Problem 1: (Bertrand competition). Total cost function of two firms selling computers is $TC_1 = TC_2 = 15q$. If these two firms compete in prices, what will be the market equilibrium price? What happens if the first firm increases the efficiency of the production and its total cost decreases to $TC_1 = 5q_1$ while the total cost of the second firm is still $TC_2 = 15q_2$?

Solution: If the two firms compete in prices they will keep undercutting each other till the price decreases to MC. Firms can not decrease the price any more because they would earn a negative profit. Marginal cost of the firms is:

$$MC = TC' = (15q)' = 15$$

So the market equilibrium price would be $P = 15$.

If the total cost of the first firm decreases to $TC_1 = 5q_1$ this firm can afford to offer computers for a lower price, therefore it will charge slightly less than marginal cost of the second firm - this will push second firm out of the market and take the whole market. Therefore the market price will be slightly below $MC_2 = 15$, e.g. 14. That means the second firm will not produce any computers because it is not profitable. The first firm will earn $P - MC_1 = 14 - 5 = 9$ for each computer sold.

Problem 2: (Cournot duopoly). Consider a market with two firms, 1 and 2 producing a homogeneous good. The market demand is $P = 130 - 2(Q_1 + Q_2)$, where Q_1 is the quantity produced by firm 1 and Q_2 is the quantity produced by firm 2. The total cost of firm 1 is $TC_1 = 10Q_1$, the one of firm 2 is $TC_2 = 10Q_2$.

(a) Find the reaction function of both firms.

(b) Find the equilibrium quantity produced by each firm by solving the system of the two reaction functions you found in (a). Sketch your solution graphically.

(c) Find the equilibrium price. Find the profit of each firm.

(d) Suppose that the two firms behave like competitive firms and compete in prices. What will be the quantity they produce and what will be the equilibrium price? Compare your results with (b) and (c). (Hint: use the fact that both firms are equal, so they must produce the same amount)

Solution:

(a) The reaction function gives firm 1s optimal choice for each possible choice by firm 2. In other words, it gives firm 1s choice given what it believes firm 2 is doing. Similarly, the reaction function of the second firms gives firm 2s optimal choice for each possible choice by firm 1. To find the reaction function of the first firm we solve profit maximization of the first firm taking the output of the second one as given.

$$\max_{Q_1} \pi_1 = TR_1 - TC_1 = P * Q_1 - 10Q_1 = [130 - 2(Q_1 + Q_2)]Q_1 - 10Q_1$$
$$\max_{Q_1}[130 - 2Q_1 - 2Q_2]Q_1 - 10Q_1$$
$$\text{FOC: } 130 - 4Q_1 - 2Q_2 - 10 = 0 \quad \Rightarrow \quad Q_1 = \frac{120 - 2Q_2}{4} = 30 - \frac{Q_2}{2}$$

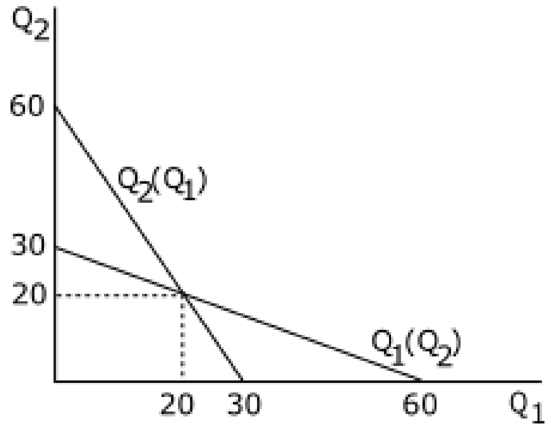
Similarly, to find the reaction function of the second firm we solve profit maximization of the second firm taking the output of the first one as given.

$$\max_{Q_2} \pi_2 = TR_2 - TC_2 = P * Q_2 - 10Q_2 = [130 - 2(Q_1 + Q_2)]Q_2 - 10Q_2$$
$$\max_{Q_2}[130 - 2Q_1 - 2Q_2]Q_2 - 10Q_2$$
$$\text{FOC: } 130 - 2Q_1 - 4Q_2 - 10 = 0 \quad \Rightarrow \quad Q_2 = \frac{120 - 2Q_1}{4} = 30 - \frac{Q_1}{2}$$

The reaction functions of both firms are the same because their total cost functions are the same. Generally, this does not have to be the case.

