AAU - Business Mathematics I

Lecture #1, February 9, 2009

1 Numbers and Sets

Set: collection of distinct objects which are called elements (numbers, people, letters of alphabet)

Two ways of defining sets:

• list each member of the set (e.g. {4,2,15,6}, {red, blue, white}, ...)

The order in which the elements of a set are listed definition is irrelevant, as are any repetitions in the list. For example,

$$\{6, 11\} = \{11, 6\} = \{11, 11, 6, 11\}$$

are equivalent, because the specification means merely that each of the elements listed is a member of the set.

• rule (e.g. A = set of even numbers, $B = \{n^2, n \in N, 0 \leqq n \leqq 5\}$, ...)

Membership:

• $4 \in A$, $15 \in \{4, 2, 15, 6\}$, $16 \in B$

• $5 \notin A$, $5 \notin B$, green $\notin$ {red, blue, white}

Cardinality: the number of members of a set

• $|A| = \infty$

• $|B| = 6$

• $|C| = 0$, where C={three sided squares}, C is an empty set

Subsets:

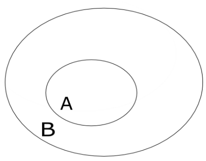
• $A \subseteq B$ if every member of A is in B as well

• if $A \subseteq B$ but $A \neq B$, then A is a proper subset of B, $A \subset B$

• $\{1, 2\} \subseteq \{1, 2, 3, 4\}$ and also $\{1, 2\} \subset \{1, 2, 3, 4\}$

• $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4\}$ but it is not true that $\{1, 2, 3, 4\} \subset \{1, 2, 3, 4\}$

• set of men is a proper subset of the set of all people

Venn diagram: 

Note: $A \subseteq A$, $\varnothing \subseteq A$ for every set A

Special Sets:

P - primes, N - natural numbers, Z - integers, $Q = \{\frac{a}{b}, a, b \in Z, b \neq 0\}$ - rational, R - real, I - irrational

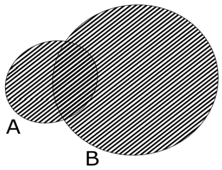
$P \subset N \subset Z \subset Q \subset R$

Note: this simple theory of sets may lead to contradictions. Russell's paradox: Barber shaves only those men who do not shave themselves. Should the barber shave himself or not? Is the barber member of a set of men who shave themselves or not.

BASIC OPERATIONS

There are ways to construct new sets from existing ones. Two sets can be "added" together, "subtracted", etc.

• **Union:** $A \cup B$ elements that belong to A or B.

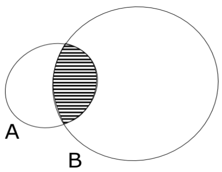


Example: $\{1, 2\} \cup \{\text{blue,red}\} = \{1, 2, \text{blue,red}\}$

Properties:

- $A \cup B = B \cup A$
- $A \subseteq A \cup B$
- $A \cup A = A$
- $A \cup \varnothing = A$

• **Intersection:** $A \cap B$ elements that belong to A and B at the same time.



Example: $\{1, 2\} \cap \{\text{blue,red}\} = \varnothing$

$\{1, 2\} \cap \{1, 2, 4, 7\} = \{2\}$

Properties:

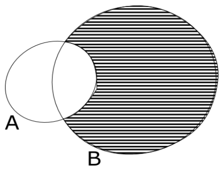
- $A \cap B = B \cap A$
- $A \cap B \subseteq A$
- $A \cap A = A$

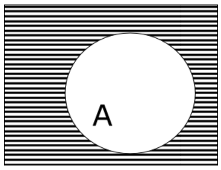
- $A \cap \varnothing = \varnothing$

- If $A \cap B = \varnothing$, then A and B are said to be disjoint.

• **Difference and Complement:** $B \setminus A$ or $B - A$: set of elements which belong to B, but not to A

In certain settings all sets under discussion are considered to be subsets of a given universal set U. Then, $U \setminus A$ is called complement of A and is denoted A' or A^C





Example: $\{1, 2, \text{green}\} \setminus \{\text{red,white,green}\} = \{1, 2\}$

$\{1, 2\} \setminus \{1, 2\} = \varnothing$

Integers $\setminus$ Even numbers = Odd numbers

Properties:

- $A \cup A^C = U$

- $A \cap A^C = \varnothing$

- $(A^C)^C = A$

- $A \setminus A = \varnothing$

- $A \setminus B = A \cap B^C$

• **Cartesian product:** $A \times B$ combining every element from A with every element from B; set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

Example: $\{1, 2\} \times \{\text{red, white,blue}\} = \{(1, \text{red}), (1, \text{white}), (1, \text{blue}), (2, \text{red}), (2, \text{white}), (2, \text{blue})\}$

Properties:

- $A \times \varnothing = \varnothing$

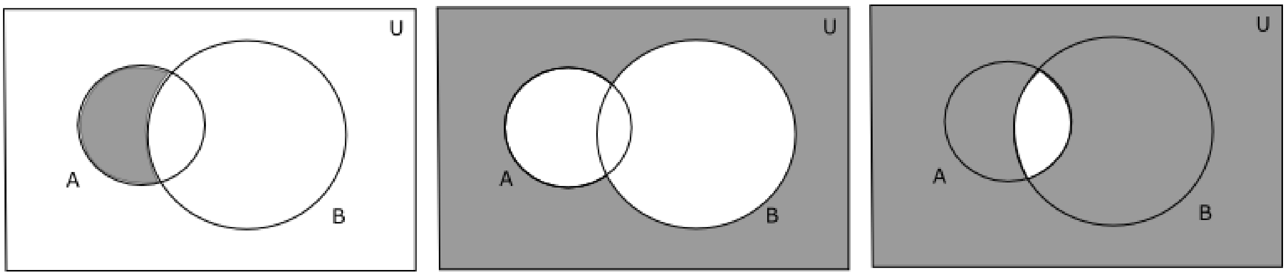
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Some identities:

