Problem 1: We consider two goods, apples (A) and bananas (B). The prices are given by $p_A = 2$ and $p_B = 4$.

(a) Suppose that Pete has an income y = 20. Derive his budget constraint and draw it into a diagram.

(b) How does the budget constraint of Pete change if

(i) a quantity tax of 2 is levied on apples

(ii) a value tax of 25% is levied on bananas

(iii) his mom forbids him to buy more than 5 apples

(iv) mom increases Petes income to y = 40

(v) prices fall by 50% ?

(c) Suppose that Nicole has 2 apples and 4 bananas. Derive her budget constraint and draw it into a diagram.

(d) How does the budget constraint of Nicole change if

(i) the price of bananas rises by 1

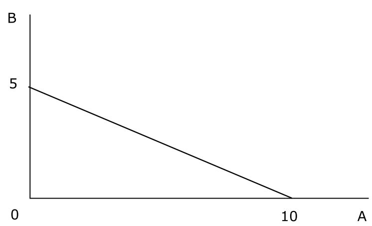
(ii) both prices fall by 50%

(iii) her initial endowment of fruits is doubled?

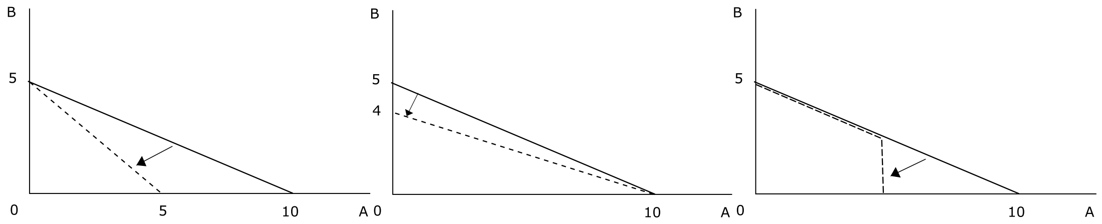
Solution:

(a) Pete's budget constraint is given by the following equation and is depicted on the figure below.

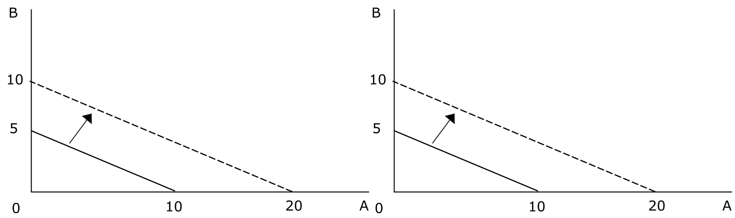
$$p_A A + p_B B = 2A + 4B = 20 \quad \Longleftrightarrow \quad B = 5 - A/2$$



(b) In case (i) a new price of apples is $p_A = 4$ and therefore Pete can afford to buy less apples. In case (ii) the price of bananas increases to $p_B = 5$. In case (iii) prices of fruit remain the same, only restriction on consumption of apples is imposed. Note that there is no difference between (iv) - doubling the income and (v) - decreasing prices to one half.



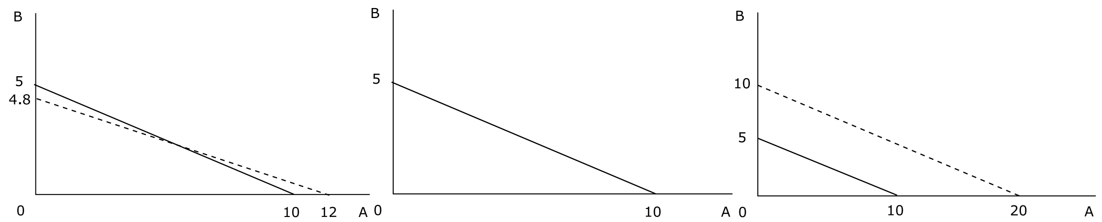
(i) (ii) (iii)



(iv) (v)

(c) Nicole has 2 apples and 4 bananas. In terms of money she has $2p_A + 4p_B = 2 * 2 + 4 * 4 = 20$. Therefore her income is the same as Pete's income and hence also the budget constraint is the same. (Nicole can decide to consume her fruit, but she might prefer a different combination than 2 apples and 4 bananas. Hence, she can decide to sell the fruit at market prices and buy a different combination.)

