

Problem 1: Suppose the demand function for corn is $Q_d = 10 - 2p$, and supply function is $Q_s = 3p - 5$. The government is concerned that the market equilibrium price of corn is too low and would like to implement a price support policy to protect the farmers. By implementing the price support policy, the government sets a support price and purchases the extra supply at the support price. In this case, the government sets the support price $p_s = 4$.

- (a) Calculate the original market equilibrium price and quantity in absence of the price support policy.
- (b) At the support price $p_s = 4$, find the quantity supplied by the farmers, the quantity demanded by the market, and the quantity purchased by the government.
- (c) Draw a diagram to show the change in the producer surplus due to the implementation of the price support policy. Calculate the change in the producer surplus.
- (d) Draw a diagram to show the change in the consumer surplus due to the implementation of the price support policy. Calculate the change in the consumer surplus.
- (e) Calculate the cost to the government to implement the price support policy. Draw a diagram to show the government cost.
- (f) Suppose now the government switches from price support policy to subsidy policy. For each unit of corn produced, the government subsidizes the farmer $s = \frac{5}{3}$. Find the new equilibrium price under this subsidy policy. How much money will the government have to spend in order to implement this subsidy policy?

Solution:

- (a) In equilibrium $Q_s = Q_d$:

$$\begin{aligned}Q_d &= 10 - 2p = 3p - 5 = Q_s \\5p &= 15 \Rightarrow p = 3\end{aligned}$$

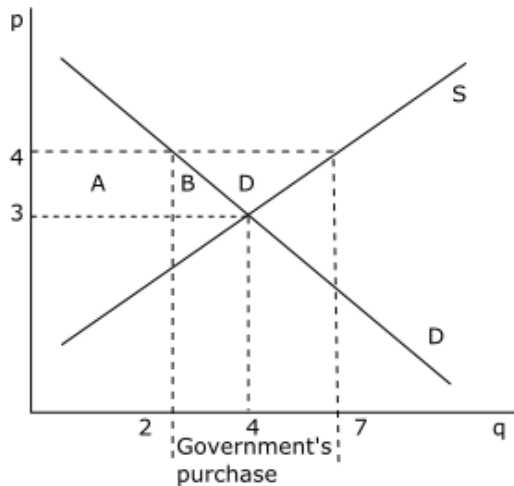
Knowing the equilibrium price we use either supply or demand equation to determine that the equilibrium quantity bought and sold is 4.

(b) When the price is $p_s = 4$, the demand is $Q_d = 10 - 2p = 2$, and supply will be $Q_s = 3p - 5 = 7$, so the government needs to buy 5.

(c) The producer's gain is:

$$1 \times 7 - \frac{1}{2}(7 - 4) = \frac{11}{2}$$

which is the area A+B+D on the graph below.



(d) The loss to consumer is:

$$2(4 - 3) + (4 - 2)(4 - 3)/2 = 3$$

which is the area A+B in the graph above.

(e) Cost to the government: $P_s Q_g = 20$, which is the rectangle area indicated in the above graph by Govs purchase \times price.

(f) Under subsidy $s = \frac{5}{3}$, the suppliers face the price $p_b + \frac{5}{3}$, and consumers face price p_b , thus:

$$10 - 2p_b = 3(p_b + s) - 5 \Rightarrow 10 = 5p_b \Rightarrow p_b = 2$$

The new amount of supply is $Q = 10 - 2p_b = 6$, which is the amount that the government needs to subsidize. The government's total budget is $s \times Q = 10$.

Problem 2: Total cost function of an individual firm facing perfect competition is given in short run by relation:

$$TC = \frac{q^3}{3} - 4q^2 + 21q$$

- Short run. Calculate the individual short run supply of this firm.
- Short run. Calculate the optimum of this firm if market price is $p=5$ Kč? (p^* ; q^* ; and corresponding profit/loss).
- Long run. Suppose now, that the same cost function applies to the long run and this is a representative firm of industry. Calculate the long run equilibrium market price (p_M^*) and corresponding quantity produced by one firm (Q^*).
- Long run. What will be the total number of firms in industry given that total quantity demanded p_M^* (ad (c)) is 420?

Solution:

- In short run the supply is given by MC curve. The optimality condition is that $MC = p$. In our example:

$$MC = TC' = q^2 - 8q + 21 = p$$

Note that in general this quadratic equation has two solutions. In such case the optimal level of output is the higher q because the lower q lies on the decreasing part of MC curve and there can never be an optimum on decreasing part of MC .

- $p=5$ Kč:

$$MC = TC' = q^2 - 8q + 21 = p = 5$$

$$q^2 - 8q + 16 = 0$$

$$q_{1,2} = \frac{8 \pm \sqrt{8^2 - 4 \times 16}}{2} = 4$$

If $p^* = 5$ and q^* then the profit of the firm is:

$$\begin{aligned} \pi &= p \times q - TC = p \times q - \frac{q^3}{3} + 4q^2 - 21q = 4 \times 5 - \frac{4^3}{3} + 4 \times 4^2 - 21 \times 4 = \\ &= 20 - \frac{64}{3} + 64 - 84 = -\frac{64}{3} < 0 \end{aligned}$$

Even if the firm produces the profit maximization level of output $q = 4$ its profit is negative. Hence, it is optimal to stop production and earn zero profit.

