Economics II - October 27, 2009

Based on H.R.Varian - Intermediate Microeconomics. A Modern Approach

GAME THEORY

Economic agents can interact strategically in a variety of ways, and many of these have been studied by using the apparatus of **game theory**. Game theory is concerned with the general analysis of strategic interaction. In this lecture we will briefly explore this subject to understand how it works and how it can be used to study economic behavior in oligopolistic markets.

Simultaneous Games

Strategic interaction can involve many players and many strategies, but we will limit ourselves to two-person games with a finite number of strategies. This will allow us to depict the game easily in a payoff matrix. Suppose that two people are playing a simple game. Person A will write one of two words on a piece of paper, "top" or "bottom." Simultaneously, person B will independently write "left" or "right" on a piece of paper. After they do this, the papers will be examined and they will each get the payoff depicted in the table below. If A says top and B says left, then we examine the top left-hand corner of the matrix. In this matrix the payoff to A is the first entry in the box, 1, and the payoff to B is the second entry, 2. Similarly, if A says bottom and B says right, then A will get a payoff of 1 and B will get a payoff of 0. The situation is summarized in the table below. This table is called **normal form** of game.

A \ B	left	right
top	1,2	0,1
bottom	2,1	1,0

From the viewpoint of person A, it is always better for him to say bottom since his payoffs from that choice (2 or 1) are always greater than their corresponding entries in top (1 or 0). Similarly, it is always better for B to say left since 2 and 1 dominate 1 and 0. Thus we would expect that the equilibrium strategy is for A to play bottom and B to play left. And this is the only Nash equilibrium (to be explained) of this game.

In this case, we have a **dominant strategy**. There is one optimal choice of strategy for each player no matter what the other player does. Whichever choice B makes, player A will get a higher payoff if he plays bottom, so it makes sense for A to play bottom. And whichever choice A makes, B will get a higher payoff if he plays left. Hence, these choices dominate the alternatives, and we have an equilibrium in dominant strategies (Bottom, Left). In game theory, **dominant strategy** occurs when one strategy is better than another strategy for one player, no matter how that player's opponents may play. Another example follows.

Prisoner's Dilemma: one Nash equilibrium.

Two suspects in a major crime are held in separate cells. If they both stay silent (deny the crime) they will serve half a year each. If both of them betray they will serve five years each. If one stays silent and the other betrays the "betrayer" is set free and the convicted prisoner has to serve ten years.

	silent	betrays
silent	0.5,0.5	10,0
betrays	0,10	5,5

Even though it would be mutually beneficial to stay silent by looking at dominant strategies (betray for both prisoners) we see that the only equilibrium is Betray,Betray.

What if there are no dominant strategies?

In the previous two examples both players had a dominant strategy and we found one equilibrium This does not have to be the case all the time. Sometimes we do not have dominant strategies for both players. For example, the game depicted in below does not have a dominant strategy equilibrium.

A \ B	left	right
top	1,2	0,1
bottom	2,0	1,1

This game does not have a dominant strategy equilibrium. A's optimal choice depends on what he thinks B will do. However, we can still find the equilibrium of the game. We will say that a pair of strategies is a **Nash equilibrium** if A's choice is optimal, given B's choice, and B's choice is optimal given A's choice. In this example the Nash equilibrium is Bottom, Right.

Game: two Nash equilibria, one of them is better for both.

A \ B	left	right
top	10,10	10,0
bottom	0,10	20,20

In this example there are two Nash equilibria but one is better for both players so we can expect that they will choose Bottom, Right equilibrium.

Battle of Sexes: two Nash equilibria, non of them is better for both.

A couple on their honeymoon in New York are separated in the crowds without having agreed on where they should go in the evening. At breakfast, they had discussed either a visit to the ballet or a boxing match. They both want to spend the evening together but a man (P1) prefers boxing match and a woman (P2) prefers ballet.

P1 \ P2	box	ballet
box	2,1	0,0
ballet	0,0	1,2

In this example there are two Nash equilibria and non of them is better for both players so we can not tell what will be tha outcome of the game.

Matching Pennies: no Nash equilibrium.

Tossing two coins. If there is the same sign on both coins (both are heads or both are tails) the first player wins, if the signs differ, the second player wins.