(d) In case (i) the price of bananas increases from 4 to 5 and Nicole's income therefore increases to $2p_A + 4p_B = 2 * 2 + 4 * 5 = 24$. In case (ii) both prices fall by 50%. Hence Nicole's income will be $2p_A + 4p_B = 2 * 1 + 4 * 2 = 10$. If Nicole spends all money on apples she can afford 10 of them and if she spends entire income on bananas she can buy 5 of them. Hence the budget set does not change. In case (iii) initial endowment of fruit doubles which is equivalent to doubling the income.



(i) (ii) (iii)

Problem 2: Antony consumes only apples and bananas. We denote be (x_A, x_B) the consumption bundle which contains x_A apples and x_B bananas. Antony's preferences are described by the utility function:

$$u(x_A, x_B) = \sqrt{x_A x_B}$$

(a) Explain the term indifference curve. Determine the indifference curves which pass through the points (16, 1) and (4, 9). Draw them in a diagram.

(b) Antony's initial endowment is (16,1). Would he exchange his initial endowment for the consumption bundle (4,9)? Indicate in your graph those consumption bundles, which Antony prefers to his initial endowment.

(c) For any consumption bundle, determine the marginal rate of substitution (MRS) for Antony. Explain the meaning of the marginal rate of substitution and check whether Antony's preferences have a falling or an increasing MRS.

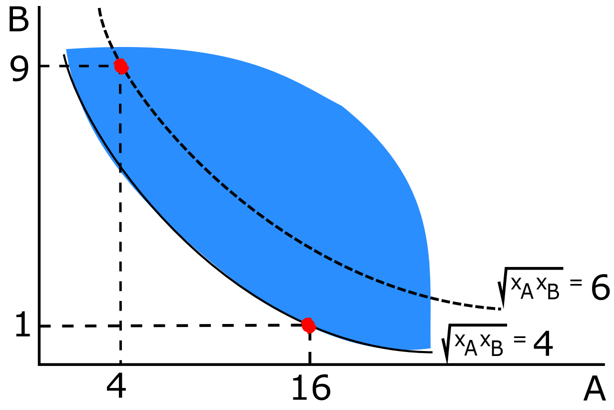
(d) Draw the budget constraint of Antony for prices $p_A = 10$ and $p_B = 20$ and income y = 240. Indicate Antony's optimal consumption bundle in your graph.

Solution:

(a) *Indifference curve* is a curve along which the consumer has constant level of utility. In other words it is a set of bundles between which the consumer is indifferent, hence the name. For example a curve given by $u(x_A, x_B) = \sqrt{x_A x_B} = 5$ is one possible indifference curve; $u(x_A, x_B) = \sqrt{x_A x_B} = 3.1415926535$ is another; and so on.

Consumption bundle (16,1) yields utility $\sqrt{x_A x_B} = \sqrt{16 * 1} = 4$ and therefore the corresponding indifference curve is given by $u(x_A, x_B) = \sqrt{x_A x_B} = 4$. Similarly, indifference curve which passes through consumption bundle (4,9) is determined by $u(x_A, x_B) = \sqrt{x_A x_B} = 6$.

(b) Yes, bundle (4,9) yields higher utility and therefore is preferred to bundle (16,1). The blue area on the figure below shows consumption bundles preferred to the initial endowment (16,1).



(c) *Marginal rate of substitution* is the slope of an indifference curve and measures the rate at which the consumer is just willing to substitute one good for the other. Or, MRS of good 2 (all other goods) for good 1 is how many dollars you would just be willing to give up spending on other goods in order to consume a little bit more of good 2. In our case:

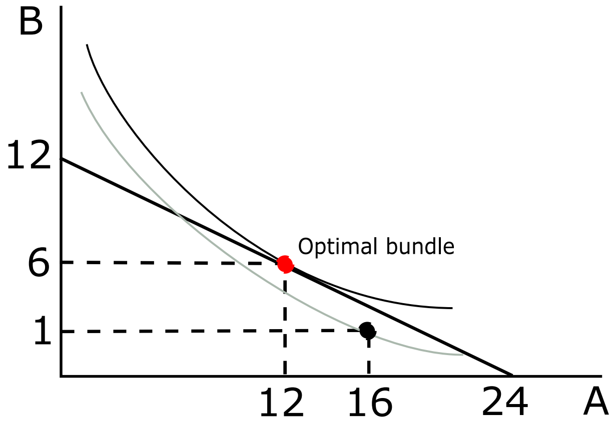
$$MRS_{AB} = \frac{MU_A}{MU_B} = \frac{\frac{\partial u}{\partial x_A}}{\frac{\partial u}{\partial x_B}} = \frac{\frac{x_B}{1/2\sqrt{x_A x_B}}}{\frac{x_A}{1/2\sqrt{x_A x_B}}} = \frac{x_B}{x_A}$$

Hence, for a consumption bundle (x_A, x_B), MRS_{AB} is equal to $-x_B/x_A$. In particular, if for example $(x_A, x_B) = (2, 4)$ and therefore $MRS_{AB} = 2$ it means, that Antony is willing to sacrifice two bananas in order to get one more apple.

MRS_{AB} shows how much of good B is consumer willing to sacrifice to get one more unit of good A.

Antony's preferences have diminishing MRS. Generally, for any strictly concave utility function the corresponding indifference curves are convex and therefore the slope of the indifference curve (which is equal to MRS) decreases (in absolute value). The more apples Antony has, the less bananas he is willing to sacrifice in order to get one more apple.

(d) Antony's budget line is given by $10x_A + 20x_B = 240$ and his optimal bundle is the point where an indifference curve is tangent to his budget line.



This problem can be solved in four different ways:

1. We use optimality condition:
$$MRS = \frac{MU_A}{MU_B} = \frac{p_A}{p_B}$$
$$\frac{x_B}{x_A} = \frac{10}{20} \Rightarrow x_A = 2x_B$$
and the budget constraint:
$$10x_A + 20x_B = 240$$
Putting the optimality condition into the budget constraint we get:
$$10(2x_B) + 20x_B = 240$$
$$40x_B = 240 \Rightarrow x_B = 6$$
$$x_A = 2x_B \Rightarrow x_A = 12$$

2. We use optimality condition:
$$\frac{MU_A}{p_A} = \frac{MU_B}{p_B} \quad \text{or} \quad \frac{MU_A}{p_A} = \frac{MU_B}{p_B}$$
(*Note:* if $\frac{MU_A}{p_A} > \frac{MU_B}{p_B}$, consumer could be better off by decreasing consumption of B and increasing consumption of A.)
and the budget constraint:
$$10x_A + 20x_B = 240$$
Analogically to the first case we get that $x_A = 12$ and $x_B = 6$.

3. We express the value of x_A from the budget constraint and plug it into the utility function. Then we maximize this function as a function of one variable:
$$10x_A + 20x_B = 240 \Rightarrow x_A = 24 - 2x_B$$

Now we use utility function:

$$\max_{\{x_A,x_B\}} u(x_A,x_B) = \max_{\{x_A,x_B\}} \sqrt{x_A x_B} = \max_{x_B} \sqrt{x_B(24-2x_B)}$$
$$FOC: \frac{24-4x_B}{2\sqrt{x_B(24-2x_B)}} = 0$$

The fraction is equal to 0 if the nominator is equal to 0, hence

$$24 - 4x_B = 0 \Leftrightarrow x_B = 6$$

Finally, using the budget constraint we get that $x_A = 12$

4. Lagrange multiplier:

$$\max_{\{x_A,x_B\}} u(x_A,x_B) = \max_{\{x_A,x_B\}} \sqrt{x_A x_B}$$
$$s.t. \quad 10x_A + 20x_B = 240$$

$$L = \sqrt{x_A x_B} - \lambda(x_A + 2x_B - 24)$$
$$\frac{\partial L}{\partial x_A} = 0 \Rightarrow \frac{x_B}{2\sqrt{x_A x_B}} - \lambda = 0$$
$$\frac{\partial L}{\partial x_B} = 0 \Rightarrow \frac{x_A}{2\sqrt{x_A x_B}} - 2\lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow x_A + 2x_B - 24 = 0$$

Putting the first two equations together we get:

$$\frac{x_B}{\sqrt{x_A x_B}} = \frac{x_A}{2\sqrt{x_A x_B}} \Rightarrow x_A = 2x_B$$

Adding the last condition ($\frac{\partial L}{\partial \lambda} = 0$) or $x_A + 2x_B - 24 = 0$ we again get the same result: $x_A = 12$, $x_B = 6$.

