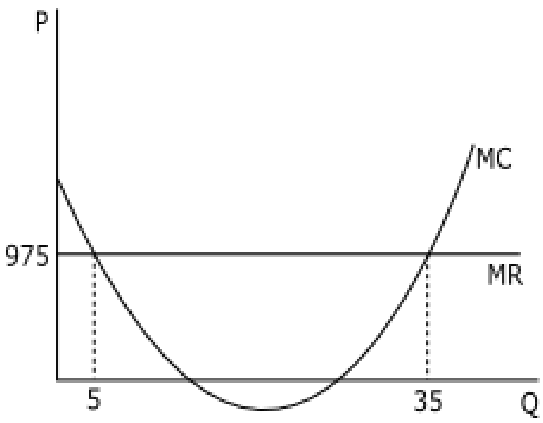
Problem 1: Long run total cost function and total revenue function of the monopolist are:

$$TC = 1500q - 60q^2 + q^3$$
$$TR = 975q$$

Find the optimal quantity of this monopolist. What will be the long run equilibrium price in the industry? How many firms there will be in a long run if the demand function is $D: P = 9600 - 2Q$?

Solution: In case of monopoly the optimal quantity is such that marginal revenue equals marginal cost (why?):

$$MR(= TR') = MC(= TC')$$
$$975 = 1500 - 120q + 3q^2$$
$$3q^2 - 120q + 525 = 0$$
$$q_{1,2} = \frac{120 \pm \sqrt{120^2 - 4 * 3 * 525}}{6} = \frac{120 \pm 90}{6} = 5, 35$$



From picture we see that the optimal level of monopoly quantity is 35. This is because in case of quantity equal to 5 - if we increase the level of output marginal revenue would be higher than marginal cost and therefore it would be possible to increase profit by increasing the level of production. So the optimal (profit maximizing) level of output is 35.

Alternative way to see this is taking second order condition (second derivative) or simply comparing the profit if the output is 5 and 35. Whichever is higher is the optimal level of output:

$$
\begin{aligned}
&\pi(q) = TR(q) - TC(q)\\
&\pi(5) = TR(5) - TC(5) = 975 * 5 - (1500 * 5 - 60 * 5^2 + 5^3) = -1250\\
&= 4875 - 7500 + 1500 - 125 = -1250\\
&\pi(35) = TR(35) - TC(35) = 975 * 35 - (1500 * 35 - 60 * 35^2 + 35^3) =\\
&= 34125 - 52500 + 73500 - 42875 = 12250
\end{aligned}
$$

Simple comparison shows that it is optimal to produce 35 units of output.

Using second order condition: first we find quantity for which profit function is in an extreme (minimum or maximum) and then we check if it is minimum or maximum.

$$
\begin{aligned}
&\pi(q) = TR(q) - TC(q) = 975 * q - (1500 * q - 60 * q^2 + q^3) = -q^3 + 60q^2 - 525q\\
&\text{FOC: } \pi(q)' = 0 \quad \Rightarrow \quad -3q^2 + 120q - 525 = 0 \quad \Rightarrow \quad q_{1,2} = 5, 35\\
&\text{SOC: } \pi(q)'' = -6q + 120 = 0\\
&\pi(5)'' = -6 * 5 + 120 = 90 > 0 \quad \Rightarrow \quad \text{Profit is minimal for } q = 5\\
&\pi(35)'' = -6 * 35 + 120 = -90 < 0 \quad \Rightarrow \quad \text{Profit is maximal for } q = 35
\end{aligned}
$$

No matter which way of finding optimal level of quantity we choose, the result is the same: $q^* = 35$.