- (c) In the long run the optimality condition requires that the profit of each firm is zero. The condition $MC = p$ still holds and in order to earn zero profit we have that $MC = p = \min ATC$.

$$ATC = \frac{TC}{q} = \frac{q^2}{3} - 4q + 21$$
$$\min ATC = \min \frac{q^2}{3} - 4q + 21$$
$$\text{FOC: } \frac{2}{3}q - 4 = 0 \Rightarrow q^* = 6$$

The equilibrium price is given by condition that $MC = p$:

$$MC = p \Rightarrow q^2 - 8q + 21 = p$$
$$6^2 - 8 \times 6 + 21 = p = 36 - 48 + 21 = 9$$

- (d) If $p_M^* = 9$ and $q^* = 6$ then there will be $\frac{420}{6} = 70$ firms.

Problem 3: Market demand is given by function: $Q = 100 - P$. Total cost function of a monopoly is given by relation $TC = Q^2$.

- (a) calculate the optimum of monopoly (P^* ; Q^* ; and corresponding profit).
- (b) depict this optimum and profit on a graph (depict correct shapes/forms of cost functions!); put numbers to it.
- (c) suppose now, that government imposed tax $t=10$ to each unit sold. Tax is being paid by the firm. Calculate the new optimum of monopoly (P^* ; Q^*).
- (d) depict this new optimum (P^* ; Q^*) on a graph.

Solution:

(a) monopoly optimum and its profit:

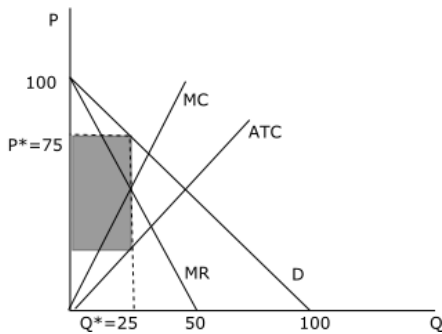
$$\pi = TR - TC = P \times Q - TC = (100 - Q)Q - Q^2 = 100Q - 2Q^2$$

$$\text{FOC: } 100 - 4Q = 0 \Rightarrow Q^* = 25 \text{ and } P^* = 100 - 25 = 75$$

For this set of price and quantity the profit of the monopolist is:

$$\pi = TR - TC = P \times Q - TC = 75 \times 25 - 25^2 = 1875 - 625 = 1250$$

(b) The profit of the monopolist is depicted as shaded rectangle on the picture below.



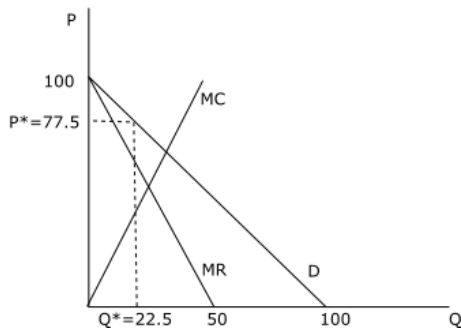
(c) unit tax $t=10$ is imposed.

$$\pi = TR - TC = (P - 10) \times Q - TC = (100 - Q - 10)Q - Q^2 = 90Q - 2Q^2$$

$$\text{FOC: } 90 - 4Q = 0 \Rightarrow Q^* = 22.5 \text{ and } P^* = 100 - 22.5 = 77.5$$

The new equilibrium price will be 77.5 but the monopolist pays the tax and keeps only 67.5

(d) Now, the optimum level of output is such that the difference between MR and MC is equal to 10.

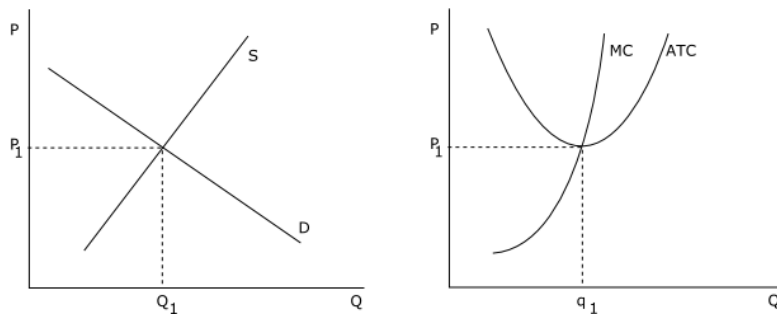


Problem 4: Bestmilk, a typical profit-maximizing dairy firm, is operating in a constant-cost, perfectly competitive industry that is in long-run equilibrium.

- (a) Draw correctly labeled side-by-side graphs for the dairy market and for Bestmilk and show: Price and output for the industry; Price and output for Bestmilk
- (b) Assume that milk is a normal good and that consumer income falls. Assume that Bestmilk continues to produce. On your graphs in part (a), show the effect of the decrease in income on each of the following in the short run: Price and output for the industry; Price and output for Bestmilk; Area of loss or profit for Bestmilk.

Solution:

- (a) Prices and quantities are depicted on the picture below.



- (b) Bestmilk will be in a loss which is represented by a shaded area on the picture below.