Problem 3: For each of the following utility functions:

$$u(x_1,x_2) = x_1 + 2x_2$$
$$u(x_1,x_2) = 2\ln x_1 + x_2$$
$$u(x_1,x_2) = \min\{x_1,x_2\}$$

(a) derive the equation describing the indifference curve for a given level of utility $\overline{u}$

(b) sketch several indifference curves in a graph

(c) derive the marginal rate of substitution (whenever possible)

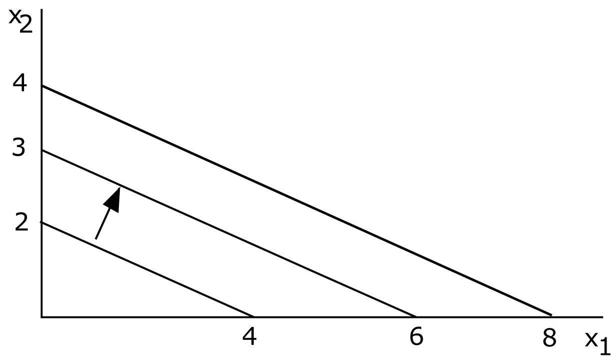
Solution:

- $u(x_1, x_2) = x_1 + 2x_2$

  (a) indifference curve is set of bundles which yield the same level of utility, in our case $\overline{u}$, therefore equation describing an indifference curve is as follows:

  $$x_1 + 2x_2 = \overline{u} \iff x_2 = \frac{\overline{u}}{2} - \frac{x_1}{2}$$

  (b)



  (c)

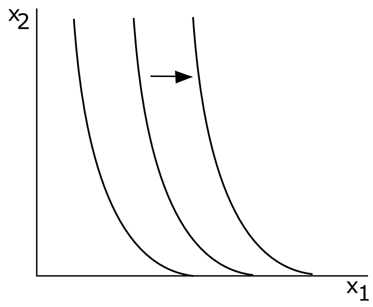
  $$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{1}{2}$$

  From the last equation we see what exactly MRS in our case is: in order to stay on the same indifference curve, consumer is willing to exchange 1 unit of x_2 for 2 units of x_1 (you can check this on the graph).

- $u(x_1, x_2) = 2 \ln x_1 + x_2$

  (a) Indifference curves are given by following equation $2 \ln x_1 + x_2 = \overline{u}$. By rearranging terms we get $x_2 = \overline{u} - 2 \ln x_1$.

  (b) Equation in part (a) determines curves depicted on the following figure:



(c)

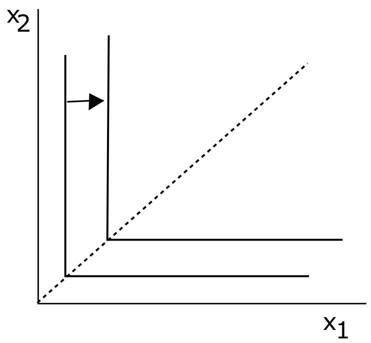
$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{\frac{2}{x_1}}{1} = \frac{2}{x_1}$$

In absolute terms, MRS is diminishing (as x_1 increases then $MRS = \frac{2}{x_1}$ decreases).

- $u(x_1, x_2) = \min\{x_1, x_2\}$

(a) IC is defined by: $x_2 =$ const if $x_1 > x_2$ and $x_1 =$ const if $x_1 < x_2$.

(b)



(c) In this case, the marginal rate of substitution is:

$$\begin{aligned} \text{MRS} = 0 \quad & \text{if} \quad x_1 > x_2 \\ \text{MRS} = \infty \quad & \text{if} \quad x_1 < x_2 \\ \text{MRS can not be determined} \quad & \text{if} \quad x_1 = x_2 \end{aligned}$$