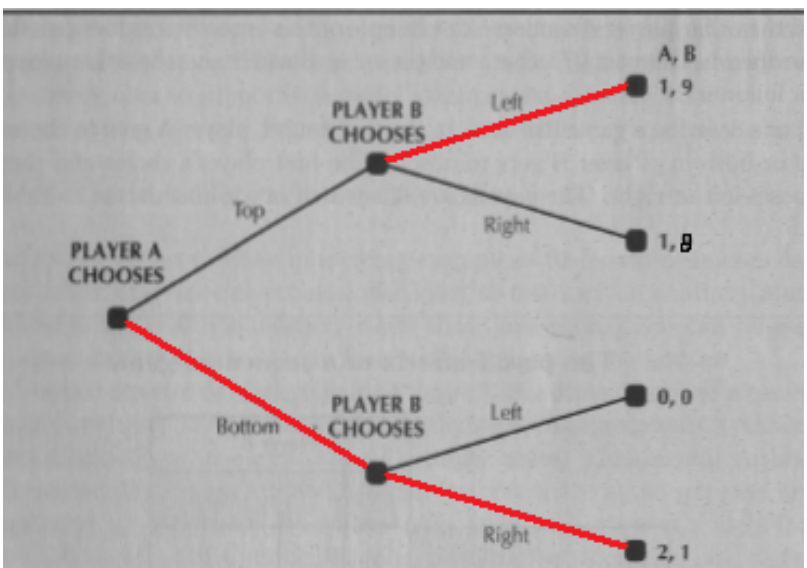
P1 \ P2	head	tail
head	1,-1	-1,1
tail	-1,1	1,-1

There is no Nash equilibrium in this game.

Sequential Games: unlike in the previous games, where two players had to make decision simultaneously now they move sequentially. The second player observes the action of the first player and decides accordingly. Look at the game below:

A \ B	left	right
top	1,9	1,8
bottom	0,0	2,1

Now we assume that player A moves first and B follows. This situation can me depicted as below and it is called **extensive form** of the game.



Extensive form of the game. This way of depicting a game indicates the order in which the players move.

To find equilibrium in sequential game we use **backward induction**. That means that we will first look at best response of the second player, player B, and then we find the optimal strategy of the first player, player A. If B is making choice in upper decision node he will choose the action randomly because his payoff is the same no matter what he does. That would give player A payoff equal to 1. If B is making choice in lower decision node he will choose the action Right because that is better for him and this would give A payoff equal to 2. Comparing these two scenarios player A will choose to play Bottom and B will respond with Right and they will get payoffs 2 and 1.

OLIGOPOLY

Different industries are organized in different ways. We can have monopolistic industry, where there is only one large firm in the market or perfect competition, where there are typically many small competitors. However, much of the world lies between these two extremes. Often there are a number of competitors in the market, but not so many as to regard each of them as having a negligible effect on price. This is the situation known as **oligopoly**. Behavior of firms in case of oligopoly can be either non-cooperative (individual profit maximization) or cooperative (cartel). It is unreasonable to expect one grand model since many different behavior patterns can be observed in the real world. What we want is a guide to some of the possible patterns of behavior and some indication of what factors might be important in deciding when the various models are applicable. Oligopoly can be analyzed using game theory approach because it is a strategic behavior of two (or few) players (firms).

We will illustrate various models of oligopoly on the following example: Market demand is given by inverse demand function $P = 120 - Q$, where Q is total production, i.e. $Q = Q_A + Q_B$; and there are two identical firms on the market with total cost $TC_A = 30Q_A$ and $TC_B = 30Q_B$.

Bertrand model of oligopoly: simultaneous competition in prices.

Bertrand oligopoly is an example of non-cooperative competition in prices. In Bertrand model firm chooses own price while taking the price of competition as given. That firm which chooses a lower price will get all the customers and firm with higher price gets nothing. Firms will keep undercutting each other as much as possible and the resulting price will be equal to marginal cost, i.e. $MC = TC' = 30$.

Cournot model of oligopoly: simultaneous competition in quantities.

Cournot oligopoly is an example of non-cooperative competition in quantities. In Bertrand competition a firm makes decision about own price while taking the price of competition as given. In Cournot competition a firm chooses own quantity to be produced and takes the quantity produced by a competitor as given.

$$\max \Pi_A = PQ_A - TC_A = (120 - Q_A - Q_B)Q_A - 30Q_A$$
$$\Pi'_A = 0 \quad \Rightarrow \quad 120 - 2Q_A - Q_B - 30 = 0 \quad \Rightarrow \quad Q_A = \frac{90 - Q_B}{2}$$
$$\max \Pi_B = PQ_B - TC_B = (120 - Q_A - Q_B)Q_B - 30Q_B$$
$$Q_B = \frac{90 - Q_A}{2}$$

Cournot model of oligopoly can be analyzed using simultaneous game approach. Both firms have to choose quantity to be produced without knowing what other firm does. That basically means solving system of two equations with two unknowns and the result is: $Q_A = Q_B = 30$ and market price is therefore $P = 60$.

