**Problem 1:** A firm is on a competitive market, i.e. takes price of the output as given. Production function is given by $f(x_1, x_2) = x_1^{1/4} x_2^{1/4}$, prices of inputs are $w_1 = 4$, $w_2 = 4$ and price of output is p = 1. Find the profit maximizing level of output using:

(a) Profit-maximization approach

(b) Cost-minimization approach

**Solution:**

(a) **Profit-maximization approach:**

We maximize profit (revenues minus costs) of the firm.

$$\max_{\{x_1,x_2\}} py - w_1x_1 - w_2x_2 \quad \rightarrow \quad \max_{\{x_1,x_2\}} 1x_1^{1/4}x_2^{1/4} - 4x_1 - 4x_2$$

$$FOC[x_1] : \frac{x_2}{4(x_1x_2)^{3/4}} - 4 = 0$$

$$FOC[x_2] : \frac{x_1}{4(x_1x_2)^{3/4}} - 4 = 0$$

Solving these two equations with two unknowns gives:

$$x_1 = x_2 = \frac{1}{256}$$

(b) **Cost-minimization approach:** Consists of two stages: First, we find minimum cost for producing any given level of output $y$. Second, we find optimal value of output $y$.

First stage: find minimum cost for arbitrary level of output $y$:

$$\min_{\{x_1,x_2\}} w_1x_1 + w_2x_2 \quad \rightarrow \quad \min_{\{x_1,x_2\}} 4x_1 + 4x_2$$

$$\text{such that } \; x_1^{1/4}x_2^{1/4} = y \quad \Rightarrow \quad x_2 = \frac{y^4}{x_1}$$

$$\min_{x_1} 4x_1 + 4\frac{y^4}{x_1}$$

$$\text{FOC: } \; 4 - 4\frac{y^4}{x_1^2} = 0 \quad \Rightarrow \quad x_1 = y^2 \; \text{ and } \; x_2 = y^2$$

So in this example, our cost function is:

$$c(y) = 4x_1 + 4x_2 = 4y^2 + 4y^2 = 8y^2$$

Second stage: find optimal level of output $y$:

$$\max_y py - c(y) \quad \rightarrow \quad \max_y y - 8y^2$$

$$\text{FOC:} \;\; 1 - 16y = 0 \quad \Rightarrow \quad y = \frac{1}{16}$$

$$x_1 = x_2 = y^2 = \frac{1}{256}$$

**Problem 2:** Take the set-up from the previous problem. Apart from that the firm has to buy certain equipment before it starts the production. This equipment cost 2000. Compute: variable costs (VC), fixed costs (FC), average variable costs (AVC), average fixed costs (AFC), average costs (AC) and marginal costs (MC).

**Solution:** Note that the cost function is given by: $c(y) = 8y^2 + 2000$

- variable costs: $VC = 8y^2$
- fixed costs: $FC = 2000$
- average variable costs: $AVC = \frac{VC}{y} = 8y$
- average fixed costs: $AFC = \frac{FC}{y} = \frac{2000}{y}$
- average costs: $AC = \frac{c(y)}{y} = AVC + AFC = 8y + \frac{2000}{y}$
- marginal costs: $MC = c'(y) = 16y$

**Problem 3:** The production function is $Q = f(L, K) = 100KL$, $w = 300$, and $r = 1200$. What are the total cost of the firm if the output is $Q = 1600$?

**Solution:** To find the total cost we need to find optimal level of $L$ and $K$, then total cost is given by $Lw + Kr$. In optimum, technical rate of substitution has to equal the ratio of prices of inputs:

$$TRS = \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{\partial f/\partial L}{\partial f/\partial K} = \frac{100K}{100L} = \frac{K}{L}$$
$$\frac{MP_L}{MP_K} = \frac{w}{r} \quad \Rightarrow \quad \frac{K}{L} = \frac{300}{1200} = \frac{1}{4} \quad \Rightarrow \quad L = 4K$$

A profit maximizing firm will always use $K$ and $L$ in the ratio of $\frac{K}{L} = \frac{1}{4}$. Furthermore, we now that the firm produces 1600 units of output, i.e. $Q = 100KL = 1600$. Putting these two conditions together we get:

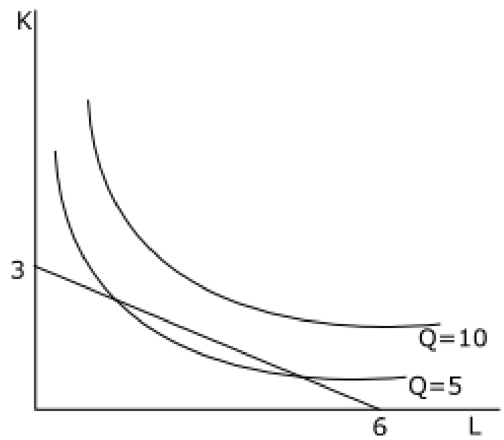
$$100KL = 1600 \text{ and } L = 4K$$
$$100K * 4K = 1600 \quad \Rightarrow \quad K^2 = 4 \quad \Rightarrow \quad K = 2 \text{ and hence } L = 8$$

Now we know how many units of labor and capital the firm uses and hence we can compute its total cost: $TC = Lw + Kr = 8 * 300 + 2 * 1200 = 4800$.

**Problem 4:** The production function is $Q = f(L, K) = K^2L$. Draw isoquants corresponding to $Q = 5$ and $Q = 10$ and isocost for $w = 1$, $r = 2$, and $C = 6$.

