Problem 1: The product market can be characterized by functions:

inverse demand function: $P = 55 - 0.1Q_D$

supply function: $Q_S = 100P$

and the representative firm has a production function: $q = 70L - L^2$ where L is number of units of labor hired at perfectly competitive labor market with equilibrium wage w=100 CZK.

(a) calculate the product market equilibrium price and quantity (P^*, Q^*)

(b) calculate the optimal number of workers hired by one firm

(c) how many units of q will firm produce in optimum?

(d) what would be the profit/loss of firm in this (ad b) short run optimum?

Solution:

(a) In the equilibrium quantity supplied equals quantity demanded: $Q_S = Q_D$

$$P = 55 - 0.1Q \quad \text{and} \quad Q = 100P$$
$$P = 55 - 0.1(100P) \quad \Rightarrow \quad 11P = 55 \quad \Rightarrow \quad P^* = 5 \quad \text{and} \quad Q^* = 500$$

(b) In optimum **the value of** marginal product equals wage. (Not marginal product equals wage but its value equals wage; marginal product is in units of production wage is in units of money - we can not compare these two, we have to compare value of marginal product with wage.)

$$VMP_L = w$$
$$P * MP_L = w$$
$$P[70L - L^2]' = P(70 - 2L) = 100 \quad \Rightarrow \quad L^* = 35 - \frac{50}{P} = 35 - \frac{50}{5} = 25$$

(c) In this optimum, the firm produces:

$$q = 70L^* - (L^*)^2 = 70 * 25 - 25^2 = 1125$$

(d) If price equals 5 and the labor used is 25 then the profit of the firm is:

$$\pi = TR - TC = P * q - w * L = 5 * 1125 - 100 * 25 = 3125$$

Note: Notice that the individual output ($q = 1125$) is larger than the industry output ($Q = 500$). This result is caused by the particular numbers in this exercise and is hard to interpret. Usually we should get the opposite result - that the individual output is smaller and hence we need more firms to satisfy the market demand.

Problem 2: Intertemporal choice. Ben earns \$110 this year and \$110 the next year. The interest rate in a bank is 10%. Ben's preferences over current and future consumption are represented by the following utility function:

$$U(c, f) = \sqrt{cf}$$

Will Ben save part of his income during the first year or will he take a loan?

Solution: Ben maximizes his utility with respect to his budget constraint. If he spends all money today - on current consumption - (including his future income) he can spend:

$$110 + \frac{110}{1 + 0.1} = 210$$

If he spends all money next year - on future consumption - (including his current income) he can spend:

$$110 + 110(1 + 0.1) = 231$$

In optimum - the combination of current and future spending has to lie on the budget line and on the highest possible indifference curve. The budget line is given by $f = 231 - 1.1c$, because the vertical intercept is 231 and the slope is given by $231/210 = 1.1$ (the slope of intertemporal budget line is always equal to $(1 + r)$). In optimum, the indifference curve touches the budget line, that means that the slope of indifference curve (MRS) has to be equal to slope of budget line (1.1):

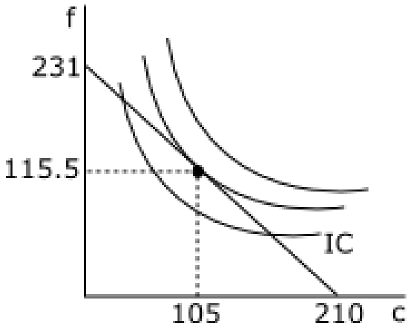
$$\text{Budget constraint}: \quad 1.1c + f = 231$$

$$MRS = \frac{MU_c}{MU_f} = \frac{\partial U/\partial c}{\partial U/\partial f} = \frac{\frac{f}{2\sqrt{cf}}}{\frac{c}{2\sqrt{cf}}} = \frac{f}{c} = 1.1 \quad \Rightarrow \quad f = 1.1c$$

$$1.1c + 1.1c = 231 \quad \Rightarrow \quad c = \frac{231}{2.2} = 105 \text{ and } f = 1.1c = 115.5$$

Ben's current spending is less than his current income and his future spending is more than future income. This means that Ben is saving. Ben saves 5 in current year and hence he can spend $5(1+r)$ more the next year, i.e he can spend 110+5(1+0.1)=115.5.

