Problem 1: There are two games (lotteries) available:

L_1 : With probability 40% you win 110$ and with probability 60% you win 115$

L_2 : With probability 40% you win 50$ and with probability 60% you win 160$

What is the expected value of these two lotteries?

Solution: Expected value is, loosely speaking, an average value of the game:

$$EV_1 = 0.4 * 110 + 0.6 * 115 = 44 + 69 = 113$$
$$EV_2 = 0.4 * 50 + 0.6 * 160 = 20 + 96 = 116$$

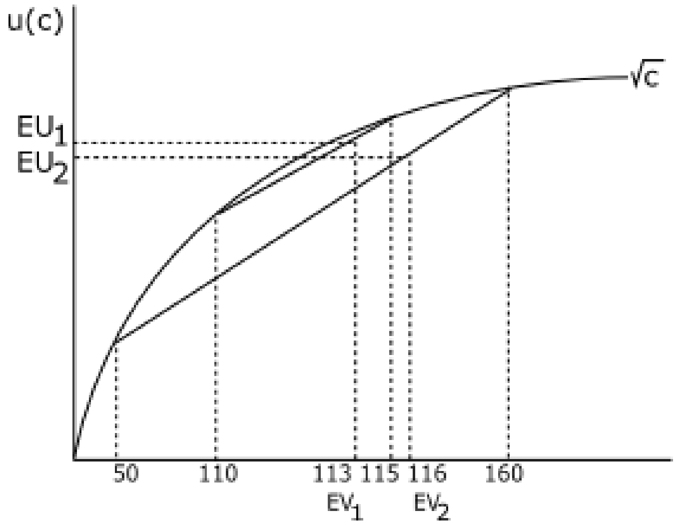
Comparing expected values it seems that L_2 is a better option.

Problem 2: A consumer has the following utility function $u(c) = \sqrt{c}$. Calculate the expected utility of the lotteries from problem 1. Which one will the consumer choose? If his default lottery is L_2 how much would he be willing to pay to be able to switch to L_1?

Solution:

$$EU_1 = 0.4\sqrt{110} + 0.6\sqrt{115} = 4.2 + 6.4 = 10.6$$
$$EU_2 = 0.4\sqrt{50} + 0.6\sqrt{160} = 2.8 + 7.6 = 10.4$$

A simple comparison of expected utilities show that L_1 is preferred to L_2 and the consumer would be willing to pay up to $EU_1 - EU_2 = 10.6 - 10.4 = 0.2$ to switch from L_2 to L_1.



Problem 3: Pat's utility function is $U(c) = \sqrt{c}$, where c is monetary value of his property. Pat owns only a computer, which is worth 1000\$. There is a 20% probability, that it will be stolen and 80%, that it will not.

(a) calculate the expected value of having a computer

(b) calculate the expected utility of this consumer (without insurance)

Suppose now, that Pat can purchase insurance, that will cover the full value of computer in case of theft.

(c) calculate the maximum insurance he is willing to pay. Explain!

(d) depict this on a graph: depict expected utility function and relevant maximum insurance he is willing to pay

(e) what is Pat's attitude towards risk - is he risk averse, risk neutral or risk loving?

Solution:

(a) $EV = 0.2 * 0 + 0.8 * 1000 = 800$

(b) $EU = 0.2\sqrt{0} + 0.8\sqrt{1000} = 8\sqrt{10}$

(c) The consumer is willing to pay at maximum such amount that his expected utility with insurance is at least as large as the expected utility without insurance. Otherwise he would not want the insurance at all:

$$\text{expected utility with insurance} = \text{expected utility without insurance}$$

$$\sqrt{1000 - I} = 0.2\sqrt{0} + 0.8\sqrt{1000}$$

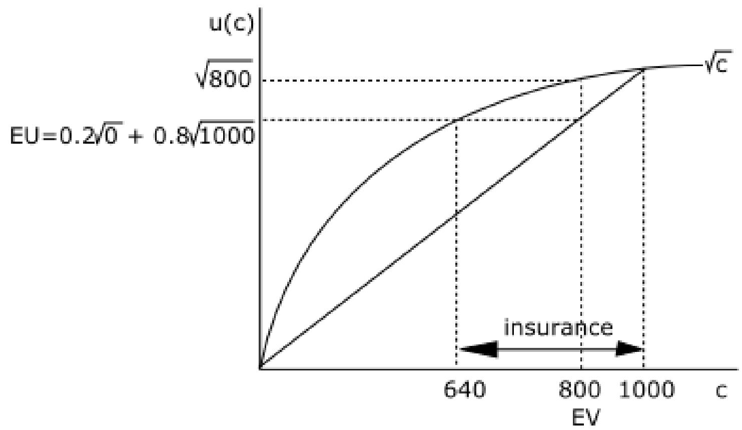
$$1000 - I = 0.64 * 1000$$

$$1000 - I = 640$$

$$I = 360$$

If the insurance costs 360 then Pat is indifferent between getting and not getting the insurance. If the insurance is cheaper Pat will buy it if it's more expensive he will not buy it.

(d) The utility function is depicted on the picture below.