Now, we will find a long run equilibrium price. In long run more firms will enter the industry. New firms will keep coming till the profit is positive. So in long run all firms will have zero profit and hence the quantity produced and price are such that the average cost is minimized (why?):

$$
\begin{aligned}
&AC = \frac{TC}{q} = 1500 - 60q + q^2\\
&\text{FOC: } AC' = 0 \quad \Rightarrow \quad -60 + 2q = 0 \quad \Rightarrow \quad q^* = 30\\
&P^* = AC = 1500 - 60 * 30 + 30^2 = 600
\end{aligned}
$$

Now we can compute the demand:

$$
\begin{aligned}
&D: \; P = 9600 - 2Q \quad \Rightarrow \quad Q = \frac{9600 - P}{2} = \frac{9600 - 600}{2} = 4500\\
&Q = nq \quad \Rightarrow \quad n = \frac{Q}{q} = \frac{4500}{30} = 150 \;\; \text{(is number of firms in the market)}
\end{aligned}
$$

Problem 2: The demand function in the industry and the total cost function of the monopoly are:

$$D: \; Q_D = 2000 - 20P$$
$$TC = 0.05Q^2 + 10000$$

Find the optimum of this monopoly (equilibrium price, quantity, and corresponding profit) and depict it on the graph. Find price elasticity of demand and interpret the result.

Solution: In optimum, the monopolist chooses profit maximizing quantity, i.e. such Q that marginal revenue equals marginal cost (why?):

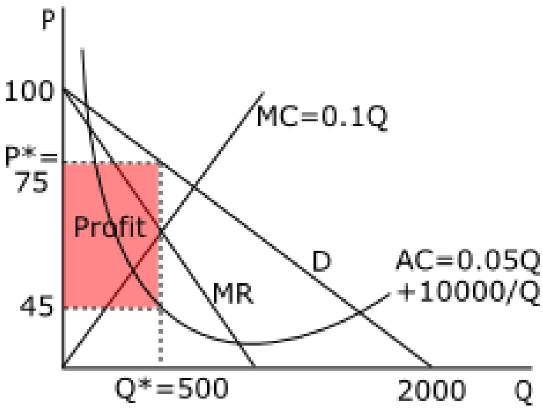
$$MR = MC$$
$$MR = TR' = (P * Q)' = \left(\frac{2000 - Q}{20} * Q\right)' = 100 - \frac{Q}{10}$$
$$MC = TC' = 0.1Q$$
$$MR = MC \quad \Rightarrow \quad 100 - \frac{Q}{10} = 0.1Q \quad \Rightarrow \quad 1000 - Q = Q \quad \Rightarrow \quad Q^* = 500$$

Use the inverse demand function to find the equilibrium price:

$$P^* = \frac{2000 - Q}{20} = \frac{2000 - 500}{20} = \frac{1500}{20} = 75$$

Then the profit of the monopolist is:

$$\pi = TR(500) - TC(500) = 75 * 500 - (0.05 * 500^2 + 10000) =$$
$$= 37500 - 22500 = 15000$$



Price elasticity of demand:

$$\varepsilon_P = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = -20\frac{75}{500} = -3$$

In absolute value price elasticity is more than 1 and therefore the demand is elastic. Is this just a coincidence, or is this a rule? This is rule - A profit maximizing monopoly will always set price on the elastic part of the demand curve. The profit maximizing monopoly will sell the quantity of output that makes $MR = MC$. MC must be positive so MR must also be positive. This is on the elastic part of the demand curve. (Marginal revenue will be positive on the elastic part of a demand curve, marginal revenue will be negative on the inelastic part of the demand curve, and marginal revenue will be zero where demand is unit elastic.)

Problem 3: Consider the monopoly from the previous problem. What happens if the monopolist has to pay unit tax $t = 20$? What happens if the regulator sets the price to $P = 60$?

