Problem 1: Assume a person has a utility function U = XY, and money income of $10,000, facing an initial price of X of $10 and price of Y of $15. If the price of X increases to $15, answer the following questions:

(a) What was the initial utility maximizing quantity of X and Y?

(b) What is the new utility maximizing quantity of X and Y following the increase in the price of X?

(c) What is the Hicks compensating variation in income that would leave this person equally well off following the price increase? What is the Slutsky compensating variation in income?

(d) Calculate the pure substitution effect and the real income effect on X of this increase in the price of X. Distinguish between the calculation of these effects using the Hicksian analysis vs. the Slutsky analysis.

Solution: In this exercise, we will seek to understand the concept of Hicks' method of decomposing a demand curve's slope into its component income and substitution effects. The Hicksian substitution effect is the change in quantity demanded that occurs when the price of the good changes, but the consumer is given enough additional income so that her total utility remains the same. The income effect is the change in the quantity demanded of a good due to the change in real income caused by the price change. The income effect is measured as the difference between the quantity demanded after the price change and the quantity demanded after the price change when income is adjusted to return the consumer to the original level of utility.

The Slutsky substitution effect is the change in quantity demanded that occurs when the price of the good changes, but the consumer is given enough additional income so that she can just afford her initial consumption. The income effect is the change in the quantity demanded of a good due to the change in real income caused by the price change. The income effect is measured as the difference between the quantity demanded after the price change and the quantity demanded after the price change when income is adjusted to keep consumer's purchasing power the same.

(a) We use optimality condition and budget constraint:

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y} \quad \Rightarrow \quad \frac{Y}{X} = \frac{10}{15} \quad \Rightarrow \quad X = 1.5Y$$
$$P_X X + P_Y Y = I \quad \Rightarrow \quad 10X + 15Y = 10000$$
$$10(1.5Y) + 15Y = 10000 \quad \Rightarrow \quad 30Y = 10000 \quad \Rightarrow \quad Y = \frac{1000}{3} \doteq 333.3$$
$$X = 1.5Y = \frac{1500}{3} = 500$$

(b)

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y} \quad \Rightarrow \quad \frac{Y}{X} = \frac{15}{15} \quad \Rightarrow \quad X = Y$$
$$P_X X + P_Y Y = I \quad \Rightarrow \quad 15X + 15Y = 10000$$
$$15X + 15X = 10000 \quad \Rightarrow \quad X = \frac{10000}{30} \doteq 333.3$$
$$X = Y \doteq 333.3$$

(c) The Hicksian compensating variation in income is that amount of money, holding the price of X constant at its higher level of \$15, that will allow the person to be as well off as they were before the price increase. In Hicksian terms of course, being equally well off means having the same level of utility. Original utility was

$$U = XY = (333.33)(500) = 166667$$

When the price ratio of X to Y had been 10/15, we observed above that Y = .67 X and X = 1.5 Y. Now, with both prices equal to \$15, the price ratio is 1:1, and Y = X, meaning that the utility maximizing bundle at the new relative price ratio on the original indifference curve must have the same quantity of X and Y. Substitution allows the following result:

$$U = XY; \quad \text{but if} \quad X = Y, U = X^2 = 166667 \text{ (original utility)} \quad \Rightarrow X = Y = 408.25$$

Note that on a graph, these would be the quantities at the tangency point of the shifted budget line (with the higher relative price of X) and the original indifference curve.

Final step is to calculate the amount of money that must be spent to achieve X = Y = 408.25, which is

