Problem 1: Suppose you're taking the escalator down to Mustek metro station. You want to take a train to Hrandcanska to go home to Kajetanka. You suddenly hear a train approaching. You are close enough to catch it if you start running, but not close enough to see which direction the train is coming from. Your belief of the train going in the direction you want to go is 0.45. If you run and catch the train that goes in your direction, you have a utility gain of 1, but if you run and the train goes in the opposite direction, you have a utility loss of 1.

(a) Without any further signals about which direction the train is going in, would you run?

(b) Now suppose that you see Tomas at the bottom of the escalator, who actually sees the direction in which the train is going (or can infer it from the movement of the people at the station), and who starts running if and only if the train goes in the same direction as he wants to go. You know that Tomas' preferences regarding running or not are the same as yours, except that he may want to go in the other direction than you do. You believe that Tomas wants to go in the same direction as you want to go with a probability p. If Tomas starts running, what is the range of p that would make you run (assume that you run if you're indifferent)? If Tomas does not start running, what is the range of p that would make you run (assume that you run if you're indifferent)?

(c) Now change the scenario a bit. Suppose that you haven't reached the escalators yet, and Tomas is just entering it. You see Tomas, but you don't see down the escalator. But Tomas does, and he sees Jakub at the bottom of the escalator, but does not see the station. The only person who sees the station is Jakub. You know that Tomas and Jakub's preferences regarding running or not are the same as yours. Both you and Tomas also know that Jakub is going to Hrandcanska, but you're not sure whether Tomas is going in the same direction or not. You believe that Tomas wants to go in the same direction as you want to go with a probability p. Now you all hear a train approaching, and if you were to run, each of you would be able to catch the train (because you're super fast). Looking down the escalator, Tomas sees whether Jakub started running, but you don't see it. Also assume, for simplicity, that if you were to start running, you would never see whether Jakub started running or not. If you see Tomas not running, what is the range of p that would make you run? If you see Tomas running, what is the range of p that would make you run?

Solution:

(a) Without any further information it is not worth running, because expected gain from running is $EG(R) = 0.45 * 1 + 0.55 * (-1) = -0.1$ as opposed to 0 if not running. Notice that it is worth running if and only if the probability that the train goes the right direction is higher or equal to 0.5.

(b) P(Tomas is going the same direction)=p. If Tomas starts running, the posterior probability of train going the right direction is:

$$P(Good\ train/Tomas\ runs) = \frac{P(Good\ train\ \&\ Tomas\ runs)}{P(Tomas\ runs)} = \frac{0.45 * p}{0.45 * p + 0.55 * (1-p)}$$

In order to induce running, this probability has to be higher or equal to 0.5, so we get:

$$\frac{0.45 * p}{0.45 * p + 0.55 * (1-p)} \geq 0.5 \Leftrightarrow 0.9p \geq 0.45p + 0.55(1-p) \Leftrightarrow p \geq 0.55$$

If Tomas does not start running our posterior probability that the train goes the right direction is the following:

$$\begin{aligned} P(Good\ train/Tomas\ does\ not\ run) &= \frac{P(Good\ train\ \&\ Tomas\ does\ not\ run)}{P(Tomas\ does\ not\ run)} = \\ &= \frac{0.45 * (1-p)}{0.45 * (1-p) + 0.55 * p} \end{aligned}$$

Again, to make running profitable, this must be higher or equal to 0.5

$$\frac{0.45 * (1-p)}{0.45 * (1-p) + 0.55 * p} \geq 0.5 \Leftrightarrow 0.9(1-p) \geq 0.45(1-p) + 0.55p \Leftrightarrow p \leq 0.45$$

(c) After careful reading of the set-up we see that it is the same as (b) so the same answer apply here as well.

