

Problem 1: Two-sided market with non transferable utility. Find the set of stable matchings for the following table of preferences:

A: b \succ c \succ d \succ a	a: D \succ B \succ C \succ A
B: a \succ d \succ c \succ b	b: C \succ A \succ B \succ D
C: c \succ d \succ b \succ a	c: B \succ D \succ A \succ C
D: a \succ c \succ d \succ b	d: A \succ D \succ C \succ B

Problem 2: Housing markets and housing allocation problem - Random serial dictatorship. There are three agents i_1, i_2, i_3 and three houses h_1, h_2, h_3 . Agent i_1 is a current tenant and he occupies house h_1 . Agents i_2, i_3 are new applicants and houses h_2, h_3 are vacant houses.

• Utilities are:

	h_1	h_2	h_3
i_1	3	4	1
i_2	4	3	1
i_3	3	4	1

• Agent i_1 has two options:

1. he can keep house h_1
2. he can give it up and enter the lottery

Which option should he chose?

Problem 3: Two-sided market with non transferable utility. In this problem we describe the mechanisms that we study in the context of constrained school choice: the Gale-Shapley Student-Optimal Stable mechanism, the Boston mechanism, and the Top Trading Cycles mechanism. We define a school choice problem by a set of schools and a set of students, each of which has to be assigned a seat at not more than one of the schools. Each student is assumed to have strict preferences over the schools and the option of remaining unassigned. Each school is endowed with a strict priority ordering over the students and a fixed capacity of seats.

Let $I = \{i_1, i_2, i_3, i_4\}$ be the set of students, $S = \{s_1, s_2, s_3\}$ be the set of schools, and $q = (1, 2, 1)$ be the capacity vector. The students' preferences P and the priority structure f are given in the table below. So, for instance, $P_{i_1} = s_2, s_1$ and $f_{s_1}(i_1) < f_{s_1}(i_2) < f_{s_1}(i_3) < f_{s_1}(i_4)$.

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	f_{s_1}	f_{s_2}	f_{s_3}
s_2	s_1	s_1	s_2	i_1	i_3	i_4
s_1	s_2	s_2	s_3	i_2	i_4	i_1
	s_3		s_1	i_3	i_1	i_2
				i_4	i_2	i_3

If the students truthfully report their preferences, find outcomes of Gale and Shapley algorithm, Boston matching algorithm, and Top trading cycles algorithm.

Problem 4: Let $I_E = \{i_1, i_2, i_3, i_4\}$, $I_N = \{i_5\}$, $H_O = \{h_1, h_2, h_3, h_4\}$, and $H_V = \{h_5, h_6, h_7\}$. Let the ordering f order the agents as $i_1 - i_2 - i_3 - i_4 - i_5$ and the preferences (from best to worst) as follows:

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
h_2	h_7	h_2	h_2	h_4
h_6	h_1	h_1	h_4	h_3
h_5	h_6	h_4	h_3	h_7
h_1	h_5	h_7	h_6	h_1
h_4	h_4	h_3	h_1	h_2
h_3	h_3	h_6	h_7	h_5
h_7	h_2	h_5	h_5	h_6
h_0	h_0	h_0	h_0	h_0

Use top trading cycles mechanism to find the outcome of this matching problem.