Ben's budget and the optimal choice is depicted on the picture below.



Problem 3: Market equilibrium - two markets. Demand functions for two goods are given by:

$$P_1 = 200 - Q_1 + 0.3P_2$$
$$P_2 = 100 - 2Q_2 + 0.5P_1$$

Suppose that the supply is fixed at the level of $Q_1 = 50$ and $Q_2 = 10$. Are these two goods substitutes or complements? What will be the equilibrium prices? Now suppose that Q_1 increases to 71. What will be new equilibrium prices?

Solution: Substitutes because if price of one good goes up the demanded quantity of the other good goes up as well.

$$P_1 = 200 - 50 + 0.3P_2 \quad \Rightarrow \quad P_1 = 150 + 0.3P_2$$
$$P_2 = 100 - 2 * 10 + 0.5P_1 \quad \Rightarrow \quad P_2 = 80 + 0.5P_1$$
$$P_1 = 150 + 0.3(80 + 0.5P_1) = 174 + 0.15P_1 \quad \Rightarrow \quad P_1 = 204; \quad P_2 = 182$$

Now, Q_1 increases to 71.

$$P_1 = 200 - 71 + 0.3P_2 \quad \Rightarrow \quad P_1 = 129 + 0.3P_2$$
$$P_2 = 100 - 2 * 10 + 0.5P_1 \quad \Rightarrow \quad P_2 = 80 + 0.5P_1$$
$$P_1 = 129 + 0.3(80 + 0.5P_1) = 153 + 0.15P_1 \quad \Rightarrow \quad P_1 = 180; \quad P_2 = 170$$

Increase in supplied quantity of one of the substitutes results in the decrease of price of both goods.

Problem 4: Consider the following story from the Second World War. There are two prisoners of war in a German camp: British (consumer A) and French (consumer B). Both of them have a right to get some weekly amount of tea (good 1) and coffee (good 2). British prisoner has the endowment $\omega_A = (1, 4)$ and French prisoner, being privileged, has the endowment $\omega_B = (5, 4)$. The two prisoners are totally separated and the direct exchange is not possible, but they succeeded to persuade a German prisoners' priest to transfer coffee and tee between them. The prisoners' preferences are given by the following utility functions:

$$u^A(x_1^A, x_2^A) = 2 \ln x_1^A + x_2^A$$
$$u^B(x_1^B, x_2^B) = 4 \ln x_1^B + x_2^B$$

where x_1^i is the amount of good 1 consumer i consumes and x_2^i the amount of good 2. Suppose that the price of good 1 is p_1 and the price of good 2 is p_2.

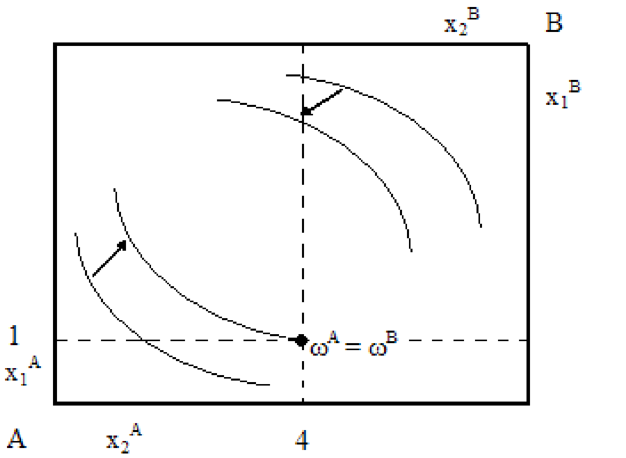
(a) Sketch the corresponding Edgeworth box. In the Edgeworth box draw several indifference curves of both agents and mark their initial endowment. Find Pareto efficient (Pareto optimal) allocations.