Solution: In optimum, the monopolist chooses profit maximizing quantity, i.e. such Q that marginal revenue equals marginal cost:

$$MR = MC$$
$$100 - \frac{Q}{10} = 0.1Q + 20 \quad \Rightarrow \quad 1000 - Q = Q + 200 \quad \Rightarrow \quad Q^* = 400$$

Use the inverse demand function to find the equilibrium price:

$$P^* = \frac{2000 - Q}{20} = \frac{2000 - 400}{20} = \frac{1600}{20} = 80$$

If regulator sets the price to be $P = 60$, then the profit maximizing quantity of the monopoly is:

$$MR = MC$$
$$60 = 0.1Q \quad \Rightarrow \quad Q_S = 600$$

But the demand is higher that that:

$$D: \; Q_D = 2000 - 20P = 2000 - 20 * 60 = 800$$

If regulator sets the price to $P = 60$, there will be excess demand on the market.

Problem 4: Monopoly: A monopolist can produce at constant average and marginal costs of $AC = MC = 5$. The firm faces a market demand curve given by $Q^D = 53 - P$.

(a) Calculate the profit-maximizing price-quantity combination for the monopolist. Also calculate the monopolists profits and consumer surplus.

(b) What output level would be produced by this industry under perfect competition if every firm could produce at the same average and marginal cost as the monopoly?

(c) Calculate the consumer surplus obtained by consumers in part (b). Show that this exceeds the sum of the monopolists profits and consumer surplus received in part (a). What is the value of the deadweight loss from monopolization?

Solution:

(a) Since $AC = MC = 5$, we know the cost function is given by $C(Q) = 5Q$. Also, we can solve for the inverse demand function to be: $P(Q) = 53 - Q$. The profit-maximizing price-quantity is given by solving the following:

$$\max_Q P(Q)Q - C(Q)$$
$$\max_Q (53 - Q)Q - 5Q$$
$$\max_Q -Q^2 + 48Q$$

Solving we have:

$$\text{F.O.C.:} \; -2Q + 48 = 0 \quad \Rightarrow \quad Q_m = 24$$

Using the demand equation and solving for P, we have:

$$P(24) = 53 - 24 = 29$$

Firms profits are then given by:

$$\pi(24)_m = 24(29 - 5) = 576$$

Consumer Surplus is given by:

$$CS_m = \frac{1}{2}(53 - 29)(24) = 288$$

(b) In perfect competition firms produce such that price is equal to marginal cost, so $P = 5$. Using the demand function, this means $Q_{pc} = 53 - 5 = 48$.

(c) If price is set to \$5 and quantity is 48, then consumer surplus is given by:

$$CS_{pc} = \frac{1}{2}(53 - 5)(48) = 1152$$

The combination of firms profits and consumer surplus from part (b) is given by:

$$\pi_m + CS_m = 576 + 288 = 864$$

Therefore, the value of deadweight loss from monopolization is given by:

$$CS_{pc} - (\pi_m + CS_m) = 1152 - 864 = 288$$

Problem 5: Suppose that a monopolist faces two markets with demand curves given by:

$$D_1(p_1) = 100 - p_1$$
$$D_2(p_2) = 100 - 2p_2$$

Assume that the monopolist's marginal cost is constant at \$20 a unit. If it can price discriminate, what price should it charge in each market in order to maximize profits? What if it can't price discriminate? Then what price should it charge?

Solution: To solve the price-discrimination problem, we first calculate the inverse demand functions:

$$p_1(y_1) = 100 - y_1$$
$$p_2(y_2) = 100 - 0.5y_2$$

Marginal revenue equals marginal cost in each market yields the two equations:

$$100 - 2y_1 = 20$$
$$50 - y_2 = 20$$

Solving we have $y_1^* = 40$ and $y_2^* = 30$. Substituting back into the inverse demand functions gives us the prices $p_1^* = 60$ and $p_2^* = 35$. If the monopolist must charge the same price in each market, we first calculate the total demand:

$$D(p) = D_1(p_l) + D_2(p_2) = 200 - 3p$$

The inverse demand curve is:

$$p(y) = \frac{200}{3} - \frac{y}{3}$$

Marginal revenue equal marginal cost gives us:

$$\frac{200}{3} - \frac{2}{3}y = 20$$

which can be solved to give $y^* = 70$ and $p^* = 43\frac{1}{3}$.