$$\$15(408.25) + \$15(408.25) = \$12247.50$$

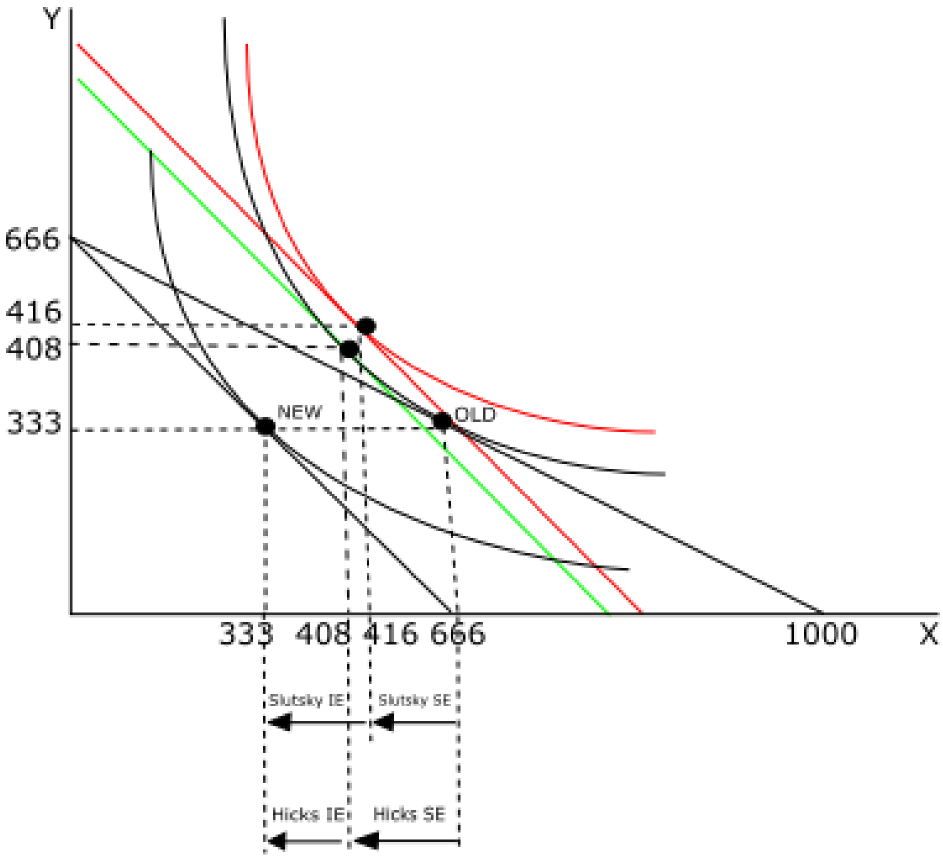
Original income was \$10,000, so the Hicksian compensating variation of income is \$12 247.50 - \$10 000 = \$2 247.50

The Slutsky compensating variation is much easier to calculate: At the new prices the money income required to consume the original X,Y bundle of X = 500, Y = 333.33 is simply: I = \$15 (500) + \$15 (333.33) = \$12 500. This is the money income required to allow a budget line at the new slope (with higher price of X) to go through the original consumption point. Since \$12 500 - \$10,000 = \$2 500, that is the Slutsky compensating variation.

(d) Finally, if no compensating variation is actually paid, the full reduction in the consumption of X is (500-333.33) = 167.67. How much of this 167.67 reduction is due to a pure substitution effect and how much is due to a real income effect. We can rely on the analysis in (c) to derive the results for both the Slutsky and the Hicksian analysis.

Hicksian analysis: We found above that if the real income effect is eliminated by "hypothetically" (in this case) giving the person another \$2 247.50, the new point on the original indifference curve is X = 408.25, Y = 408.25. Therefore, the movement along that original indifference curve representing the pure substitution effect is 500 - 408.25 = 91.75. Then, the remaining change in X of 408.25 - 333.33 = 74.92 is the real income effect (the result of now taking that \$2 247.50 away from the person, so there is a parallel shift to the left to the lower indifference curve at X = 333.33 and Y = 333.33).

Slutsky derivation of substitution and income effects: We found above that the elimination of the real income effect as defined by Slutsky would require a "hypothetical" increase in I of \$2 500 to I = \$12 500. We also know as stated earlier that with $P_X = P_Y$, the first order condition requires that Y = X. Thus, we can calculate the point on the higher utility indifference curve that can be achieved with \$12,499.95 (and the more steeply sloped budget line incorporating the higher price of X) as \$12 500 = \$15 X + \$15 Y, or since X = Y, \$12 500 = 15 X + 15 X, so X = 416.67, and Y = 416.67. Therefore, the pure substitution effect related to X is 500 - 416.67 = 83.33 and the real income effect is then the "residual" of 416.67 - 333.33 = 83.34 (essentially equal, just a rounding difference).



Problem 2: Suppose that the price elasticity, ϵ, for cigarettes is 4, the price of cigarettes is \$3 per pack and we want to reduce smoking by 20%. What should we do?

Solution: First, recognize that we need to raise the price. Then, figure out by how much:

$$4 = \epsilon = \frac{\Delta Q/Q}{\Delta P/P} = \frac{0.2}{\Delta P/3} \quad \Rightarrow \quad \Delta P = \frac{0.2 * 3}{4} = 0.15$$

In order to decrease smoking by 20% we need to increase price by 15 cents.

Problem 3: Consumer consumes two goods with their prices $P_X = 10$, $P_Y = 80$ and has income $I = 5000CZK$. The demand function is given by $X = 80 - 0.8P_X^2 - 0.5P_Y + 0.04I$.

(a) Are X and Y substitutes or complements?

