Problem 1: Suppose that the utility function of the consumer is $U(x,y) = 2xy - x$, price of x is $P_x = 3$ and income $I = 64$.

(a) Derive the demand for good y (as a function of its price P_y)

For the further part of the problem assume that the price of y is $P_y = 4$

(b) Find the optimal consumption of good y

(c) Calculate price elasticity at this optimum

(d) If you were monopolist in this market would you rise/lower/maintain the price of y (with respect to price elasticity of demand)? Explain!

Solution:

(a) The demand for good y is a result of utility maximization:

$$\max_{\{x,y\}} 2xy - x$$
$$\text{s.t.} \quad 3x + P_y y = 64$$

We use optimality condition $MRS = \frac{P_x}{P_y}$:

$$MRS = \frac{2y-1}{2x} = \frac{3}{P_y} = \frac{P_x}{P_y} \quad \Rightarrow \quad P_y(2y-1) = 6x$$
$$3x + P_y y = 64$$

Combining the two equations (optimality condition and budget constraint) we get:

$$P_y(2y-1) = 128 - 2P_y y$$
$$y = \frac{128 + P_y}{4P_y} = \frac{128}{4P_y} + \frac{1}{4} = \frac{32}{P_y} + \frac{1}{4} \quad \rightarrow \textbf{ demand function for } y$$

(b) If the price of y, P_y, is equal to 4, then the demand is:

$$y = \frac{32}{P_y} + \frac{1}{4} = \frac{32}{4} + \frac{1}{4} = 8.25$$

(c)

$$\varepsilon_{P_y} = \frac{\Delta y/y}{\Delta P_y/P_y} = \frac{\Delta y}{\Delta P_y}\frac{P_y}{y} = -\frac{32}{{P_y}^2}\frac{P_y}{y} = -\frac{32}{P_y}\frac{1}{y} = -\frac{32}{4}\frac{1}{8.25} = -\frac{32}{33}$$

(d) Increase the price. Increase in price will cause decrease in demand but not so large. So overall profit would increase. Generally, if price elasticity is (in absolute value) less than 1 the price should be increased. If it is more than 1 then the price should be decreased.

Problem 2: Calculating Slutsky's substitution and income effect. Suppose that the consumer has a demand function for beer of the form

$$x_1 = 10 + \frac{I}{10p_1}$$

His income I is \$120 per week and the price of beer is $p_1 = \$3$. Determine the substitution and income effect if the price of beer falls to \$2.

Solution: Original demand for beer is:

$$x_1 = 10 + \frac{I}{10p_1} = 10 + \frac{120}{10 * 3} = 14$$

If price falls to \$2, the demand is:

$$x_1 = 10 + \frac{120}{10 * 2} = 16$$

So the total effect of the change in price is +2 beers per week.

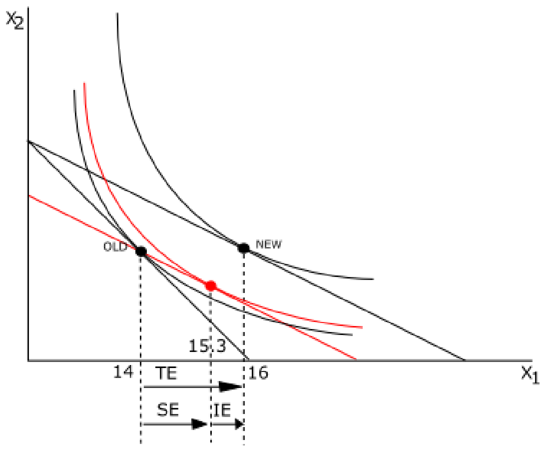
In order to calculate substitution effect, we must first calculate how much income would have to change in order to make the original consumption of beer just affordable when the price of beer is \$2. The price of one beer went go by \$1, the original consumption was 14 beers per week, hence we need to decrease consumers income by $14 * 1 = \$14$.

Thus the level of income necessary to keep purchasing power constant is \$120 - \$14 = \$106. What is the consumer's demand for beer at the new price \$2 and this level of income?

$$x_1 = 10 + \frac{I}{10p_1} = 10 + \frac{106}{10 * 2} = 15.3$$

Thus the substitution effect is 15.3 - 14 = 1.3.

Income effect can be easily calculated using total effect and substitution effect. The total change in demand caused by the decrease in the price is 2 ($14 \rightarrow 16$), change of 1.3 is due to substitution effect, hence the income effect is 0.7.



Problem 3: Suppose that the price elasticity, ϵ, for cigarettes is -4, the price of cigarettes is \$3 per pack and we want to reduce smoking by 20%. What should we do?

Solution: First, recognize that we need to raise the price. Then, figure out by how much:

$$-4 = \epsilon = \frac{\Delta Q/Q}{\Delta P/P} = \frac{-0.2}{\Delta P/3} \quad \Rightarrow \quad \Delta P = \frac{0.2 * 3}{4} = 0.15$$

In order to decrease smoking by 20% we need to increase price by 15 cents.

Problem 4: Consumer consumes two goods with their prices $P_X = 10$, $P_Y = 80$ and has income $I = 5000CZK$. The demand function is given by $X = 80 - 0.8P_X^2 - 0.5P_Y + 0.04I$.

