Problem 1: Suppose, that the utility function is given by:

$$U(x, y) = 5 \ln x + 8 \ln y$$
$$P_x = 5$$
$$P_y = 8$$

(a) Derive equation of Engel's curve for x (Hint: this is something we have done when we were deriving the demand for x with given P_y and income; but here you are looking for function $x = f(\text{income; at given } P_x, P_y)$

(b) Depict this curve on the graph as relation between x, I (with x on horizontal and I on vertical axis; find at least 3 points).

(c) What type of good is x - normal or inferior? Prove and explain.

(d) If the answer in part (c) is normal (inferior) sketch a graph for the case of inferior (normal) good.

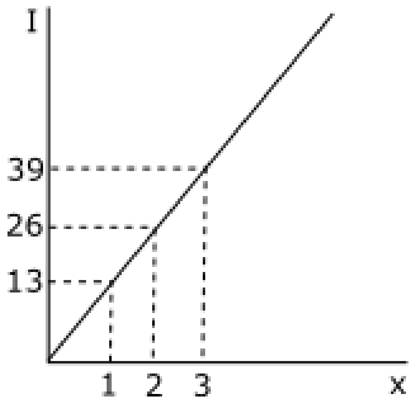
Solution:

(a) Consumer maximizes his utility so the optimality condition has to hold. Also, we have a budget constraint that has to hold.

$$\text{optimum: } MRS = \frac{MU_x}{MU_y} = \frac{5/x}{8/y} = \frac{5y}{8x} = \frac{5}{8} = \frac{P_x}{P_y} \quad \Rightarrow \quad x = y$$

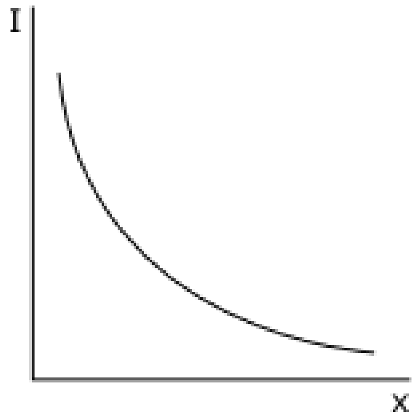
$$\text{BL: } xP_x + yP_y = I \quad \Rightarrow \quad 5x + 8y = I \quad \Rightarrow \quad 13x = I \quad \Rightarrow \quad I = \frac{I}{13}$$

(b) Engel's curve is depicted on picture below.



(c) Based on the income curve x is normal good. Because as income increases the consumption of good x increases as well.

(d) The income curve for an inferior good is decreasing function (as income goes up consumption goes down).



Problem 2: Intertemporal choice is something we haven't dealt with during our seminars. But, after reading related chapter in any of recommended textbooks, you will find out, that it can be fully described by indifference analysis. Go through it and then solve this problem:

The preference about current and future consumption can be described by utility function:

$$U(c, f) = 11c + 10f$$

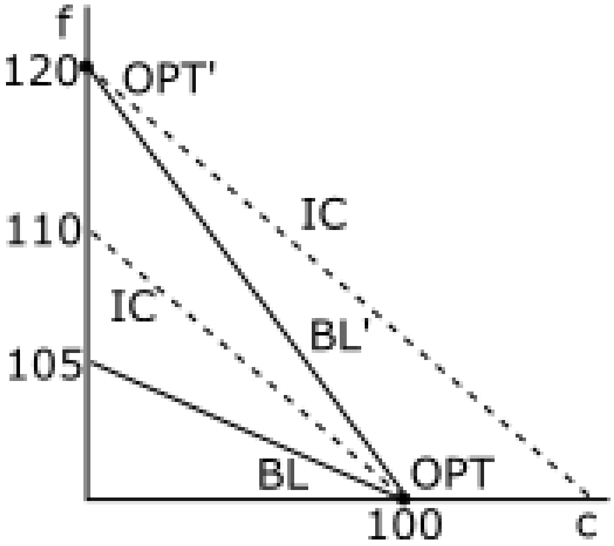
where c = current consumption

f = future consumption

We further know that the interest rate is 5% and we have \$100 that can be used either on current or future consumption (or any of their combination)

(a) Depict this situation in indifference system (it means: depict budget line, at least one of IC - with at least three points; mark intersections with axis); what will happen if r changes to 20%? Depict and explain!

