Problem 1: Suppose the agent has a utility of wealth w and effort a given by $u(w,a) = \sqrt{w} - a/2$. The reservation utility is given by 0. The effort a is chosen from the set $A = [0, \infty)$, and conditional on this effort, the distribution of profits is governed by the exponential distribution with parameter $1/a$, whose density on $[0, \infty)$ is given by

$$f(\pi|a) = \frac{e^{-\pi/a}}{a}.$$

Note that under this distribution, $E(\pi|a) = a$, and $Var(\pi|a) = a^2$. Assume throughout the problem that the first-order approach is valid. Your calculations may also be simplified by the observation that $f_a/f = \partial \ln(f)/\partial a$.

(a) Write out the principal's problem when the agent's effort is *observable.* Clearly state the objective, principal's choice variables, and any constraints and their meaning.

(b) Solve for the optimal compensation scheme when the agent's effort is *observable.* What is the welfare of the principal and the agent in this case?

(c) Write out, in two stages, the principal's problem when the agent's effort is *unobservable.* In the first stage, compensation cost is minimized for a fixed effort level. Write down both the original constraints, and their modification using the first-order approach. In the second stage, optimal effort level is chosen. In each stage, clearly state the objective, principal's choice variables, and any constraints and their meaning.

(d) Using the first-stage program, characterize the optimal compensation scheme for a fixed effort level a when the agent's effort is *unobservable* in terms of the two multipliers. Don't pin down the multipliers at this stage. What is the sign of the multiplier on the participation constraint? Why? Assuming that it is not negative, what is the sign of the multiplier on the incentive constraint? Does it depend on a? Why? What is the shape of the compensation schedule?

(e) Using the participation and the incentive constraint, solve for the wage cost minimizing compensation schedule for a fixed effort level a. That is, pin down the two multipliers. How does the compensation schedule depend on a? Derive the expected cost function for inducing an effort level a.

(f) Using the second-stage program, pin down the optimal level of effort and the profit-maximizing compensation schedule. Assume that is the manager is indifferent among multiple levels of effort, he will pick whatever level is intended by the principal. How do the effort level and the welfare differ from the first-best? Why?

(g) Now suppose that there is another signal π_2 of the manager's effort that does not on its own contribute to the principal's payoff. It is distributed according to the same exponential distribution shown above. In addition, conditional on effort, the two profit levels are independent. Effort is unobservable. What is the compensation schedule a function of in this case? Using the two-stage approach, solve for the optimal compensation schedule for a fixed effort level a, the expected cost of effort, and then pin down the optimal level of effort and the final form of the compensation schedule. Does total compensation only depend on the sum of $\pi_1 + \pi_2$? How do the effort level and the welfare differ from the case with one signal only? Why?

Problem 2: Based on *Aggarwal and Samwick: Executive Compensation, Strategic Competition, and Relative Performance Evaluation: Theory and Evidence (1999).* The main idea is to find optimal compensation scheme for managers in a model of duopoly and Bertrand competition (competition in prices). First, principal (owner of the company) chooses a compensation scheme. Then the agent (manager of the company) chooses the price to be charged.

As a benchmark model we use standard example of duopoly and the compensation of manager is an increasing function of firm's profit. In this case, the manager has an incentive to earn as high profit as possible, so he will choose a profit maximizing price. The demand is given by $D_i(p_i, p_j) = A - bp_i + ap_j$ and profit is given by $\pi_i = (p_i - c)(A - bp_i + ap_j) + \varepsilon$, where ε is some noise with distribution $N(0, \sigma^2)$, so it is not possible to determine prices from observed profit. In our example, $A = 2$, $c = 1$, $a = 1$ and $b = 2$.

Assume standard Bertrand competition and find equilibrium prices and corresponding profit for both firms. Then repeat this exercise for the case, where manager's compensation depends on both own firm's profit and competing firm's profit in the following way: $w_i = k_i + \alpha_i \pi_i + \beta_i \pi_i$. In which case is the profit higher? Would the situation be the same in case of Cournot competition?