = \frac{MU_Y}{p_Y}$$

(*Note:* if $\frac{MU_X}{p_X} > \frac{MU_Y}{p_Y}$, consumer could be better off by decreasing consumption of Y and increasing consumption of X.)