• $A \setminus B = A \cap B^C$

• $(A \cup B)^C = A^C \cap B^C$

• $(A \cap B)^C = A^C \cup B^C$



$A \setminus B$ $\quad$ $(A \cup B)^C$ $\quad$ $(A \cap B)^C$

Example: Do the following identities hold?

- $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$ holds
- $(A \cap B^C) \cup C = A \cap B$ does not hold

(to see this draw Venn diagrams)

Example: 116 out of 129 students are regularly having lunch or supper. 62 students eat at most one of these meals. Further, there are 47 more students having lunch than students having supper. How many students are regularly having both lunch and supper? How many of them eat only supper and how many of them eat just lunch? [Hint: use Venn diagrams]

Solution: We will use the following notation:

x - students eating only lunch

y - students eating only supper

z - students eating both lunch and supper

w - students not eating any meal

Then the set-up of the problem says:

• $x + y + z = 116$

• $x + y + w = 62$

• $x = y + 47$

We know that 129 - 116 students do not eat any meal, so w = 129 - 116 = 13.

Now we know that $x + y = 62 - 13 = 49$ and $x - y = 47$. This is true only for $x = 48$ and $y = 1$. Last, $x + y + z = 116$ implies that $z = 116 - 48 - 1 = 67$. Therefore, 67 students regularly eat both lunch and supper, 48 eat only lunch and 1 eats only supper.

Numbers

complex (C) — imaginary
╲ real (R) — irrational (I)
╲ rational (Q) — non-integer
╲ integer (Z) — natural (N)
╲ 0
╲ negatives of natural

Real numbers (R): represented on real line with origin 0

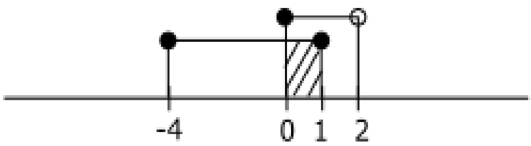
Intervals: subsets of a real line

closed - e.g. [2,5] - 2 and 5 belong to the interval

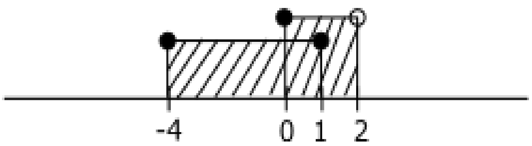
open - e.g. (3,9) - 3 and 9 do not belong to the interval

Intersection: $[-4, 1] \cap [0, 2) = [0, 1]$

Union: $[-4, 1] \cup [0, 2) = [-4, 2)$



Intersection



Union

Example:

- $A = [-5, 3], B = [1, 10] \ \rightarrow \ A \cap B = [1, 3], \ A \cup B = [-5, 10]$
- $A = (-\infty, 2), B = (0, 4] \ \rightarrow \ A \cap B = (0, 2), \ A \cup B = (-\infty, 4]$
- $A = [-2, 8], B = (3, 10), C = (9, 15) \ \rightarrow \ A \cap B \cap C = \emptyset, \ A \cup B \cup C = [-2, 15)$

Logic

A statement - a declarative sentence that is either true or false.

A simple statement - one that does not contain any other statement as a part.

Examples of sentences that are (or make) statements:

• ”Socrates is a man.”

• ”A triangle has three sides.”

• ”Paris is the capital of England.”

The first two (make statements that) are true, the third is (or makes a statement that is) false.

Examples of sentences that are not (or do not make) statements:

- "Who are you?"
- "Run!"

Negation - In logic and mathematics, negation or "not" is an operation on logical values, which changes true to false and false to true. Intuitively, the negation of a proposition holds exactly when that proposition does not hold.

Notation: statement - p, can be true or false; if p is true then "NOT p" or "$\sim p$" or "$\rceil p$" is false.

Examples of negations of previous statements:

- "Socrates is not a man." or "Socrates is a woman."
- "A triangle does not have three sides." or "A triangle has at most two or at least four sides."
- "Paris is not the capital of England."

Notation: $\forall$ - for all/every/each; $\exists$ - there exists/at least one

Examples: A is a set of all natural numbers larger than 10: $A = \{\forall x \in N; x > 10\}$

There exists at least one number for which the square root is negative: $\exists x; \sqrt{x} < 0$

Examples: Decide if the following statements are true or false and find negations:

- Math teacher is nice. True. Math teacher is not nice.
- There are **no** unemployed people in the CR. False. There is **at least** one unemployed person in the CR.
- **At least four** students in class are women. True. **At most three/less than four** students in class are women.
- Today is Tuesday. False. Today is not Tuesday.
- **At most three** students have blue eyes. True. **At least four/more than three** students have blue eyes.
- Every day has 25 hours. False. At least one day does not have 25 hours.
- **All** cars are red. False. **At least one** car is **not** red.
- The semester begins today. False. The semester does not begin today.