**Solution:** Rearranging terms in the production function we get:

$$K = \sqrt{\frac{Q}{L}}$$
$$Q = 5: \quad K = \sqrt{\frac{5}{L}}$$
$$Q = 10: \quad K = \sqrt{\frac{10}{L}}$$



**Problem 5:** Total cost function of an individual firm facing perfect competition is given by relation:

$$TC(Q) = Q^3 - 20Q^2 + 150Q$$

The market price is equal to \$22. Find the optimal level of production of this firm. What is its profit/loss? Draw your solution in a graph.

**Solution:** In optimum the firm chooses the level of production such that its marginal cost is equal to the market price:

$$MC = TC' = 3Q^2 - 40Q + 150 = 22 = P$$
$$Q_{1,2} = \frac{40 \pm \sqrt{40^2 - 4 * 3 * 128}}{2 * 3} = \frac{16}{3}, 8$$

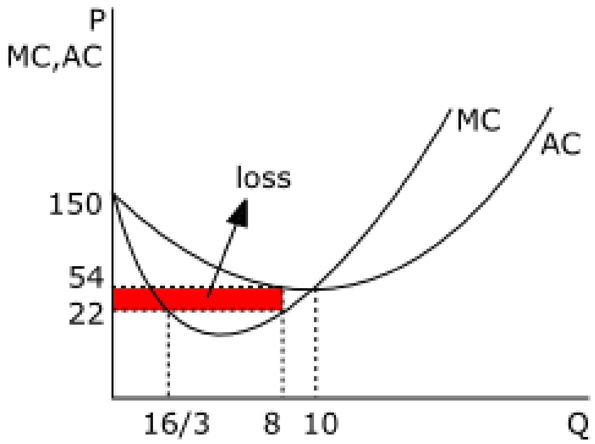
The optimum can never by on decreasing part of marginal cost curve (Why?) so the solution is that $Q = 8$.

To see if the firm makes positive or negative profit we compute average cost:

$$AC = Q^2 - 20Q + 150 = 8^2 - 20 * 8 + 150 = 54$$

We see that in its optimal level of production the company's $AC$ is higher than the price $P$. That means that the profit is negative and it is better to stop the production and produce $Q = 0$. To see that the profit really is negative we can compute it:

$$\pi = P * Q - TC(Q) = 22 * 8 - (8^3 - 20 * 8^2 + 150 * 8) = -256 < 0$$



**Problem 6:** Total cost function of an individual firm facing perfect competition is given in short run by relation:

$$TC(Q) = \frac{Q^3}{3} - 2Q^2 + 5Q$$

(a) Short run. Calculate the individual short run supply of this firm.

(b) Short run. Calculate the optimum of this firm if market price is P=10CZK? (P*; Q*; and corresponding profit/loss).

(c) Long run. Suppose now, that the same cost function applies to the long run and this is a representative firm of industry. Calculate the long run equilibrium market price ($P_M^*$) and corresponding quantity produced by one firm (Q*).

(d) Long run. What will be the total number of firms in industry given that total quantity demanded is 30?

**Solution:**

(a) The price is taken as given and in short run the firm produces a profit maximizing level of output if $MC = P$.

$$MC = TC' = \left(\frac{Q^3}{3} - 2Q^2 + 5Q\right)' = Q^2 - 4Q + 5$$

So the short run supply is implicitly given by the equation $Q^2 - 4Q + 5 = P$. So far we don't have any information about the price so we can not provide any more specific solution.

(b) Now we know that the price is P=10CZK.

$$Q^2 - 4Q + 5 = 10$$
$$Q^2 - 4Q - 5 = 0$$
$$Q_{1,2} = \frac{4 \pm \sqrt{4^2 - 4(-5)}}{2} = \frac{4 \pm 6}{2} = 5, -1$$

If the market price is 10CZK the profit maximizing level of production is $Q^* = 5$. In this case the profit of the firm is:

$$\pi = P * Q - TC = 5 * 10 - \left(\frac{5^3}{3} - 2 * 5^2 + 5 * 5\right) = 50 - \frac{5^3}{3} + 50 - 25 > 0$$

(c) The condition for a long run is that the profit is equal to 0. This holds if price is equal to the minimum average cost, because than the optimality condition $MC = P$ leads to price being equal to average cost and hence the profit is 0.

$$AC = \frac{TC}{Q} = \frac{\frac{Q^3}{3} - 2Q^2 + 5Q}{Q} = \frac{Q^2}{3} - 2Q + 5$$

$$AC' = 0 \quad \Leftrightarrow \quad \frac{2}{3}Q - 2 = 0 \quad \Rightarrow \quad Q^* = 3$$

$$P_M^* = AC(Q = 3) \quad \Rightarrow \quad P_M^* = \frac{3^2}{3} - 2 * 3 + 5 \quad \Rightarrow \quad P_M^* = 2$$

Another way to find a long run solution is to put $AC = MC$:

$$AC = MC \quad \Leftrightarrow \quad \frac{Q^2}{3} - 2Q + 5 = Q^2 - 4Q + 5$$

$$\frac{2}{3}Q^2 - 2Q = 0 \quad \Rightarrow \quad Q(Q-3) = 0 \quad \Rightarrow \quad Q = 0 \text{ or } Q = 3$$

We take the solution that lies on the increasing part of MC curve, i.e. $Q = 3$.

(d) If $Q_M$=20 and each firm produces $Q = 3$ units of production then there are $\frac{30}{3} = 10$ firms in the industry.