AAC - Business Mathematics I

Lecture #1, October 6, 2007

Katarína Kálovcová

1 Numbers and Sets

Set: collection of distinct objects which are called elements (numbers, people, letters of alphabet)

Ways of defining sets:

- list each member of the set (e.g. {4,2,15,6}, {red, blue, white}, ...)
- rule (e.g. A = set of even numbers, $B = \{n^2, n \in N, 0 \leqq n \leqq 5\}$, ...)

Membership:

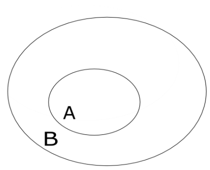
- $4 \in A$, $15 \in \{4, 2, 15, 6\}$, $16 \in B$
- $5 \notin A$, $5 \notin B$, green $\notin$ {red, blue, white}

Cardinality: the number of member of a set

- $|A| = \infty$
- $|B| = 6$
- $|C| = 0$, where C={three sided squares}

Subsets:

- $A \subseteq B$ if every member of A is in B as well
- if $A \subseteq B$ but $A \neq B$, then A is a proper subset of B, $A \subset B$
- $\{1, 2\} \subseteq \{1, 2, 3, 4\}$ and also $\{1, 2\} \subset \{1, 2, 3, 4\}$
- $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4\}$ but it is not true that $\{1, 2, 3, 4\} \subset \{1, 2, 3, 4\}$
- set of men is a proper subset of the set of all people

Venn diagram: 

Note: $A \subseteq A$, $\varnothing \subseteq A$ for every set A

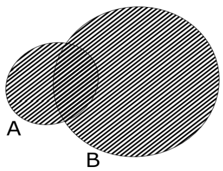
Special Sets:

P - primes, N - natural numbers, Z - integers, $Q = \{\frac{a}{b}, a, b \in Z, b \neq 0\}$ - rational, R - real, I - irrational

$P \subset N \subset Z \subset Q \subset R$

BASIC OPERATIONS

• **Union:** $A \cup B$ elements that belong to A or B.

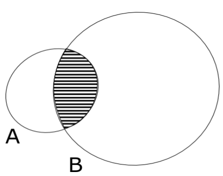


Example: $\{1,2\} \cup \{\text{blue,red}\} = \{1,2,\text{blue,red}\}$

Properties:

- $A \cup B = B \cup A$
- $A \subseteq A \cup B$
- $A \cup A = A$
- $A \cup \varnothing = A$

• **Intersection:** $A \cap B$ elements that belong to A and B at the same time.



Example: $\{1,2\} \cap \{\text{blue,red}\} = \varnothing$

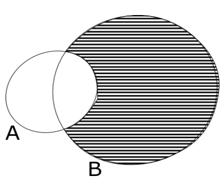
$\{1,2\} \cap \{1,2,4,7\} = \{2\}$

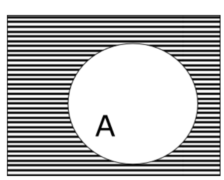
Properties:

- $A \cap B = B \cap A$
- $A \cap B \subseteq A$
- $A \cap A = A$
- $A \cap \varnothing = \varnothing$

• **Difference and Complement:** $B \setminus A$ or $B - A$: set of elements which belong to B, but not to A

In certain settings all sets under discussion are considered to be subsets of a given universal set U. Then, $U \setminus A$ is called complement of A and is denoted A' or A^C





Example: $\{1, 2, \text{green}\} \setminus \{\text{red,white,green}\} = \{1, 2\}$

$\{1, 2\} \setminus \{1, 2\} = \varnothing$

Integers $\setminus$ Even numbers = Odd numbers

Properties:

- $A \cup A^C = U$
- $A \cap A^C = \varnothing$
- $(A^C)^C = A$
- $A \setminus A = \varnothing$
- $A \setminus B = A \cap B^C$

• **Cartesian product:** $A \times B$ combining every element from A with every element from B; set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

Example: $\{1, 2\} \times \{\text{red, white,blue}\} = \{(1, \text{red}), (1, \text{white}), (1, \text{blue}), (2, \text{red}), (2, \text{white}), (2, \text{blue})\}$

Properties:

- $A \times \varnothing = \varnothing$
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Some identities:

• $A \setminus B = A \cap B^C$

• $(A \cup B)^C = A^C \cap B^C$

• $(A \cap B)^C = A^C \cup B^C$

Numbers

complex (C) — imaginary
＼ real (R) — irrational (I)
＼ rational (Q) — non-integer
＼ integer (Z) — natural (N)
＼ 0
＼ negatives of natural

Real numbers (R) - represented on real line with origin 0

Intervals - subsets of a real line

closed - e.g. [2,5] - 2 and 5 belong to the interval

open - e.g. (3,9) - 3 and 9 do not belong to the interval

Intersection - $[-4, 1] \cap [0, 2) = [0, 1]$

Union - $[-4, 1] \cup [0, 2) = [-4, 2)$

Logic

A simple statement - one that does not contain any other statement as a part (p, q, can be true or false; if p is true then "NOT p" or " $\sim p$" or "$\rceil p$" is false)

A compound statement - one with two or more simple statements as parts

An operator - joins simple statements into compounds

Compound statements:

statement	symbol	how we read it
Conjunction	$p \wedge q$	both p and q are true
Disjunction	$p \vee q$	either p or q is true or both
Implication	$p \Rightarrow q$	if p is true then q is true
Equivalence	$p \Leftrightarrow q$	p and q are either both true or both false

Every statement has its truth value, i.e. every statement is either true or false. Truth value of a compound statement can be derived based on truth values of its parts (simple statements)

Truth table - complete list of the possible truth values of a statement:

p	q	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Examples: Look at the 3^{rd} column and 3^{rd} row in the table above. Interpretation: if p is true and q is false, than conjunction is false; e.g.: A day has 24 hours and an hour has 70 minutes. Here, p is "A day has 24 hours" and q is "an hour has 70 minutes". p is true, q is false and the compound statement - conjunction - is false, because conjunction requires both simple statements to be true ("p and q" means p is true and at the same time q is true).

Now, let's look at the 5^{th} column and 4^{th} row in the table above. Interpretation: if p is false and q is true, than implication is true; e.g.: If it doesn't rain, I'll go out with you. Here, p is "it doesn't rain" and q is "I'll go out with you". I make a promise to go out only if it does not rain, I don't say a word about what I'll do if it does rain. So $false \Rightarrow true$ is a true statement as well as $false \Rightarrow false$. The only case when implication is $false$ is $true \Rightarrow false$ - it does not rain, but I will not go out. This is the only case when the original statement was a lie.

Examples: Statements and their negations:

- Today it is Sunday $\not\Leftrightarrow$ Today it is not Sunday
- All people have black hair $\not\Leftrightarrow$ At least one person does not have black hair
- At least one student is a girl $\not\Leftrightarrow$ None of students is a girl
- John and Susan are sick $\not\Leftrightarrow$ Either John or Susan is not sick
- If a firm has smart CEO then it makes a profit $\not\Leftrightarrow$ Firm has smart CEO and it does not make a profit
- Profit of Microsoft is either \$1000 or \$5000 $\not\Leftrightarrow$ Profit of Microsoft is neither \$1000 nor \$5000