(b) Find the market demand functions x_1^A, x_1^B.

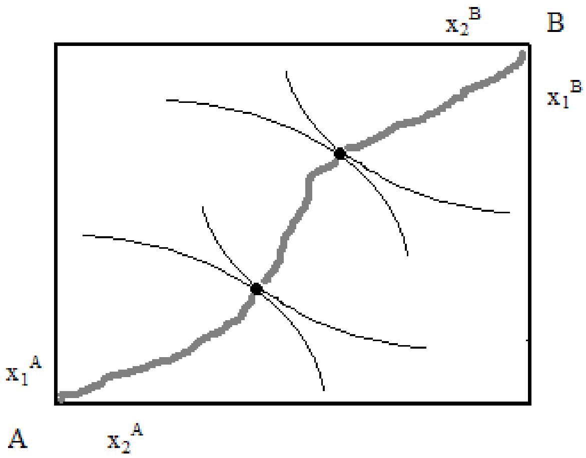
(c) Find the competitive equilibrium (prices and allocations) for this prisoners' economy.

Solution:

1.



Edgeworth box with indifference curves and endowment



Pareto efficient allocations

2. To find consumers' demand functions we solve their optimization problems:

$$\begin{aligned}
\text{Consumer A:} \quad & \max_{\{x_1^A, x_2^A\}} 2\ln x_1^A + x_2^A \\
& s.t. \;\; p_1 x_1^A + p_2 x_2^A = p_1 + 4p_2 \\
\text{Consumer B:} \quad & \max_{\{x_1^B, x_2^B\}} 4\ln x_1^B + x_2^B \\
& s.t. \;\; p_1 x_1^B + p_2 x_2^B = 5p_1 + 4p_2
\end{aligned}$$

We choose good 1 to be a numeraire, therefore $p_1 = 1$ and for simplicity we denote $p_2 = p$. Furthermore, we use equalities in budget constraints.

$$\begin{aligned}
\text{Consumer A:} \quad & \max_{\{x_1^A, x_2^A\}} 2\ln x_1^A + x_2^A \\
& s.t. \;\; x_1^A + p x_2^A = 1 + 4p \\
\text{Consumer B:} \quad & \max_{\{x_1^B, x_2^B\}} 4\ln x_1^B + x_2^B \\
& s.t. \;\; x_1^B + p x_2^B = 5 + 4p
\end{aligned}$$

Now we plug budget constraints into the objective functions and take the first order conditions:

$$\begin{aligned}
\text{Consumer A:} \quad & \max_{x_2^A} 2\ln(1 + 4p - p x_2^A) + x_2^A \\
& \text{FOC:} \frac{2(-p)}{1 + 4p - p x_2^A} + 1 = 0 \Rightarrow x_2^A = \frac{2p+1}{p} \\
\text{Consumer B:} \quad & \max_{x_2^B} 4\ln(5 + 4p - p x_2^B) + x_2^B \\
& \text{FOC:} \frac{4(-p)}{5 + 4p - p x_2^B} + 1 = 0 \Rightarrow x_2^B = \frac{5}{p}
\end{aligned}$$

Therefore the demand functions are $x_2^A = \frac{2p+1}{p}$ and $x_2^B = \frac{5}{p}$. And from budget constraints we get the demands $x_1^A = 2p$ and $x_1^B = 4p$.