(b) Find the optimum combination of future and current consumption (remember, this is strictly analogical problem to our x, y problems, only you have to think about "price of future consumption").



Solution: The budget line will be a line with the slope $1 + r$. So if $r = 5\%$ ($r = 20\%$) the slope will be 1.05 (110). These two budget lines are depicted on the picture below.

The utility function suggests that the two goods are perfect substitutes, i.e. the indifference curves will be straight lines. To see that we can find marginal rate of substitution:

$$MRS = \frac{MU_c}{MU_f} = \frac{11}{10}$$

Marginal rate of substitution is constant, so indeed indifference curves are straight lines with slope 11/10.

To find the optimum we look for a highest possible indifference curve within the budget set. The optimum is depicted on the picture below for both $r = 5\%$ and $r = 20\%$. Note that in this case we can not use optimality condition that the indifference curve touches budget line and they have the same slope. Because here, the slopes of IC a BL are always different. In these cases we have a corner solution (consumption of only one good).

Problem 3: Suppose, production of a firm can be described by function:

$$Q(L, K) = K^{1/3}L^{2/3}$$

(a) Derive the total cost function in a general form with respect to any prices of labor and capital w and r, respectively. (Hint: you gave to go through the optimality conditions and get the function of TC in form $TC = f(w; r; Q)$)

(b) Suppose now, that in short run your K is fixed at level of 1000. How many units of labor do you need to employ to produce $Q = 1000$? For the rest of the problem suppose that $w = 100$ and $r = 100$. What would be the optimal combination of L and K for $Q = 1000$ if you could change both variables (K, L)?

(c) Compare TC necessary to produce $Q = 1000$ in short run and in long run. Explain.

Solution:

(a) The total cost function gives us the cost of the cheapest combination of K and L such that we produce a level of output Q. Therefore we have to use the optimality condition that $MRTS = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{w}{r}$

$$MRTS = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{\frac{2}{3}\left(\frac{K}{L}\right)^{1/3}}{\frac{1}{3}\left(\frac{L}{K}\right)^{2/3}} = \frac{2K}{L} = \frac{w}{r}$$

$$K = L\frac{w}{2r}, \quad L = K\frac{2r}{w}$$

To find the optimal level of K and L we use production function $Q(L, K) = K^{1/3}L^{2/3}$ and the optimality condition:

$$Q = K^{1/3}L^{2/3} \;\Rightarrow\; Q = K^{1/3}\left(K\frac{2r}{w}\right)^{2/3} \;\Rightarrow\; K = Q\left(\frac{w}{2r}\right)^{2/3}$$

$$Q = K^{1/3}L^{2/3} \;\Rightarrow\; Q = \left(L\frac{w}{2r}\right)^{1/3} L^{2/3} \;\Rightarrow\; L = Q\left(\frac{2r}{w}\right)^{1/3}$$

Now when we know the cost minimizing combination of capital and labor we can write down the cost function:

$$TC = wL + rK = Q\left(w\left(\frac{2r}{w}\right)^{1/3} + r\left(\frac{w}{2r}\right)^{2/3}\right)$$

(b) If K is fixed at level of 1000 and Q is supposed to be 1000 then to find the necessary level of L we solve:

$$Q(L, K) = K^{1/3} L^{2/3}$$
$$1000 = 1000^{1/3} L^{2/3} \quad \Rightarrow \quad L^{2/3} = 100 \quad \Rightarrow \quad L = 100^{3/2} = 1000$$

Then the short run total cost equals to:

$$TC = wL + rK = 100 \times 1000 + 100 \times 1000 = 200000$$

(c) In long run the level of capital is not fixed but is chosen according to the optimality condition:

$$K = Q\left(\frac{w}{2r}\right)^{2/3} = 1000\left(\frac{100}{2 \times 100}\right)^{2/3} = 630$$
$$L = Q\left(\frac{2r}{w}\right)^{1/3} = 1000\left(\frac{2 \times 100}{100}\right)^{1/3} = 1260$$

And the long run total cost equals to:

$$TC = wL + rK = 100 \times 630 + 100 \times 1260 = 189000$$

Total cost in the long run is lower because the firm is not limited by fixed level of capital and have better options to optimize.