(b) Find the equilibrium quantity produced by each firm by solving the system of the two reaction functions you found in a). Sketch your solution graphically.

$$Q_1 = 30 - \frac{Q_2}{2}$$
$$Q_2 = 30 - \frac{Q_1}{2} = 30 - \frac{30 - \frac{Q_2}{2}}{2} = 30 - \frac{60 - Q_2}{4} = 15 + \frac{Q_2}{4} \Rightarrow 3Q_2 = 60 \Rightarrow Q_2^* = 20$$
$$Q_1^* = 30 - \frac{Q_2}{2} = 30 - \frac{20}{2} = 20$$



(c) Now when we know the level of output of each firm we can use the demand function to find the equilibrium price:

$$P = 130 - 2(Q_1 + Q_2) = 130 - 2(20 + 20) = 130 - 80 = 50$$

And the profit of the firms is:

$$\pi_1 = TR_1 - TC_1 = P * Q_1 - 10Q_1 = 50 * 20 - 10 * 20 = 800$$
$$\pi_2 = 800$$

(d) If these two firms compete in prices they will both charge price equal to marginal cost. And then we use the demand function to find the quantity demanded.

$$P = MC_1 = MC_2 = (TC_2)' = (10Q_2)' = 10$$
$$P = 130 - 2(Q_1 + Q_2) \quad \Rightarrow \quad Q_1 + Q_2 = 60$$

Since the firms are identical they will produce 30 units of output each.

Problem 3: (Collusive Cournot duopoly). Consider the same duopoly in problem 2. However, now assume that the two firms agree to produce half of the monopoly quantity each.

(a) Find the quantity Q that maximizes the industry profits.

(b) Find the equilibrium price in the market.

(c) Calculate the profits for each firm. Compare your result with the one in problem 2 part (c).

(d) Now suppose that firm 2 produces half of the monopoly quantity but firm 1 deviates from the agreement and produces according to its reaction function you have found in problem 2 part (a). Show that the deviation of firm 1 is profitable.

Solution:

(a) In this part we find the monopoly level of output:

$$\max_Q \pi = \max_Q P * Q - TC = \max_Q (130 - 2Q)Q - 10Q$$
$$\text{FOC:} \;\; 130 - 4Q - 10 = 0 \quad \Rightarrow \quad Q^* = 30$$

This means that both firms will produce 15 units of output, i.e. $Q_1 = Q_2 = 15$.

(b) Now, using the demand function the equilibrium price in the market is as follows:

$$P = 130 - 2Q = 130 - 2 * 30 = 70$$

(c) The profit of the firms is:

$$\pi_1 = TR_1 - TC_1 = P * Q_1 - 10Q_1 = 70 * 15 - 10 * 15 = 900$$
$$\pi_2 = 900$$

If the two firms collude and agree to produce lower total quantity for higher prices (compared to outcome of Cournot duopoly) they will earn higher profits.

(d) The output of the second firm is $Q_2 = 15$ and the first firm will break the agreement and choose a profit maximizing level of output:

$$\max_{Q_1} \pi_1 = \max_{Q_1} P * Q_1 - TC_1 = \max_{Q_1} (130 - 2(Q_1 + 15))Q_1 - 10Q_1$$

We do not need to solve this maximization problem again because we already did it when looking for reaction functions. So we can use firs 1's reaction function to find the optimal level of output:

$$Q_1 = 30 - \frac{Q_2}{2} = 30 - \frac{15}{2} = 22.5$$
$$\pi_1 = P * Q_1 - TC_1 = (130 - 2(Q_1 + Q_2))Q_1 - 10Q_1 =$$
$$= (130 - 2(22.5 + 15)) - 10 * 22.5 = 55 * 22.5 - 10 * 22.5 = 1012.5$$

So if the first firm breaks the agreement it can make a higher profit - this is the reason why collusion is very often broken by one of the firms.

Problem 4: (Stackelberg oligopoly). Suppose that the two firms choose quantity, but one firm moves first and the second firm observes firm 1s choice. Firm A enters the market first and decides how many iPhones it will produce. Later firm B enters the market and makes the decision about its own level of production after observing A's level of output. The market demand function is $P = 12 - Q$, total costs of firms are $TC_A = 2Q_A + 12$ and $TC_B = 4Q_B + 1$.

(a) How many iPhones will each firm produce?

(b) What will be the market equilibrium price?

(c) What will be the profit of each firm?

