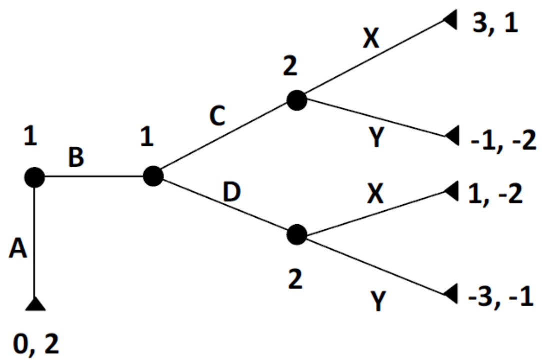
VSE - Introduction to Game Theory
Problem set #2 - Due Wednesday, December 2, 2015 – Suggested Solution

Teamwork is an important part of this course. Therefore, please **work in groups of up to 4 students.** Each student can only be in one group. Each group submits **one copy** of problem set with the names of all members.

Homework can be delivered: (1) by email or (2) personally during the lecture or office hours.

No late submissions will be accepted.

Problem 1 [22 points]: List the strategies of the following game and find Subgame perfect Nash equilibria and Nash equilibria in pure strategies. Is there any NE that is not SPNE?

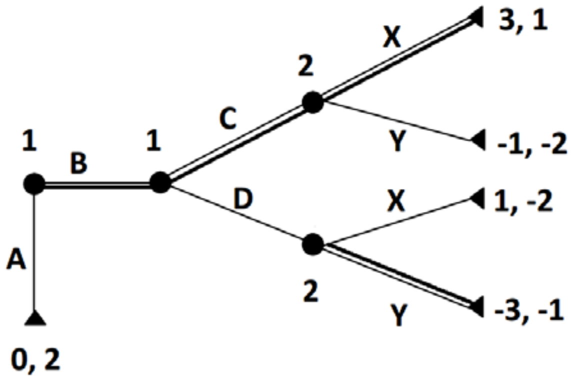


Solution: Payer 1 has two decision nodes and four strategies: AC, AD, BC, BD. Similarly, Player 2 also has two decision nodes and four strategies XX, XY, YX, YY.

1 \ 2	XX	XY	YX	YY
AC	0,2	0,2	0,2	0,2
AD	0,2	0,2	0,2	0,2
BC	3,1	3,1	-1,-2	-1,-2
BD	1,-2	-3,-1	1,-2	-3,-1

There are four Nash equilibria in this game: (AC,YY), (AD,YY), (BC,XX) and (BC,XY).

The SPNE of this game is (BC,XY).



Problem 2 [28 points]: Consider the following game of incomplete information between Pat and Mat. Pat's type is known but Mat may be either an H type (with probability p) or an L type (with probability 1-p). The payoffs to this simultaneous game of incomplete information are as follows:

H type

P1\P2	X	Y
A	1,1	0,0
B	0,0	0,0

L type

P1\P2	X	Y
A	0,0	0,0
B	0,0	2,2

a) Suppose that p = 0.75. Find all pure strategy Bayes-Nash equilibria.
b) Suppose that p = 0.25. Find all pure strategy Bayes-Nash equilibria.

Solution:

a) For p=0.75 we construct the following payoff table:

1 \ 2	XX	XY	YX	YY
A	0.75,1,0	0.75,1,0	0,0,0	0,0,0
B	0,0,0	0.5,0,2	0,0,0	0.5,0,2

Nash equilibria in this game: (A,XX), (A,XY), (B,YY).

b) For p=0.25 we construct the following payoff table:

1 \ 2	XX	XY	YX	YY
A	0.25,1,0	0.25,1,0	0,0,0	0,0,0
B	0,0,0	1.5,0,2	0,0,0	1.5,0,2

Nash equilibria in this game: (A,XX), (B,XY), (B,YY).

Problem 3 [24 points]: Find the following Nash equilibria in a second-price sealed-bid auction:

a) Player 1 (player with the highest valuation) obtains the object.
b) Player 1 (player with the highest valuation) obtains the object – different from (a).
c) Player 4 (player with the fourth highest valuation) obtains the object.

