AAC - Business Mathematics I
Lecture #13, January 18, 2008
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13 Review Lecture

Equations and Inequalities

Problem 1: Solve the absolute value inequality. Write the solution set using interval notation: $|4x+7|<5$

[A] $\left(-3,-\frac{1}{2}\right)$ [B] $\left(-\infty,-\frac{1}{2}\right)$ [C] $(-\infty,-3)$ [D] $(-\infty,-3)\cup\left(-\frac{1}{2},\infty\right)$

Solution:

$$\begin{aligned}
&|4x+7|<5\\
&-5<4x+7<5 \quad /-7\\
&-12<4x<-2 \quad /\div 4\\
&-3<x<-1/2 \quad \Rightarrow \quad x\in(-3,-1/2) \quad \rightarrow \quad [A]
\end{aligned}$$

Problem 2: Write as a single interval, using interval notation: $(-\infty,1)\cap(-10,5)$

[A] $(-\infty,-5)$ [B] $(1,5)$ [C] $(-10,1)$ [D] $(-\infty,-10)$

Solution: $(-\infty,1)\cap(-10,5)=(-\infty,5) \quad \rightarrow \quad [A]$

Problem 3: Solve the following inequality for x. Express the solution set using interval notation: $\frac{1}{2}x-4<\frac{1}{3}x+5$

[A] $(-\infty,9)$ [B] $(-\infty,54)$ [C] $(-\infty,6)$ [D] $(-\infty,\frac{54}{4})$

Solution:

$$\begin{aligned}
&\frac{1}{2}x-4<\frac{1}{3}x+5\\
&\frac{1}{2}x-\frac{1}{3}x<5+4\\
&\frac{1}{6}x<9\\
&x<54 \quad \Rightarrow \quad x\in(-\infty,54) \quad \rightarrow \quad [B]
\end{aligned}$$

Problem 4: Solve the compound inequality for x. Express the solution set using interval notation: $8 \le 5 - x$ or $3x - 2 > 10$

[A] $\emptyset$ [B] $[-3, 4)$ [C] $(-\infty, -3] \cup (4, \infty)$ [D] $[-3, \infty)$

Solution:

$$\begin{aligned}
&8 \le 5 - x \quad \text{or} \quad 3x - 2 > 10 \\
&-3 \le -x \quad \text{or} \quad 3x > 12 \\
&x \le 3 \quad \text{or} \quad x > 4 \quad \Rightarrow \quad x \in (-\infty, 3] \quad \text{or} \quad x \in (4, \infty) \quad \rightarrow \quad [C]
\end{aligned}$$

Exponents and Logarithms

Problem 5: Solve for x: $\log(3x - 9) = \log(2x - 6)$

[A] 3 [B] all real numbers [C] -3 [D] No solution

Solution:

$$\begin{aligned}
&\log(3x - 9) = \log(2x - 6) \\
&3x - 9 = 2x - 6 \\
&3x - 2x = -6 + 9 \\
&x = 3 \quad \rightarrow \quad [A]
\end{aligned}$$

Problem 6: Find the exact solution for x: $(1.3)^{2x} = 4$

$[A]\ \frac{1}{2} \ln\left(\frac{4}{1.3}\right)$ $[B]\ \frac{20}{13}$ $[C]\ \frac{1}{2}[\ln(4) - \ln(1.3)]$ $[D]\ \frac{\ln(4)}{2\ln(1.3)}$

Solution:

$$\begin{aligned}
&(1.3)^{2x} = 4 \\
&\ln\left[(1.3)^{2x}\right] = \ln 4 \\
&(2x)\ln[(1.3)] = \ln 4 \\
&2x = \frac{\ln 4}{\ln[(1.3)]} \\
&x = \frac{\ln 4}{2\ln[(1.3)]} \quad \rightarrow \quad [D]
\end{aligned}$$

Problem 7: Solve the equation for x: $\left(\frac{2}{3}\right)^{x+1} = \frac{8}{27}$

[A] 1 [B] 2 [C] 3 [D] 4

Solution:

$$\left(\frac{2}{3}\right)^{x+1} = \frac{8}{27}$$
$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{2}{3}\right)^{3}$$
$$x+1=3$$
$$x=2 \quad \rightarrow \quad [B]$$

Matrices and Determinants

Problem 8: When the system of linear equations is solved, what is the value of x and y?