(d) Compare this to Cournot duopoly outcome.

Solution: Searching for a Stackelberg equilibrium works in two stages. First stage: Firm A chooses its output. Second stage: Firm B observes A's action and chooses its output. We solve this problem backwards - first solve firm B's maximization problem and then A's problem.

(a)

$$\max_{Q_B} \pi_B = TR_B - TC_B = P * Q_B - (4Q_B + 1) = (12 - Q_A - Q_B)Q_B - 4Q_B - 1$$

$$\text{FOC:} \;\; 12 - Q_A - 2Q_B - 4 = 0 \quad \Rightarrow \quad Q_B = 4 - \frac{Q_A}{2}$$

This is the reaction function of firm B, i.e. whatever is the A's output, firm B will produce $Q_B = 4 - \frac{Q_A}{2}$.

Firm A knows this and will include this piece of information into its decision making:

$$\max_{Q_A} \pi_A = TR_A - TC_A = P * Q_A - (2Q_A + 12) = (12 - Q_A - Q_B)Q_A - 2Q_A - 12$$

$$\text{and:} \;\; Q_B = 4 - \frac{Q_A}{2}$$

$$\max_{Q_A} \left[12 - Q_A - \left(4 - \frac{Q_A}{2}\right)\right] Q_A - 2Q_A - 12$$

$$\text{FOC:} \;\; 8 - Q_A - 2 = 0 \quad \Rightarrow \quad Q_A = 6$$

$$Q_B = 4 - \frac{Q_A}{2} = 4 - \frac{6}{2} = 1$$

(b) When we know the level of output of both firms we can use demand function to determine the equilibrium price:

$$P = 12 - Q = 12 - Q_A - Q_B = 12 - 6 - 1 = 5$$

(c) Profits of the firms are:

$$\pi_A = P * Q_A - TC_A = 5 * 6 - (2 * 6 + 12) = 6$$
$$\pi_B = P * Q_B - TC_B = 5 * 1 - (4 * 1 + 1) = 0$$

(d) Cournot duopoly:

$$\max_{Q_A} \pi_A = P * Q_A - TC_A = (12 - Q_A - Q_B)Q_A - (2Q_A + 12)$$
FOC: $12 - 2Q_A - Q_B - 2 = 0$
reaction function: $Q_A = 5 - \frac{Q_B}{2}$
$$\max_{Q_B} \pi_B = P * Q_B - TC_B = (12 - Q_A - Q_B)Q_B - (4Q_B + 1)$$
FOC: $12 - Q_A - 2Q_B - 4 = 0$
reaction function: $Q_B = 4 - \frac{Q_A}{2}$

Solving the system of 2 reaction functions we get:

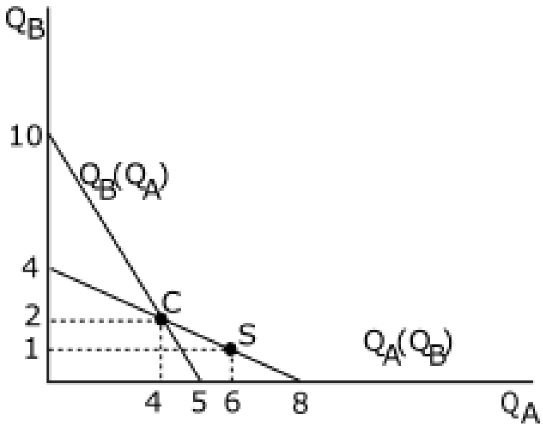
$$Q_A = 5 - \frac{Q_B}{2}$$
$$Q_B = 4 - \frac{Q_A}{2} = 4 - \frac{5 - \frac{Q_B}{2}}{2} = 4 - \frac{10 - Q_B}{4} \quad \Rightarrow \quad Q_B = 2$$
$$Q_A = 5 - \frac{Q_B}{2} = 5 - \frac{2}{2} = 4$$

Then, using the demand function, we get:

$$P = 12 - Q_A - Q_B = 12 - 2 - 4 = 6$$

Profits of the firms are:

$$\pi_A = P * Q_A - TC_A = 6 * 4 - (2 * 4 + 12) = 4$$
$$\pi_B = P * Q_B - TC_B = 6 * 2 - (4 * 2 + 1) = 3$$



Q_B
10
$Q_B(Q_A)$
4
C
2
S
1
$Q_A(Q_B)$
4
5
6
8
Q_A