(a) Are X and Y substitutes or complements?

(b) Is X normal or inferior good?

(c) What is price elasticity of demand for good X? What information does this give to the producer of good X?

(d) What is cross elasticity of demand for good X if price of Y changes?

(e) What is income elasticity of demand for good X?

Solution:

(a) If P_Y increases the demand for good X decreases, hence X and Y are complements.

(b) If income increases the demand for good X increases as well, hence X is a normal good.

(c) Price elasticity measures change in the demand for good X caused by the change in its price. So first we need to know what the demand is with original price and income.

$$X = 80 - 0.8P_X^2 - 0.5P_Y + 0.04I = 80 - 0.8 * 100 - 0.5 * 80 + 0.04 * 5000 = 160$$

Price elasticity is given by

$$\epsilon_P = \frac{dX/X}{dP_X/P_X} = \frac{dX}{dP_X}\frac{P_X}{X} = -1.6P_X\frac{P_X}{X} = -1.6 * 10\frac{10}{160} = -1$$

Price elasticity is -1 which means that 1% increase (decrease) in price of X will lead to 1% decrease (increase) in quantity sold. Hence the change in price does not change the revenue.

(d) $\epsilon_C = \dfrac{dX/X}{dP_Y/P_Y} = \dfrac{dX}{dP_Y}\dfrac{P_Y}{X} = -0.5\dfrac{P_Y}{X} = -0.5\dfrac{80}{160} = -\dfrac{1}{4}$

Cross elasticity is negative what means that increase (decrease) in P_Y causes decrease (increase) in consumption of good X which implies that X and Y are complements.

(e) $\epsilon_I = \dfrac{dX/X}{dI/I} = \dfrac{dX}{dI}\dfrac{I}{X} = 0.04\dfrac{I}{X} = 0.04\dfrac{5000}{160} = \dfrac{200}{160} = 1.25$

Income elasticity is positive what implies that good X is normal good (rather luxury than necessary).

Problem 5: Consider the following utility function: $U(x_1, x_2) = 3x_1 + 2x_2$, where x_1, x_2 are the consumption of good 1 (food) and good 2 (clothing) respectively. Let p_1, p_2 and I denote the corresponding prices and income. Suppose that $p_1 = 2, p_2 = 1$. Derive the optimal choice of food and clothing. Draw the Engel curves for food and clothing. Is food a normal good? What about clothing?

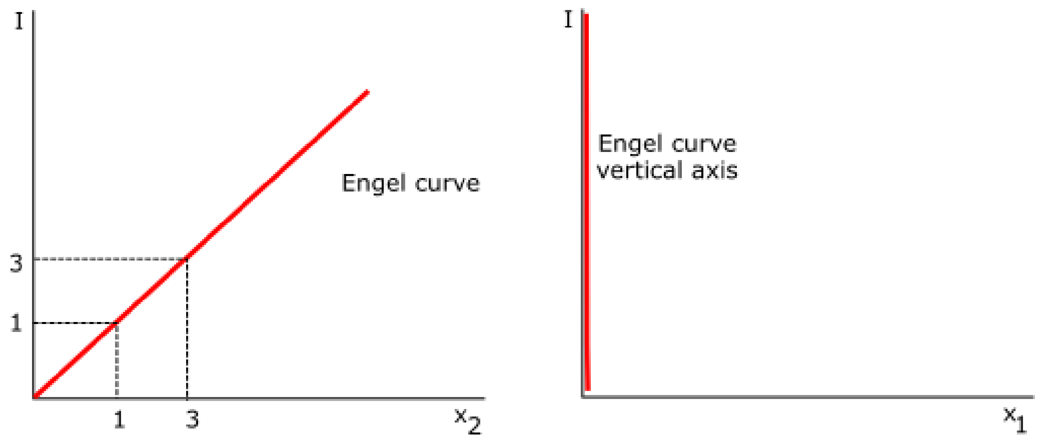
Solution: The utility function is $U(x_1, x_2) = 3x_1 + 2x_2$. This means that x_1 and x_2 are perfect substitutes. The optimal choices are typically corner solutions and we need to decide which good to choose. We first calculate the marginal utilities: $MU_1 = 3$ and $MU_2 = 2$. Now, we compare the marginal utility to price ratios for both goods:

$$\frac{MU_1}{p_1} = \frac{3}{2} < \frac{2}{2} = \frac{MU_2}{p_2}$$

Therefore the consumer will only consume the second good - x_2. Plug $x_1 = 0$ into the budget constraint to get:

$$p_1 x_1 + p_2 x_2 = I \quad \Rightarrow \quad 2 \cdot 0 + 1 \cdot x_2 = I \quad \Rightarrow \quad x_2 = I \text{ and } x_1 = 0$$

So the Engel curve of good 1 (food) is the vertical axis on the graph (zero consumption irrespective of the level of income). The Engel curve of good 2 (clothing) is depicted on the picture below.



Clothing is a normal good. Food is NOT a normal good, NOR an inferior good, since its demand does not change when income changes.