Problem 1: We consider two goods, apples (A) and bananas (B). The prices are given by $p_A = 2$ and $p_B = 4$.

(a) Suppose that Pete has an income y = 20. Derive his budget constraint and draw it into a diagram.

(b) How does the budget constraint of Pete change if

(i) a quantity tax of 2 is levied on apples

(ii) a value tax of 25% is levied on bananas

(iii) his mom forbids him to buy more than 5 apples

(iv) mom increases Petes income to y = 40

(v) prices fall by 50% ?

(c) Suppose that Nicole has 2 apples and 4 bananas. Derive her budget constraint and draw it into a diagram.

(d) How does the budget constraint of Nicole change if

(i) the price of bananas rises by 1

(ii) both prices fall by 50%

(iii) her initial endowment of fruits is doubled?

Problem 2: Antony consumes only apples and bananas. We denote be (x_A, x_B) the consumption bundle which contains x_A apples and x_B bananas. Antony's preferences are described by the utility function:

$$u(x_A, x_B) = \sqrt{x_A x_B}$$

(a) Explain the term indifference curve. Determine the indifference curves which pass through the points (16, 1) and (4, 9). Draw them in a diagram.

(b) Antony's initial endowment is (16,1). Would he exchange his initial endowment for the consumption bundle (4,9)? Indicate in your graph those consumption bundles, which Antony prefers to his initial endowment.

(c) For any consumption bundle, determine the marginal rate of substitution (MRS) for Antony. Explain the meaning of the marginal rate of substitution and check whether Antony's preferences have a falling or an increasing MRS.

(d) Draw the budget constraint of Antony for prices $p_A = 10$ and $p_B = 20$ and income y = 240. Indicate Antony's optimal consumption bundle in your graph.

Problem 3: For each of the following utility functions:

$$u(x_1, x_2) = x_1 + 2x_2$$
$$u(x_1, x_2) = 2\ln x_1 + x_2$$
$$u(x_1, x_2) = \min\{x_1, x_2\}$$

(a) derive the equation describing the indifference curve for a given level of utility $\overline{u}$

(b) sketch several indifference curves in a graph

(c) derive the marginal rate of substitution (whenever possible)