Problem 2: Assume you need to hire one employee for your firm. You advertise the job opening and 3 prospective employees apply for the position. Your policy is never to hire an employee before interviewing her. Suppose that the constant cost of interviewing is 20 per interview. For any of the three applicants, your priors are that the value to you of hiring this particular applicant continuously uniformly distributed in the interval from 100 to 200.

You will hire at most one of the prospective employees, and your objective is to maximize your value net of interviewing costs. What should your hiring strategy be?

Solution:

(a) After the first interview we can decide whether to accept the first applicant or reject her for good (can not hire her any more). After the second interview we again decide whether to accept the second applicant or reject her. If we reject the second candidate we hire the third one for sure.

At the second interview, if we wait to hire the third prospective employee, our expected net gain is:

$$E[V_3] - 20 = 150 - 20 = 130$$

Therefore, we reject an applicant with value less than 130 and accept all others. The probability that the applicant's value is less than 130 and we reject her is 0.3 and the probability that the applicant's value is higher than 130 and we accept her is 0.7. Hence the expected value of the second applicant (if hired) is (130+200)/2=165.

At the first interview, if we reject the first applicant and wait, the expected net value is:

$$EV = 130 * 0.3 + 0.7 * 165 - 20 = 134.5$$

So at the first interview we reject those who have lower value than 134.5, and accept all others.

(b) Alternative interpretation of the problem is that after each interview we are allowed to go back to any applicant and decide to accept her. In this case, at the end of interviewing we will accept the applicant with the highest value. We only do another interview if the expected marginal benefit is at least as much as the marginal cost. Intuitively, the cut-off value in this case should be higher than in part (a), because we have many more options here and therefore should be more likely to reject an applicant. Let's denote the maximum value of up to now interviewed applicants v:

$$\text{MB} = P(\text{find a better applicant}) * E(\text{marginal value of this applicant}) = 20 = \text{MC}$$
$$\frac{200 - v}{100}\left(\frac{v + 200}{2} - v\right) = 20$$
$$v^2 + 400v - 36000 = 0 \quad \Rightarrow \quad v \approx 136.75$$

After each interview we accept (reject) the applicant if her value is higher (lower) than 136.75.

Problem 3: You are sequentially meeting 2 potential dates. You know that they are both interested. The quality of your match with either of them is x, which is uniformly distributed between 0 and 1, independently for the two dates. You know that if you reject the first date, you can still go back to him/her with probability p after you have met the second date. You want to maximize the quality of the match you end up with.

(a) If $p = 0$, at least how good must the match with the first date be for you to accept it straight away and not meet the second date?

(b) As a function of p, how good must the match with the first date be for you to accept it straight away and not meet the second date?

Solution:

(a) 0.5

(b) Suppose the first date is of quality x. If rejecting her, the expected value of continuation is $x[px + (1-p)x/2] + (1-x)(1+x)/2 = (px^2+1)/2$. The first term represents the case when the second date turns out to be worse than the first date. The second term represents the case when the second date turns out to be better than the first date. This is less than x (taking the first date) if $px^2 - 2x + 1 < 0$, or $x > (2-(4-4p)^{0.5})/2p = (1-(1-p)^{0.5})/p$.

Problem 4: You are going to sequentially interview up to three job candidates. The value to you of hiring any particular candidate is uniformly distributed between 100 and 200, independently for each candidate. The cost of interviewing is 6 for the first candidate, 8 for the second candidate, and 10 for the third candidate. After each interview, you either hire the candidate and quit the interviewing process, or you reject the candidate and keep interviewing. Before you start interviewing, what is your expected overall interviewing cost?

Solution: Go from the end of the tree by backward induction. If you get to interview #3, you take anyone. Therefore from the point of view of interview #2, the expected value for continuing to interview #3, disregarding the sunk costs of interviews #1 and #2, is 150-10=140. That will be the acceptance threshold for interview #2. As a result, from the point of view of interview #1, the expected value of continuing to interview #2, disregarding the sunk cost of interview #1, is 0.4*140+0.6*(140+200)/2-8=56+102-8=150. That will be your acceptance threshold for interview #1. Therefore the probability of conducting exactly one interview is 0.5, exactly two interviews 0.5*0.6=0.3, and exactly three interviews 0.5*0.4=0.2. Therefore the expected cost of interviewing is 6*0.5+14*0.3+24*0.2=3+4.2+4.8=12.