Stackelberg model of oligopoly: sequential competition in quantities.

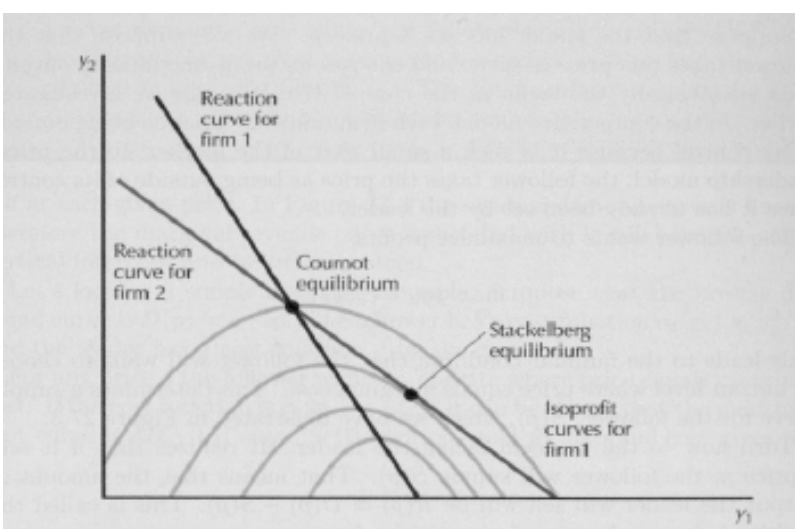
When one firm decides about its choices for prices and quantities it may already know the choices made by the other firm. If one firm gets to set its price before the other firm, we call it the **price leader** and the other firm the **price follower**. Similarly, one firm may get to choose its quantity first, in which case it is a **quantity leader** and the other is a **quantity follower**. In the case of quantity leadership, one firm makes a choice before the other firm. This is called the **Stackelberg model**.

Stackelberg competition consists of two stages: First, the leader sets the quantity to be produced. Second, the follower observes the action of the leader and chooses the optimal level of production. So when the leader makes a decision about the quantity to be produced he anticipates followers reaction and hence chooses the quantity accordingly. Notice, that compared to Cournot competition the Stackelberg leader can never be worse off. (Leader can choose the Cournot quantity and then for the follower it is optimal to produce the same amount and we have Cournot outcome. So leader either chooses Cournot quantity or if he chooses a different level of production he must be better off otherwise he wouldn't do that).

Stackelberg model of oligopoly can be analyzed as sequential game because follower observes what leader does and behaves accordingly. The difference from Cournot model is that in this case the leader knows (can expect) what will follower do. So firm A is looking for the optimal level of production while knowing that $Q_B = \frac{90-Q_A}{2}$.

$$\begin{aligned}
\max \Pi_A &= PQ_A - TC_A = (120 - Q_A - Q_B)Q_A - 30Q_A = \\
&= \left(120 - Q_A - \frac{90 - Q_A}{2}\right) Q_A - 30Q_A = \left(75 - \frac{Q_A}{2}\right) Q_A - 30Q_A \\
\Pi'_A &= 0 \quad \Rightarrow \quad 75 - Q_A - 30 = 0 \quad \Rightarrow \quad Q_A = 45 \\
Q_B &= \frac{90 - Q_A}{2} = 22.5
\end{aligned}$$

And the resulting market price will be $P = 52.5$.



Stackelberg equilibrium. Firm 1, the leader, chooses the point on firm 2's reaction curve that touches firm 1's lowest possible isoprofit line, thus yielding the highest possible profits for firm 1.

Summary:

- Output is greater with Cournot duopoly than monopoly, but lower than perfect competition.
- Price is lower with Cournot duopoly than monopoly, but not as low as with perfect competition.

Bertrand versus Cournot

Although both models have similar assumptions, they have very different implications:

- Since the Bertrand model assumes that firms compete on price and not output quantity, it predicts that a duopoly is enough to push prices down to marginal cost level, meaning that a duopoly will result in perfect competition.
- Neither model is necessarily "better." The accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the industry situation.
- If capacity and output can be easily changed, Bertrand is a better model of duopoly competition. If output and capacity are difficult to adjust, then Cournot is generally a better model.

Stackelberg versus Cournot

- The Stackelberg and Cournot models are similar because in both, competition is on quantity.
- The first move gives the leader in Stackelberg a crucial advantage.
- There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.
- In Cournot competition, it is the simultaneity of the game (the imperfection of knowledge) that results in neither player (ceteris paribus) being at an advantage.