**Problem 1:** A firm is on a competitive market, i.e. takes price of the output as given. Production function is given by $f(x_1, x_2) = x_1^{1/4} x_2^{1/4}$, prices of inputs are $w_1 = 4$, $w_2 = 4$ and price of output is p = 1. Find the profit maximizing level of output using:

(a) Profit-maximization approach

(b) Cost-minimization approach

**Solution:**

(a) **Profit-maximization approach:**

We maximize profit (revenues minus costs) of the firm.

$$\max_{\{x_1,x_2\}} py - w_1x_1 - w_2x_2 \quad \rightarrow \quad \max_{\{x_1,x_2\}} 1x_1^{1/4}x_2^{1/4} - 4x_1 - 4x_2$$

$$FOC[x_1] : \frac{x_2}{4(x_1x_2)^{3/4}} - 4 = 0$$

$$FOC[x_2] : \frac{x_1}{4(x_1x_2)^{3/4}} - 4 = 0$$

Solving these two equations with two unknowns gives:

$$x_1 = x_2 = \frac{1}{256}$$

(b) **Cost-minimization approach:** Consists of two stages: First, we find minimum cost for producing any given level of output $y$. Second, we find optimal value of output $y$.

First stage: find minimum cost for arbitrary level of output $y$:

$$\min_{\{x_1,x_2\}} w_1x_1 + w_2x_2 \quad \rightarrow \quad \min_{\{x_1,x_2\}} 4x_1 + 4x_2$$

$$\text{such that } \; x_1^{1/4}x_2^{1/4} = y \quad \Rightarrow \quad x_2 = \frac{y^4}{x_1}$$

$$\min_{x_1} 4x_1 + 4\frac{y^4}{x_1}$$

$$\text{FOC: } \; 4 - 4\frac{y^4}{x_1^2} = 0 \quad \Rightarrow \quad x_1 = y^2 \; \text{ and } \; x_2 = y^2$$

So in this example, our cost function is:

$$c(y) = 4x_1 + 4x_2 = 4y^2 + 4y^2 = 8y^2$$

Second stage: find optimal level of output $y$:

$$\max_y py - c(y) \quad \rightarrow \quad \max_y y - 8y^2$$
$$\text{FOC: } 1 - 16y = 0 \quad \Rightarrow \quad y = \frac{1}{16}$$
$$x_1 = x_2 = y^2 = \frac{1}{256}$$

**Profit maximization $\leftrightarrow$ Cost minimization.** If a firm is maximizing profits and if it chooses to supply some output $y$, then it must be minimizing the cost of producing $y$. If this were not so, then there would be some cheaper way of producing y units of output, which would mean that the firm was not maximizing profits in the first place. This simple observation turns out to be quite useful in examining firm behavior.

**Problem 2:** Take the set-up from the previous problem. Apart from that the firm has to buy certain equipment before it starts the production. This equipment cost 2000. Compute: variable costs (VC), fixed costs (FC), average variable costs (AVC), average fixed costs (AFC), average costs (AC) and marginal costs (MC).

**Solution:** Note that the cost function is given by: $c(y) = y^2 + 2000$

- variable costs: $VC = y^2$
- fixed costs: $FC = 2000$
- average variable costs: $AVC = \frac{VC}{y} = y$
- average fixed costs: $AFC = \frac{FC}{y} = \frac{2000}{y}$
- average costs: $AC = \frac{c(y)}{y} = AVC + AFC = y + \frac{2000}{y}$
- marginal costs: $MC = c'(y) = 2y$

**Problem 3:** Monopoly: A monopolist can produce at constant average and marginal costs of $AC = MC = 5$. The firm faces a market demand curve given by $Q^D = 53 - P$.

(a) Calculate the profit-maximizing price-quantity combination for the monopolist. Also calculate the monopolists profits and consumer surplus.

(b) What output level would be produced by this industry under perfect competition if every firm could produce at the same average and marginal cost as the monopoly?

(c) Calculate the consumer surplus obtained by consumers in part (b). Show that this exceeds the sum of the monopolists profits and consumer surplus received in part (a). What is the value of the deadweight loss from monopolization?

**Solution:**

(a) Since $AC = MC = 5$, we know the cost function is given by $C(Q) = 5Q$. Also, we can solve for the inverse demand function to be: $P(Q) = 53 - Q$. The profit-maximizing price-quantity is given by solving the following:

$$\max_Q P(Q)Q - C(Q)$$
$$\max_Q (53 - Q)Q - 5Q$$
$$\max_Q -Q^2 + 48Q$$

Solving we have:

$$\text{F.O.C.:} \; -2Q + 48 = 0 \quad \Rightarrow \quad Q_m = 24$$

Using the demand equation and solving for $P$, we have:

$$P(24) = 53 - 24 = 29$$

Firms profits are then given by:

$$\pi(24)_m = 24(29 - 5) = 576$$

Consumer Surplus is given by:

$$CS_m = \frac{1}{2}(53 - 29)(24) = 288$$

(b) In perfect competition firms produce such that price is equal to marginal cost, so $P = 5$. Using the demand function, this means $Q_{pc} = 53 - 5 = 48$.

(c) If price is set to \$5 and quantity is 48, then consumer surplus is given by:

$$CS_{pc} = \frac{1}{2}(53 - 5)(48) = 1152$$

The combination of firms profits and consumer surplus from part (b) is given by:

$$\pi_m + CS_m = 576 + 288 = 864$$

Therefore, the value of deadweight loss from monopolization is given by:

$$CS_{pc} - (\pi_m + CS_m) = 1152 - 864 = 288$$

**Problem 4:** Suppose that a monopolist faces two markets with demand curves given by:

$$D_1(p_1) = 100 - p_1$$
$$D_2(p_2) = 100 - 2p_2$$

Assume that the monopolist's marginal cost is constant at \$20 a unit. If it can price discriminate, what price should it charge in each market in order to maximize profits? What if it can't price discriminate? Then what price should it charge?

**Solution:** To solve the price-discrimination problem, we first calculate the inverse demand functions:

$$p_1(y_1) = 100 - y_1$$
$$p_2(y_2) = 100 - 0.5y_2$$

Marginal revenue equals marginal cost in each market yields the two equations:

$$100 - 2y_1 = 20$$
$$50 - y_2 = 20$$

Solving we have $y_1^* = 40$ and $y_2^* = 30$. Substituting back into the inverse demand functions gives us the prices $p_1^* = 60$ and $p_2^* = 35$. If the monopolist must charge the same price in each market, we first calculate the total demand:

$$D(p) = D_1(p_l) + D_2(p_2) = 200 - 3p$$

The inverse demand curve is:

$$p(y) = \frac{200}{3} - \frac{y}{3}$$

Marginal revenue equal marginal cost gives us:

$$\frac{200}{3} - \frac{2}{3}y = 20$$

which can be solved to give $y^* = 70$ and $p^* = 43\frac{1}{3}$.