Problem 5: You are sequentially meeting 2 potential dates. You know that they are both interested. The quality of your match with either of them is x, which is uniformly distributed between 0 and 1, independently for the two dates. You cannot go back to the first date once you have rejected him/her. But you can start dating the first person and once you meet the second person, you can break up with the first person and move to the second person. However, such breakup has an emotional cost that reduces your enjoyment of the second match by c. You want to maximize the quality of the match net of any emotional costs.

(a) If $c = 0$, at least how good must the match with the first date be for you to start dating this person?

(b) If $c = 0.2$, at least how good must the match with the first date be for you to start dating this person? (Hint: $0.4^{0.5} = 0.632$).

Solution:

(a) If you can break up costlessly, then you start dating anyone you meet on the first date, so your threshold is zero.

(b) Suppose you get a match of at least 0.8 on the first date. Then not taking it is worse than taking it, since you only expect 0.5 on the second match. Also, you will never break up with a person like that when you meet the next person. Now suppose you get a match of $x < 0.8$ on the first date. If you don't take it, you expect 0.5 from the second date. If you take it, though, you will only break up if the new match is better than $x + 0.2$. Therefore your expected value is $(x + 0.2)x + (1 - (x + 0.2))[(1 + (x + 0.2))/2 - 0.2] = (x + 0.2)x + (0.8 - x)(0.8 + x)/2 = (x^2 + 0.4x + 0.64)/2$.

This is better that 0.5 if and only if $x^2+0.4x-0.36 > 0$ giving $x = -0.2+0.4^{0.5} = 0.432$.

In behavioral economics, hyperbolic discounting refers to the empirical finding that people generally prefer smaller, sooner payoffs to larger, later payoffs when the smaller payoffs would be imminent; but when the same payoffs are distant in time, people tend to prefer the larger, even though the time lag from the smaller to the larger would be the same as before.

Problem 6: Hyperbolic discounting. Consider a consumer faced with a ”vice” good like potato chips, which they are tempted to consume rapidly.

The consumer can buy a large (2-serving) or small (1-serving) pack at period 0. In period 1, she must decide how much to consume. If she bought only the small pack, she consumes one serving. If she bought the large pack, she can consume two servings right away, or one serving and save another serving for the future (which is automatically consumed in period 2).

Assume there is positive utility in period 1 from consumption, and negative utility in period 2 (a reduced-form expression for poor health, say). Because the large size has some production economies, it is cheaper, which is reflected in higher immediate consumption utility. The Table below shows numerical utilities. (If she chooses to eat 1 serving from the large pack in period 1, then she gets utility of +3 in period 1, and -2 in period 2, from the second pack.)

Consider a quasi-hyperbolic framework, i.e a discreet time discount function is $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots\}$. For simplicity assume $\delta = 1$ to focus attention on the β term. Analyze the optimal consumption decisions of three types of agents: Exponential ($\beta = 1, \beta' = 1$); naive hyperbolic ($\beta < 1, \beta' = 1$); and sophisticated hyperbolic ($\beta < 1, \beta' = \beta$). Exponential agent has all the time discount rate 1. For naive hyperbolic consumer from the perspective of Period 0 the relative value of a serving at Period 2 to the value at Period 1 is $\beta/\beta = 1$. However, from the perspective of Period 1, the relative value of a serving at Period 2 to the value at Period 1 is $\beta/1 = \beta$. So the value of consumption in Period 2 is lower relative to consumption in Period 1. The same is true for sophisticated hyperbolic consumer but he is aware of this future temptation and therefore he will take this into consideration when deciding what pack to buy.