(b) Is X normal or inferior good?

(c) What is price elasticity of demand for good X? What information does this give to the producer of good X?

(d) What is cross elasticity of demand for good X if price of Y changes?

(e) What is income elasticity of demand for good X?

Solution:

(a) If P_Y increases the demand for good X decreases, hence X and Y are complements.

(b) If income increases the demand for good X increases as well, hence X is a normal good.

(c) Price elasticity measures change in the demand for good X caused by the change in its price. So first we need to know what the demand is with original price and income.

$$X = 80 - 0.8P_X^2 - 0.5P_Y + 0.04I = 80 - 0.8 * 100 - 0.5 * 80 + 0.04 * 5000 = 160$$

Price elasticity is given by

$$\epsilon_P = \frac{dX/X}{dP_X/P_X} = \frac{dX}{dP_X}\frac{P_X}{X} = -1.6P_X\frac{P_X}{X} = -1.6 * 10\frac{10}{160} = -1$$

Price elasticity is -1 which means that 1% increase (decrease) in price of X will lead to 1% decrease (increase) in quantity sold. Hence the change in price does not change the revenue.

(d) $\epsilon_C = \dfrac{dX/X}{dP_Y/P_Y} = \dfrac{dX}{dP_Y}\dfrac{P_Y}{X} = -0.5\dfrac{P_Y}{X} = -0.5\dfrac{80}{160} = -\dfrac{1}{4}$

Cross elasticity is negative what means that increase (decrease) in P_Y causes decrease (increase) in consumption of good X which implies that X and Y are complements.

(e) $\epsilon_I = \dfrac{dX/X}{dI/I} = \dfrac{dX}{dI}\dfrac{I}{X} = 0.04\dfrac{I}{X} = 0.04\dfrac{5000}{160} = \dfrac{200}{160} = 1.25$

Income elasticity is positive what implies that good X is normal good (rather luxury than necessary).

Problem 4: (not covered during the session due to time constraint; will not be required on the test)

Peter's utility from CDs is given by $TU_X = 1000X - 10X^2$, where X is number of CDs bought per year. The price of a CD is 400 CZK and Peter's income is 200000 CZK per year.

(a) How many CDs will Peter buy?

(b) Determine Peter's consumer surplus.

(c) Use indifference analysis to illustrate Peter's decision making and his consumer surplus.

(d) How does consumer surplus change if the price of a CD increases to 500 CZK?

(e) How many CDs in maximum is Peter willing to buy?

Solution:

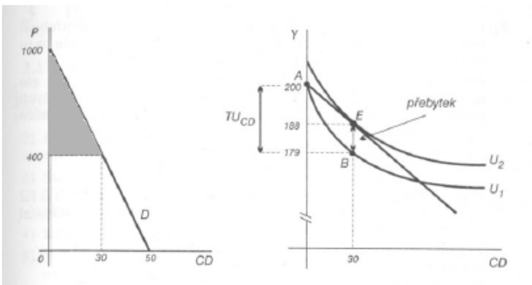
(a) Peter buys more CDs only if the marginal utility from additional CD is more or equal to the price:

$$MU_X = P_X \quad \Rightarrow \quad 1000 - 20X = 400 \quad \Rightarrow \quad X = 30$$

(b) Algebraically, consumer surplus is given by

$$CS = TU - P_X X = 1000 * 30 - 10 * 30^2 - 30 * 400 = 9000$$

Graphically, consumer surplus is a shaded triangle on the picture below (left). The area is $(1000 - 400) * 30/2 = 9000$



(c) Starting point is point A - no consumption of CDs. If Peter buys 30 CDs, point B is a point where Peter's utility is the same as in the starting point A. If Peter buys 30 CDs and spends 179 000 CZK on other goods he is as well as in the starting point A. However, in the equilibrium point of consumption, E, Peter gets 30 CDs and can spend 188 000 CZK on other goods. Hence Peter's consumer surplus is 188 000 CZK - 179 000 CZK = 9000 CZK. The situation is depicted on the picture above (right).

(d) New optimality condition is:

$$MU_X = P_X \quad \Rightarrow \quad 1000 - 20X = 500 \quad \Rightarrow \quad X = 25$$

New consumer surplus is as follows:

$$CS = TU - P_X X = 1000 * 25 - 10 * 25^2 - 25 * 500 = 6250 CZK$$

(e) Here we look at how many CDs Peter decides to buy if the price of CD is zero. Therefore, the maximum number of CDs is 50. Alternative way to determine the maximum amount of CDs is that Peter will keep buying CDs as long as his marginal utility from additional CD is positive. Again, this holds for X=50.