Note that to be able to solve auction problems we need to express person's willingness to obtain an object in monetary units. Therefore if player values an object for 100 CZK then he is indifferent between not winning the auction and winning and getting the object for 100 CZK).

Solution: First, we order n players according to their valuations: $v_1 > v_2 > v_3 > \ldots > v_n$. Player one is the one with the highest valuation v_1 and so on. Each player i submits a bid b_i.

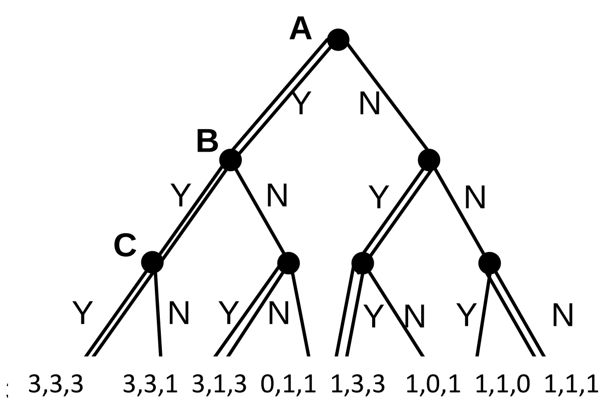
a) One possible Nash equilibrium of this game is $(b_1,b_2,b_3,b_4,b_5,\ldots,b_n) = (v_1, v_2,\ldots,v_n)$. Player 1 wins the object and pays the second highest price which is v_2. This winning player has no incentive to change the action. Any of the other players would have to bid more than v_1 in order to win the auction and then the second highest bid and paid price would be v_1. This would mean negative payoff for a winner (or zero payoff in case of player 1 winning the auction). This means that no player wants to change their action and we have a Nash equilibrium.
b) Another possible Nash equilibrium of this game is $(b_1,b_2,b_3,b_4,b_5,\ldots b_n) = (v_2, 0, 0,\ldots,0)$. Player 1 wins the object and pays the second highest price which is 0. This winning player has no incentive to change the action. Any of the other players would have to bid more than v_2 in order to win the auction and then the second highest bid and paid price would be v_2. This would mean negative payoff for a winner (or zero payoff in case of player 1 winning the auction). This means that no player wants to change their action and we have a Nash equilibrium.
c) One possible Nash equilibrium of this game is $(b_1,b_2,b_3,b_4,b_5,\ldots,b_n) = (0,0,0,v_1,0,\ldots,0)$. Player 4 with the fourth highest valuation wins the object and pays the second highest price which is 0. This winning player has no incentive to change the action. Any of the other players would have to bid more than v_1 in order to win the auction and then the second highest bid and paid price would be v_1. This would mean negative payoff for a winner (or zero payoff in case of player 1 winning the auction). This means that no player wants to change their action and we have a Nash equilibrium.

Problem 4 [26 points]: Amy, Barbara and Carol are independently thinking of getting a tattoo. They are meeting at a party in a few days. Each has to decide whether or not to go to the tattoo parlor. For each of them the most preferred outcome is one where she shows up with a tattoo and at least one of her friends also does (being indifferent between being one of two or one of three). The least preferred outcome is to be the only one of the three with a tattoo. The middle-ranked outcomes are the ones where she does not have a tattoo (she is indifferent among those outcomes).

a) Find the Nash equilibrium/a in pure strategies.
b) Find subgame perfect NE if this game is sequential and each friend informs the others about whether she got a tattoo or not. The order of players is Amy, Barbara, Carol.
c) What if there are five friends instead of three? How would NE change?

Solution:

a) In case of three players, it is not possible to construct one table through which we can find NE, we would need more tables. Alternatively, we can analyze or possibilities to find NE:
    - All three girls get a tattoo: NE, that's the most preferable outcome, no reason to change it.
    - No girl gets a tattoo: NE, changing this action would lead to be the only one with tattoo – the least preferred outcome.
    - One girl gets a tattoo: not NE, one girl with a tattoo would be better off not having it.
    - Two girls get a tattoo: not NE, one girl with no tattoo would be better off getting it.
b) Subgame perfect NE in this game is (Y, YY, YYYN).



c) No change in NE, there would be two NE: all girls get tattoo and no one does.