$x-2y=4$, $3x-y=-3$

Solution:

$$x-2y=4 \quad \Rightarrow \quad x=4+2y \quad \text{plug in the second equation:}$$
$$3(4+2y)-y=-3$$
$$12+6y-y=-3$$
$$5y=-15$$
$$y=-3 \quad \text{plug in the first equation to find the value of } x$$
$$x=4+2y=4+2(-3)=4-6=-2$$
$$\Rightarrow \quad x=-2,\ y=-3$$

Problem 9: Find product of the following matrices:

$$A=\begin{pmatrix} 1 & 4 \\ -2 & 0 \end{pmatrix} \qquad B=\begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$$

Solution:

$$AB = \begin{pmatrix} 1 & 4 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 + 4\cdot 2 & 1\cdot(-1) + 4\cdot 2 \\ -2\cdot 3 + 0\cdot 2 & -2\cdot(-1) + 0\cdot 2 \end{pmatrix} = \begin{pmatrix} 11 & 7 \\ -6 & 2 \end{pmatrix}$$

Problem 10: Find the following determinants:

(a) $\begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 2 & 2 & 3 \\ 3 & -1 & -1 \\ -2 & 1 & 0 \end{vmatrix}$

Solution:

$$(a) \quad \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} = 1\cdot 1 - (-2)(-3) = 1 - 6 = -5$$

$$(b) \quad \begin{vmatrix} 2 & 2 & 3 \\ 3 & -1 & -1 \\ -2 & 1 & 0 \end{vmatrix} = 2(-1)^{1+1}[(-1)\cdot 0 - 1\cdot(-1)] + 2(-1)^{1+2}[3\cdot 0 - (-1)(-2)] +$$

$$+3(-1)^{1+3}[3\cdot 1 - (-1)(-2)] = 2 + 4 + 3 = 9$$

Problem 11: Solve the following systems using (i) matrix method, (ii) inverse matrix, and (iii) Cramer's rule:

$x + y = 1, \;\; 3x - 4y = -18$

Solution: Matrix method:

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 3 & -4 & -18 \end{array}\right) \begin{array}{c} \times(-3) \\ \swarrow \end{array} \sim \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -7 & -21 \end{array}\right) \div(-7) \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right) \begin{array}{c} \nwarrow \\ \times(-1) \end{array} \sim \left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right) \Rightarrow \begin{array}{c} x = -2 \\ y = 3 \end{array}$$

Inverse matrix:

$$A = \begin{pmatrix} 1 & 1 \\ 3 & -4 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 3 & -4 & 0 & 1 \end{array}\right) \begin{array}{c} \times(-3) \\ \swarrow \end{array} \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -7 & -3 & 1 \end{array}\right) \div(-7) \sim$$

$$\sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 3/7 & -1/7 \end{array}\right) \begin{array}{c} \nwarrow \\ \times(-1) \end{array} \sim \left(\begin{array}{cc|cc} 1 & 0 & 4/7 & 1/7 \\ 0 & 1 & 3/7 & -1/7 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 4/7 & 1/7 \\ 3/7 & -1/7 \end{pmatrix}$$

Check:

$$AA^{-1} = \begin{pmatrix} 1 & 1 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} 4/7 & 1/7 \\ 3/7 & -1/7 \end{pmatrix} = \begin{pmatrix} 1 \cdot 4/7 + 1 \cdot 3/7 & 1 \cdot 1/7 + 1 \cdot (-1/7) \\ 3 \cdot 4/7 + (-4) \cdot 3/7 & 3 \cdot 1/7 - 4 \cdot (-1/7) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$AX = B \longrightarrow \begin{pmatrix} 1 & 1 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -18 \end{pmatrix}$$

$$X = A^{-1}B \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4/7 & 1/7 \\ 3/7 & -1/7 \end{pmatrix} \begin{pmatrix} 1 \\ -18 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} \cdot 1 + \frac{1}{7} \cdot (-18) \\ \frac{3}{7} \cdot 1 - \frac{1}{7} \cdot (-18) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \Rightarrow \begin{array}{c} x = -2 \\ y = 3 \end{array}$$

Cramer's rule:

$$x = \frac{\begin{vmatrix} 1 & 1 \\ -18 & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3 & -4 \end{vmatrix}} = \frac{14}{-7} = -2$$

$$y = \frac{\begin{vmatrix} 1 & 1 \\ 3 & -18 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3 & -4 \end{vmatrix}} = \frac{-21}{-7} = 3$$