3. Competitive equilibrium consists of equilibrium prices (only price p_2 needs to be determined since we set the price p_1 equal to 1) and allocations $\{x_1^A, x_2^A\}, \{x_1^B, x_2^B\}$. In equilibrium both markets (market for good 1 and market for good 2) clear:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \quad \Leftrightarrow \quad 2p + 4p = 6 \quad \Rightarrow \quad p = 1$$

Here, we check if market for good 2 clears for price p=1 as well:

$$\frac{2p+1}{p} + \frac{5}{p} = 8 \quad \Rightarrow \quad p = 1$$

Hence, the competitive equilibrium is:

$$\{x_1^A, x_2^A\} = (2,3); \ \{x_1^B, x_2^B\} = (4,5); \ p_1 = 1; \ p_2 = 1.$$

Problem 5: *This problem was not covered during the seminar. It is similar to the previous problem but this time the indifference curves do not have standard shape as before.* Consider a following two-consumers (A,B) and two-goods (x,y) economy. The utility function of the first consumer is

$$u^A = 2x^A + y^A$$

and his endowment is $\omega^A = (4,0)$. The second consumer has the following utility function

$$u^B = \min\{x^B, 2y^B\}$$

and endowment $\omega^B = (0,3)$.

1. Find the competitive equilibrium and graph it in the Edgeworth box.
2. Find Pareto efficient allocations and graph them in the Edgeworth box.

Solution:

1. Competitive equilibrium consists of equilibrium prices p_x, p_y and allocations $\{x^A, y^A\}$, $\{x^B, y^B\}$. Consumers solve following optimization problems:

$$\text{Consumer A:} \quad \max_{\{x^A,y^A\}} 2x^A + y^A \quad s.t.\ x^A + py^A \leq 4$$

$$\text{Consumer B:} \quad \max_{\{x^B,y^B\}} \min\{x^B, 2y^B\} \quad s.t.\ x^B + py^B \leq 3p$$

   where we choose good x to be numeraire and denote $p_y = p$.

In Consumer A'a optimization problem we replace inequality constraint with equality and plug it into the function to be maximized:

$$\text{Consumer A:} \quad \max_{y^A} 2(4 - py^A) + y^A$$

$$FOC: -2p + 1 = 0 \Rightarrow p = \frac{1}{2}$$

In Consumer B's optimization problem it must hold that $x^B = 2y^B$.

$$\text{Consumer B:} \quad \max_{\{x^B, y^B\}} \min\{x^B, 2y^B\}$$

$$s.t. \ 2y^B + py^B = 3p$$

$$\text{since } \ p = \frac{1}{2}, \ \text{we get: } y^B = \frac{3}{5}$$

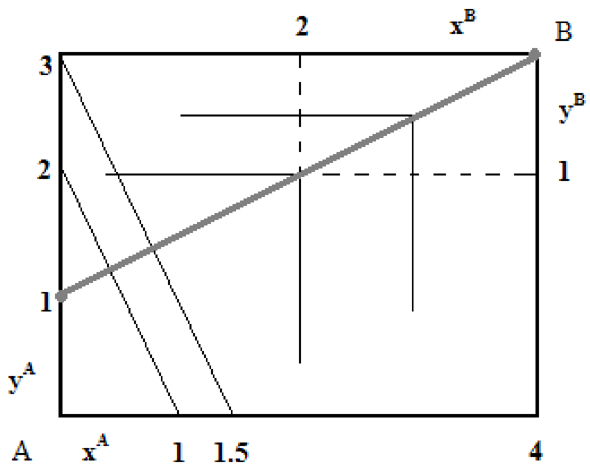
$$\text{and from budget constraint: } \ x^B = \frac{6}{5}$$

Finally, we determine x^A and y^A from market clearing conditions:

$$x^A + x^B = 4 \Rightarrow x^A = 4 - \frac{6}{5} = \frac{14}{5}$$

$$y^A + y^B = 3 \Rightarrow y^A = 3 - \frac{3}{5} = \frac{12}{5}$$

Hence, the competitive equilibrium is: $\{p_x, p_y\} = \{1, 1/2\}; \{x^A, y^A\} = \{14/5, 12/5\}$; $\{x^B, y^B\} = \{6/5, 3/5\}$.



The set of Pareto efficient allocations is given by:

$$x^A \ \text{ and } \ y^A = 1 + \frac{x^A}{2}$$

$$x^B = 4 - x^A \ \text{ and } \ y^B = 3 - y^A$$