Problem 1: The demand for yak butter is $q = 120 - 4p$ and the supply is $q = 2p - 30$, where p is the price measured in dollars per hundred pounds and q is the quantity measured in hundred pound units.

(a) On one graph, draw the demand curve and the supply curve for yak butter.

(b) Write down the equation that you would solve to find the equilibrium price.

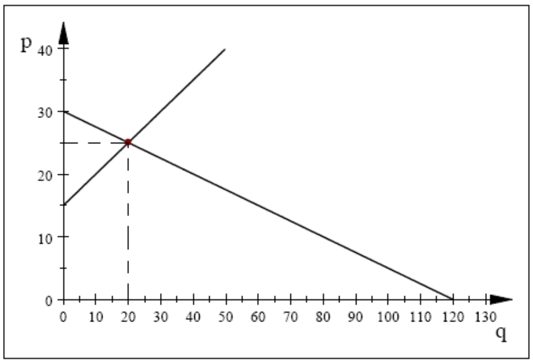
(c) What is the equilibrium price of yak butter? What is the equilibrium quantity? Show the equilibrium price and quantity on the graph and label them p_1 and q_1.

(d) A terrible drought strikes the central Ohio steppes, traditional homeland of the yaks. The supply schedule shifts to $2p - 60$. The demand schedule remains as before. Draw the new supply schedule. Write down the equation that you would solve to find the new equilibrium price of yak butter.

(e) What is the new equilibrium price of yak butter? What is the new equilibrium quantity? Show the equilibrium price and quantity on the graph and label them p_2 and q_2.

Solution:

(a) Graph with demand and supply curve:



(b)

$$D(p^*) = 120 - 4p^* = 2p^* - 30 = S(p^*)$$

(c) Solving the equation in (b):

$$\begin{aligned}120 - 4p_1 &= 2p_1 - 30\\ 150 = 6p_1 \quad &\Rightarrow p_1 = \frac{150}{6} = 25\end{aligned}$$

Evaluating the demand (or supply) equation at this price:

$$q_1 = 120 - 4p_1 = 20$$

(d) The equation is:

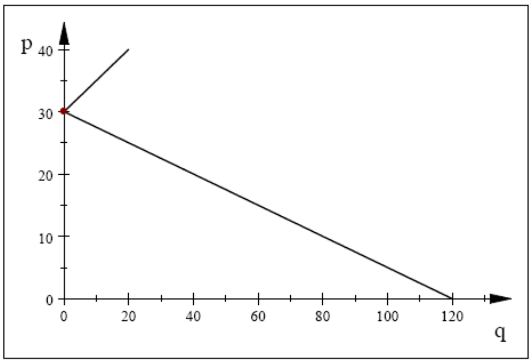
$$D(p^*) = 120 - 4p^* = 2p^* - 60 = S(p^*)$$

(e) Solving the equation in (b):

$$p_2 = \frac{180}{6} = 30$$

Evaluating the demand (or supply) equation at this price:

$$q_2 = 120 - 4p_2 = 0$$



Problem 2: Consider the following curves.

Supply: $P = 4Q$

Demand: $P = 150 - Q$

(a) Give a definition of a competitive equilibrium.

(b) Calculate competitive equilibrium.

(c) Calculate producer surplus, consumer surplus and total surplus.

(d) Suppose now there is no price floor, but the government impose taxes \$5 per unit sold. Calculate consumer surplus, producer surplus, government revenue, total surplus and deadweight loss.

(e) Illustrate the situation in (d) graphically. Does it matter whether the tax is imposed on the producers or the consumers? Explain.

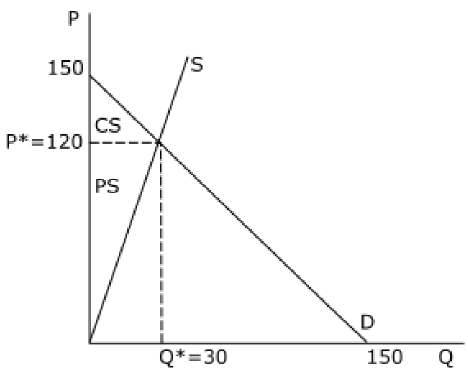
Solution:

(a) Market Equilibrium is a pair of price and quantity (P^*, Q^*) such that at the equilibrium price P^*, the quantity demanded equals the quantity supplied (i.e. market clears).

