Problem 1: Robinson-Crusoe economy. Consider Robinson-Crusoe sitting on his island who is trying to survive by either gathering bananas (x denotes the number of bananas) or going fishing (y is the number of fish). The technologies which are available for these two production processes are given by:

$$x = \sqrt{L_x}$$
$$y = 0.5\sqrt{L_y}$$

where L_x is the amount of labor (time) used for gathering bananas and L_y the time devoted to fishing. Notice that there are diminishing returns in both activities. Total time (L) is constrained to 15 hours per day. So $L_x + L_y = L = 15$.

Further we assume that Robinson Crusoe's preferences are described by a Cobb-Douglas utility function:

$$U(x, y) = \sqrt{x}\sqrt{y}$$

(a) Construct the production possibility frontier (PPF).

(b) Sketch several indifference curves and find marginal rate of substitution (MRS).

(c) Find the optimal combination of two goods and optimal allocation of time.

Problem 2: Two countries, Brazil and Vietnam, can each produce two goods: coffee and tea. In one year, Brazil can produce 40 units of coffee (millions of 60kg-bags) or 200 units (tons) of tea or a mix of the two. In one year, Vietnam can produce 15 units of coffee or 150 units of tea or a mix of the two. For both countries, the marginal returns are decreasing. This means that using more and more resources to produce coffee brings less and less additional output of tea and vice versa for both countries.

(a) What are Brazil's opportunity cost of producing coffee? How about Vietnam's opportunity cost of tea production?

(b) Suppose that each country prefers to consume the commodity which is produced with relatively higher costs. Which of the commodities is it (for both Brazil and Vietnam)?

(c) Sketch the optimal points of consumption on a graph with tea on horizontal and coffee on vertical axis.

(d) Show graphically, that there exist prices (coffee for tea), that both countries would accept (they will both get to higher IC).