2 Algebraic Expressions and Polynomials

Algebraic expressions are formed using constants, variables and operators

e.g. $\sqrt{x^3+5}$, $x+y-7$, $(2x-y)^2$, ...

Polynomials are special algebraic expressions which include only addition, subtraction, multiplication and raising to a natural number powers

e.g. $4x^3-2x+7$ (polynomial of 3^{rd} degree), $x^3-3x^2y+xy^2+2y^7$ (7^{th} degree), $2x^3y^2-5x-2y^2$ (5^{th} degree), ...

BASIC OPERATIONS

Addition: $(3x^3+2x+1)+(7x^2-x+3) = 3x^3+7x^2-x+4$

Subtraction: $(3x^3+2x+1)-(7x^2-x+3) = 3x^3+7x^2+3x-2$

Multiplication: $(2x-3)(3x^2-2x+3) = 2x(3x^2-2x+3)-3(3x^2-2x+3) = 6x^3-4x^2+6x-9x^2+6x-9 = 6x^3-13x^2+12x-9$

Special products: $(a+b)^2 = a^2+2ab+b^2$ NOT a^2+b^2!!!

$(a-b)^2 = a^2-2ab+b^2$ NOT a^2-b^2!!!

$a^2-b^2 = (a+b)(a-b)$

Factoring: Factor of an algebraic expression is one of two or more algebraic expressions whose product is the given algebraic expression; e.g. $x^2-4=(x+2)(x-2)$. $(x+2)$ and $(x-2)$ are factors.

RATIONAL EXPRESSIONS: BASIC OPERATIONS

Rational expressions are fractional expressions whose numerator and denominator are polynomials

Simplify:

$$\frac{x^2-6x+9}{x^2-9} = \frac{(x-3)^2}{(x+3)(x-3)} = \frac{x-3}{x+3} \quad \text{for all} \quad x \neq \pm 3$$

Reduce to the lowest terms:

$$\frac{6x^4(x^2+1)^2-3x^2(x^2+1)^3}{x^6} = \frac{(x^2+1)^2[6x^4-3x^2(x^2+1)]}{x^6} =$$
$$= \frac{(x^2+1)^2 3x^2[2x^2-x^2-1]}{x^6} = \frac{(x^2+1)^2(x^2-1)}{x^4} \quad \text{for all} \quad x \neq 0$$

Least common denominator: is found as follows: Factor each denominator completely; identify each different prime factor from all the denominators; form a product using each different factor to the highest power that occurs in any one denominator. This product is the LCD.

Example: $$\frac{x^2}{x^2+2x+1} + \frac{x-1}{3x+3} - \frac{1}{6} = \frac{x^2}{(x+1)^2} + \frac{x-1}{3(x+1)} - \frac{1}{6} =$$
$$= \frac{6x^2 + 2(x+1)(x-1) - (x+1)^2}{6(x+1)^2} = \frac{7x^2+2x-3}{6(x+1)^2}$$

More problems:

$$x^a x^b = x^{a+b} \qquad x^2x^4 = x^6 \qquad 2^2 2^3 = 4.8 = 32 = 2^5$$
$$(x^a)^b = x^{ab} \qquad (x^2)^3 = x^6 \qquad (2^2)^3 = 4^3 = 64 = 2^6$$
$$x^{-a} = \frac{1}{x^a} \qquad x^{-2} = \frac{1}{x^2} \qquad 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$
$$x^{1/2} = \sqrt{x} \qquad 9^{1/2} = \sqrt{9} = 3$$