Combinatorial Mathematics

Problem 12: Evaluate:

$$(a)\ \frac{14!}{12!}$$

$$(b)\ \frac{7!}{7!(7-7)!}$$

$$(c)\ \frac{8!}{3!(8-3)!}$$

$$(d)\ \frac{5!}{2!3!}$$

Solution:

$$(a)\ \frac{14!}{12!} = \frac{14 \cdot 13 \cdot 12!}{12!} = 14 \cdot 13 = 182$$

$$(b)\ \frac{7!}{7!(7-7)!} = \frac{7!}{7!0!} = \frac{7!}{7!1} = 1$$

$$(c)\ \frac{8!}{3!(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3!5!} = \frac{8 \cdot 7 \cdot 6}{3!} = \frac{8 \cdot 7 \cdot 6}{6} = 8 \cdot 7 = 56$$

$$(d)\ \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2!3!} = \frac{5 \cdot 4}{2} = 10$$

Problem 13: A deli serves sandwiches with the following options: 3 kinds of bread, 5 kinds of meat, and lettuce or sprouts. How many different sandwiches are possible, assuming one item is used out of each category?

Solution:

O_1 : Choose kind of brad
N_1 : 3
O_2 : Choose kind of meat
N_2 : 5
O_3 : Choose vegetable
N_3 : 2

Using multiplication principle we have that the number of sandwiches is: $3 \cdot 5 \cdot 2 = 30$

Problem 14: How many 5-digit zip codes are possible? How many of these codes contain no repeated digits?

Solution: There are ten numbers available (0, 1, 2, ... 9). If repeated digits are allowed, then there are 10 digits possible for each digit in the zip code and hence there are $10 \times 10 \times 10 \times 10 \times 10 = 10^5$ zip codes. If codes contain no repeated digits, then there are 10 digits possible for the first digit in the zip code, 9 possible numbers for the second digit in the zip code, ... and 6 possible numbers for the last digit in the zip code. Therefore there are $10 \times 9 \times 8 \times 7 \times 6$ zip codes with no repeated digits.

Financial Mathematics

Problem 15: If \$2500 is invested in the account that pays interest compounded continuously at a rate of 4%, how long will it take to double the investment? Hint: $A = Pe^{rn}$

[A] 2.8 years [B] 13.0 years [C] 14.2 years [D] 17.3 years

Solution:

$$\begin{aligned}
&A = Pe^{rn} \\
&5000 = 2500e^{0.04n} \\
&2 = e^{0.04n} \\
&\ln 2 = \ln e^{0.04n} \\
&\ln 2 = (0.04n) \ln e = 0.04n \\
&n = \frac{\ln 2}{0.04} = 17.3 \ \text{ years} \quad \rightarrow \quad [D]
\end{aligned}$$

Problem 16: If \$4500 is deposited in a bank account paying 8% compounded quarterly, then how much interest will be earned at the end of 6 years? Hint: $A = P\left(1 + \frac{i}{t}\right)^{nt}$

[A] \$353,235.81 [B] \$24,035.31 [C] \$7237.97 [D] \$2737.97

Solution:

$$\begin{aligned}
&A = P\left(1 + \frac{i}{t}\right)^{nt} \\
&A = 4500\left(1 + \frac{0.08}{4}\right)^{4 \cdot 6} = 4500 \cdot 1.02^{24} = 7237.97
\end{aligned}$$

The amount of money in a bank account will be \$7237.97 what means that the interest earned is $\$7237.97 - \$4500 = \$2737.97 \ \rightarrow \ [D]$

Problem 17: Celia has invested \$2500 at 11% yearly interest. How much must she invest at 12% so that the interest from both investments totals \$695 after a year?

[A] \$50.40 [B] \$2928.75 [C] \$1596.67 [D] \$3500.00

Solution: The interest from the first investment is:

$$A = P(1+i)^n$$
$$A = 2500(1+0.11)^1 = 2500 \cdot 1.11 = 2775$$

Hence the interest from the first investment is \$2775 − \$2500 = \$275.

The total investment is supposed to be \$695 what means that the interest from the second investment has to be \$695 − \$275 = \$420.

$$P + 420 = P(1+i)^n$$
$$P + 420 = P(1+0.12)^n$$
$$P + 420 = P \cdot 1.12^1$$
$$420 = P \cdot 1.12 - P = 0.12P$$
$$P = \frac{420}{0.12} = 3500 \quad \rightarrow \quad [D]$$

Problem 18: After a 7% increase in salary, Laurie makes \$1016.50 per month. How much did she earn per month before the increase?

[A] \$950 [B] \$1087.66 [C] \$945.35 [D] \$871.29

Solution: Let's denote the original salary S. The according to the setup we have that:

$$S + 0.07S = 1016.50$$
$$1.07S = 1016.50$$
$$S = \frac{1016.50}{1.07} = 950 \quad \rightarrow \quad [A]$$