Purchase Decision Consumption Decision	Instantaneous Utility in Period 1	Instantaneous Utility in Period 2
Small		
1 serving	2.5	-2
Large		
1 serving	3	-2
2 servings	6	-7

For each agent, figure out:

(i) What will they expect to do, at time 0, if they buy either the large or small packages?

(ii) Given your answer in (i), which package will the period-0 "self" purchase, for each of the three types?

(iii) After they buy their optimal package, how much will they consume in period 1?

(iv) Which of the type's (if any) plans embedded in (i) are actually violated in (iii)

(v) Suppose agents could purchase external commitment, in which they could only consume 1 of the 2 servings in the large pack in period 1, at a price of $P > 0$ (think of this as buying pre-packaged dietary portions of food). Which agents would commit at time zero to pay P, and how much would they pay?

Solution:

(i) If consumers buy small package, they have no other choice than consume it in Period 1. If they buy large package, they can choose whether to consume 1 or 2 servings in Period 1:

Exponential discounting:

U(Large, 1 serving) $= 3 + (-2) = 1$

U(Large, 2 servings) $= 6 + (-7) = -1$

Hence, consumer expects to consume 1 serving at Period 1

Naive hyperbolic discounting: Here, the consumer expects to have the discount rate $\beta < 1$ for consumption in Period 1 so given the large package bought in Period 0:

U(Large, 1 serving) $= 3\beta + (-2)\beta = \beta$

U(Large, 2 servings) $= 6\beta + (-7)\beta = -\beta$

Hence, consumer expects to consume 1 serving at Period 1.

Sophisticated hyperbolic discounting: Here, the consumer knows, that when the Period 1 comes, she will not be patient any more:

U(Large, 1 serving) $= 3 + (-2)\beta = 3 - 2\beta$

U(Large, 2 servings) $= 6 + (-7)\beta = 6 - 7\beta$

Hence, if $\beta > 0.6$ the consumer expects to consume 1 serving in Period 1, and if $\beta < 0.6$ the consumer expects to consume 2 servings in Period 1. For the rest of the problem, assume that β is less than 0.6.

(ii) The first two types of consumers (exponential and naive hyperbolic discounting) will choose to buy a large pack. The sophisticated consumer knows that the optimal solution is to buy a large pack and consume 1 serving in Period 1 but at the same time she knows, that when Period 1 comes, she will be greedy and eat everything up. So instead, she buys a small pack leaving her with utility $2.5\beta - 2\beta = 0.5\beta$ instead of buying large pack eating it all and having utility $6\beta - 7\beta = -\beta$.

(iii) Here, the consumer with exponential discounting bought large pack and will consume 1 serving in Period 1. The sophisticated hyperbolic consumer bought small pack and eats it in Period 1. Naive hyperbolic consumer bought large pack and when she decides how much to consume in Period 1 she compares:

U(Large, 1 serving) $= 3 + (-2)\beta = 3 - 2\beta$ and

U(Large, 2 servings) $= 6 + (-7)\beta = 6 - 7\beta$.

For $\beta < 0.6$ she will eat both servings in Period 1.

(iv) The plan is violated only for naive hyperbolic discounting consumer. This is cause by her ignorance of future temptation.

(v) Exponential and naive hyperbolic consumer buys large pack and expects to consume 1 serving in the first period so they will not buy any external commitment. The sophisticated hyperbolic consumer, however, buys the external commitment, because it is better to buy large pack and consume only 1 serving in Period 1 than to buy small pack. The difference in utility is

U(large, 1 serving) - U(small) $= \beta - 0.5\beta = 0.5\beta$.

Sophisticated hyperbolic agent would commit at time zero to pay up to $P = 0.5\beta$.

Note: If $\beta > 0.6$ than naive as well as sophisticated consumer consume only 1 serving in Period 1, they will all buy large pack and not pay for external commitment.