(b) To solve for market equilibrium we need to find solution to the following system:

$$P = 4Q$$
$$P = 150 - Q$$
$$4Q = 150 - Q \quad \Rightarrow \quad Q^* = 30, P^* = 120$$

(c) Consumer surplus and producer surplus is the area of triangles depicted on the picture below:



$$\text{CS} = \frac{1}{2}30 \times 30 = 450$$
$$\text{PS} = \frac{1}{2}30 \times 120 = 1800$$
$$\text{Welfare} = \text{Total surplus} = \text{CS} + \text{PS} = 2250$$

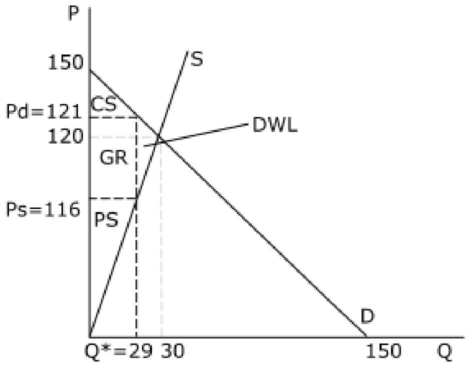
(d) First we need to find competitive equilibrium with taxes, because it is needed for surplus computation. We need to find price paid by buyers P_d, price received by sellers P_s, and quantity Q. Irrespective of who pays the tax (producer or consumer) the difference between P_d and P_s is \$5. So $P_s = P_d - 5$. Now the system of two equations is as follows:

$$P_d - 5 = 4Q$$
$$P_d = 150 - Q$$
$$4Q + 5 = 150 - Q \quad \Rightarrow \quad Q^* = 29, \quad P_d = 121, \quad P_s^* = 116$$

Now we can compute surpluses:

$$\text{CS} = \frac{1}{2}29 \times 29 = 420.5$$
$$\text{PS} = \frac{1}{2}116 \times 29 = 1682$$
$$\text{Government Revenue} = 5 \times 29 = 145$$
$$\text{Welfare} = 420.5 + 1682 + 145 = 2247.5 < 2250$$
$$\text{DWL} = 2.5$$

(e) No, it does not matter. Look at the graph of the market, and put the tax on the graph. The tax puts a wedge between the price paid by buyers and the price received by sellers. No matter who formally pays the tax, the costs of the tax are borne by both sides of the transaction, and who pays what share depends on the relative elasticities (slopes of demand and supply curve). If the demand is relative inelastic in comparison to supply (the reaction of consumers on change in price is subtle) most of the tax will be paid by consumers. If on the other hand consumers are very sensitive to changes in price and producers are not, most of the tax will be paid by producers.



Problem 3: Consider the following story from the Second World War. There are two prisoners of war in a German camp: British (consumer A) and French (consumer B). Both of them have a right to get some weekly amount of tea (good 1) and coffee (good 2). British prisoner has the endowment $\omega_A = (1, 4)$ and French prisoner, being privileged, has the endowment $\omega_B = (5, 4)$. The two prisoners are totally separated and the direct exchange is not possible, but they succeeded to persuade a German prisoners' priest to transfer coffee and tee between them. The prisoners' preferences are given by the following utility functions:

$$u^A(x_1^A, x_2^A) = 2 \ln x_1^A + x_2^A$$
$$u^B(x_1^B, x_2^B) = 4 \ln x_1^B + x_2^B$$

where x_1^i is the amount of good 1 consumer i consumes and x_2^i the amount of good 2. Suppose that the price of good 1 is p_1 and the price of good 2 is p_2.

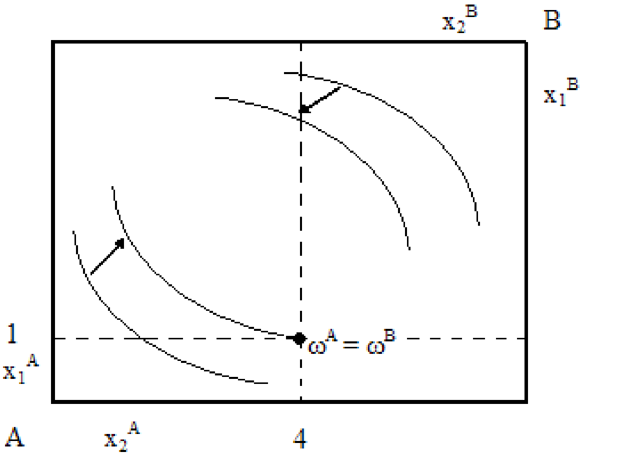
(a) Sketch the corresponding Edgeworth box. In the Edgeworth box draw several indifference curves of both agents and mark their initial endowment. Find Pareto efficient (Pareto optimal) allocations.