(e) Pat is risk averse because he has concave utility function, i.e. he prefers certain outcome to lottery with the same expected value.

Problem 4: Do the previous problem for Mat with utility function $U(x) = x^2$. Explain any difference between your answers (if any).

Solution:

(a) $EV = 0.2 * 0 + 0.8 * 1000 = 800$

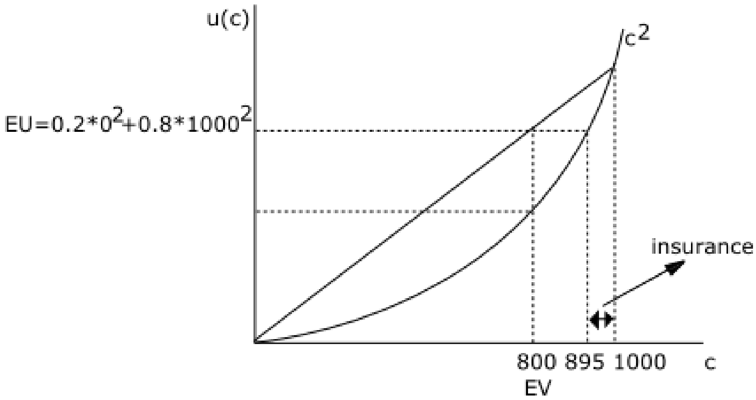
(b) $EU = 0.2 * 0^2 + 0.8 * 1000^2 = 8 * 1000^2$

(c) The consumer is willing to pay at maximum such amount that his expected utility with insurance is at least as large as the expected utility without insurance. Otherwise he would not want the insurance at all:

$$\begin{aligned}
&\text{expected utility with insurance} = \text{expected utility without insurance} \\
&(1000 - I)^2 = 0.2 * 0^2 + 0.8 * 1000^2 \\
&\vdots \\
&I = 105
\end{aligned}$$

If the insurance costs 105 then Mat is indifferent between getting and not getting the insurance. If the insurance is cheaper Mat will buy it if it's more expensive he will not buy it.

(d) The utility function is depicted on the picture below.



(e) Mat is risk loving because he has convex utility function, i.e. he prefers lottery to the certain outcome with its expected value.

Think about the previous problem with risk-neutral consumer with the utility function $u(c) = 2c$.

A risk neutral consumer with linear utility function will be willing to pay for the insurance the following amount:

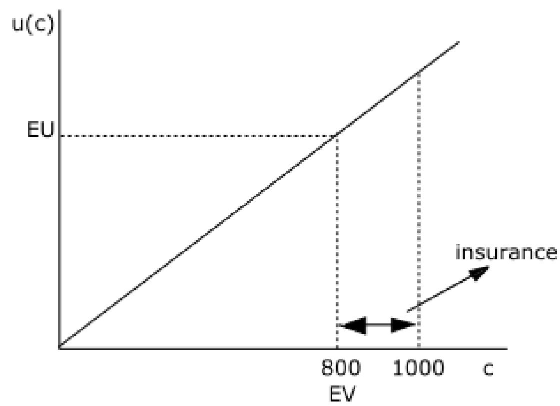
expected utility with insurance = expected utility without insurance

$2(1000 - I) = 0.2(2 * 0) + 0.8(1000)$

$1000 - I = 0.8 * 1000$

$I = 0.2 * 640$

$I = 200$



Problem 5: Jane has utility function over her net income $U(I) = \sqrt{I}$

(a) What are Jane's preferences towards risk? Is she risk averse, risk neutral or risk loving? [Briefly explain your answer]

(b) Jane drives to work every day and she spends a lot of money on parking meters. Often the thought of cheating and not paying for parking crosses her mind. However, she knows that there is a 1/4 probability of being caught on a given day if she cheats, and that the cost of the ticket is \$36. Her daily income is \$100. What is the maximum amount of she will be willing to pay for one day parking?

(c) Paul also faces the same dilemma every single day. But he has a utility function $U(I) = I$. His daily income is also \$100. What are Pauls preferences towards risk? Is he risk averse, risk neutral or risk loving?

(d) If the price of one day parking is \$9.25, will Paul cheat or pay the parking meter? Will Jane cheat or pay the parking meter?

Solution:

(a) We know that Jane is risk averse by the concavity of her utility function. She prefers an amount for sure than any lottery that has that amount as expected value but involves some risk.

(b) She will be indifferent between paying x for parking or facing the cheating lottery when the following equation holds: $U(100 - x) = 0.25 * U(100 - 36) + 0.75 * U(100)$

$$\sqrt{100 - x} = 0.25 * \sqrt{64} + 0.75 * \sqrt{100}$$
$$\sqrt{100 - x} = 0.25 * 8 + 0.75 * 10 = 2 + 7.5 = 9.5$$
$$100 - x = 90.25$$
$$x = 9.75$$

(c) Paul is risk neutral since he has linear utility function over income.

(d) Paul will cheat since: $U(100 - 9.25) < 0.25 * U(100 - 36) + 0.75 * U(100)$
$90.75 < 16 + 75 = 91$

Jane will not cheat since the price of one day parking is lower than what she is willing to pay to avoid the risk of getting caught.