(b) Find the market demand functions x_1^A, x_1^B.

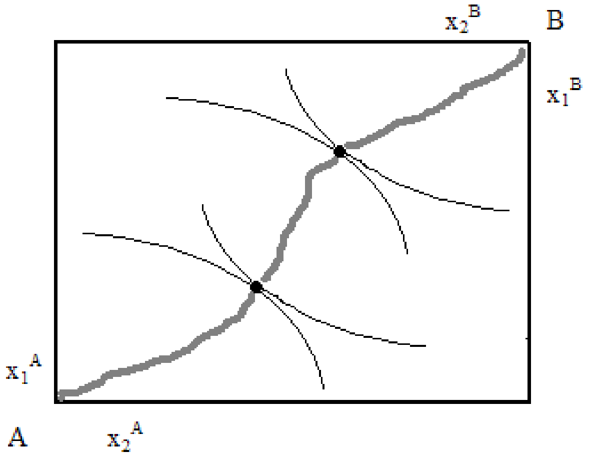
(c) Find the competitive equilibrium (prices and allocations) for this prisoners' economy.

Solution:

1.



Edgeworth box with indifference curves and endowment



Pareto efficient allocations

2. To find consumers' demand functions we solve their optimization problems:

$$\text{Consumer A:} \quad \max_{\{x_1^A, x_2^A\}} 2\ln x_1^A + x_2^A$$
$$s.t. \;\; p_1 x_1^A + p_2 x_2^A \leq p_1 + 4p_2$$
$$\text{Consumer B:} \quad \max_{\{x_1^B, x_2^B\}} 4\ln x_1^B + x_2^B$$
$$s.t. \;\; p_1 x_1^B + p_2 x_2^B \leq 5p_1 + 4p_2$$

We choose good 1 to be a numeraire, therefore $p_1 = 1$ and for simplicity we denote $p_2 = p$. Furthermore, we use equalities in budget constraints.

$$\text{Consumer A:} \quad \max_{\{x_1^A, x_2^A\}} 2\ln x_1^A + x_2^A$$
$$s.t. \;\; x_1^A + p x_2^A = 1 + 4p$$
$$\text{Consumer B:} \quad \max_{\{x_1^B, x_2^B\}} 4\ln x_1^B + x_2^B$$
$$s.t. \;\; x_1^B + p x_2^B = 5 + 4p$$

Now we plug budget constraints into the objective functions and take the first order conditions:

$$\text{Consumer A:} \quad \max_{x_2^A} 2\ln(1 + 4p - p x_2^A) + x_2^A$$

$$\text{FOC:}\frac{2(-p)}{1+4p-px_2^A}+1=0 \Rightarrow x_2^A=\frac{2p+1}{p}$$

Consumer B: $$\max_{x_2^B} 4\ln(5+4p-px_2^B)+x_2^B$$

$$\text{FOC:}\frac{4(-p)}{5+4p-px_2^B}+1=0 \Rightarrow x_2^B=\frac{5}{p}$$

Therefore the demand functions are $x_2^A = \frac{2p+1}{p}$ and $x_2^B = \frac{5}{p}$. And from budget constraints we get the demands $x_1^A = 2p$ and $x_1^B = 4p$.

3. Competitive equilibrium consists of equilibrium prices (only price p_2 needs to be determined since we set the price p_1 equal to 1) and allocations $\{x_1^A, x_2^A\}, \{x_1^B, x_2^B\}$. In equilibrium both markets (market for good 1 and market for good 2) clear:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \quad \Leftrightarrow \quad 2p + 4p = 6 \quad \Rightarrow \quad p = 1$$

Here, we check if market for good 2 clears for price p=1 as well:

$$\frac{2p+1}{p} + \frac{5}{p} = 8 \quad \Rightarrow \quad p = 1$$

Hence, the competitive equilibrium is:

$$\{x_1^A, x_2^A\} = (2,3);\ \{x_1^B, x_2^B\} = (4,5);\ p_1 = 1;\ p_2 = 1.$$