



1. Suppose that you need \$ 14500 for a new car which you want to buy in 5 years. In your bank the annual rate is 7% and it is compounded annually. How much money do you have to deposit today to be able to afford your car in 5 years?

Solution:

$$\begin{aligned}A &= P(1 + r)^t \\14500 &= P(1 + 0.07)^5 \\P &= \frac{14500}{(1 + 0.07)^5} \approx 10338\end{aligned}$$

2. Solve the following exponential equations:

(a) $7^{3x+1} = 49^x$

(b) $2^{x^2-7x+10} = 2^{2x-10}$

(c) $3^{x^2-3x+2} = 1$

Solution:

(a) $7^{3x+1} = 49^x$

$$7^{3x+1} = 7^{2x}$$

$$3x + 1 = 2x$$

$$x = -1$$

(b) $2^{x^2-7x+10} = 2^{2x-10}$

$$x^2 - 7x + 10 = 2x - 10$$

$$x^2 - 9x + 20 = 0$$

$$D = b^2 - 4ac = 81 - 80 = 1$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{9 \pm 1}{2} = 4, 5$$

(c) $3^{x^2-3x+2} = 1$

$$3^{x^2-3x+2} = 3^0$$

$$x^2 - 3x + 2 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 2}}{2} = \frac{3 \pm 1}{2} = 1, 2$$

3. Solve the following logarithmic equations:

(a) $\log_2 1 = \log_2 3x - 4$

(b) $10^{\log_{10}(x+2)} = x^2 - 4$

(c) $\log_{10} |x - 3| = \log_{10} 1$

Solution:

(a) $\log_2 1 = \log_2 3x - 4$

$$0 = \log_2 3x - 4$$

$$4 = \log_2 3x$$

$$2^4 = 3x$$

$$x = \frac{16}{3}$$

(b) $10^{\log_{10}(x+2)} = x^2 - 4$

$$x + 2 = x^2 - 4$$

$$x^2 - x - 6 = 0$$

$$D = b^2 - 4ac = 1 + 24 = 25$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{1 \pm \sqrt{25}}{2} = 3, -2$$

-2 can not be a solution, because logarithm is only defined for positive numbers

(c) $\log_{10} |x - 3| = \log_{10} 1$

$$|x - 3| = 1$$

$$x - 3 = \pm 1$$

$$x_{1,2} = 2, 4$$

4. Find inverse to the following functions and sketch both (function and its inverse) graphically.

(a) $y = 1 - 2x$

(b) $y = \frac{1}{x-2}$

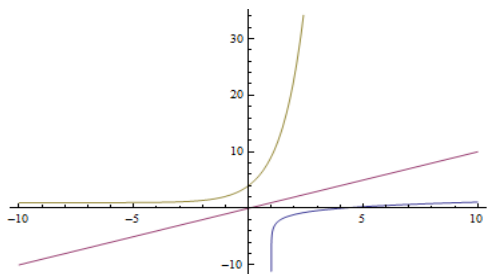
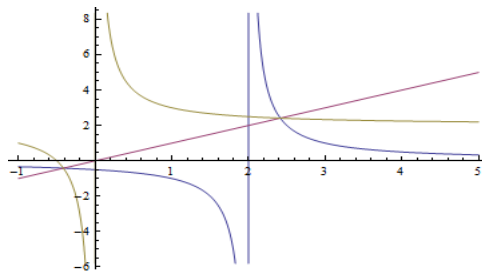
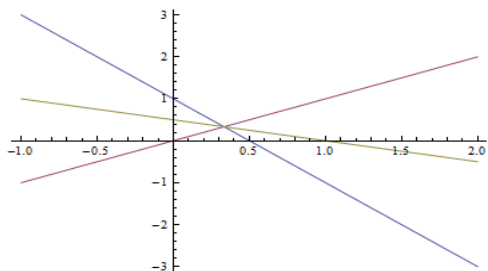
(c) $y = \ln \frac{x-1}{3}$

Solution:

(a) $x = 1 - 2y \Rightarrow y = \frac{1-x}{2}$

(b) $x = \frac{1}{y-2} \Rightarrow y = \frac{1}{x} + 2$

(c) $x = \ln \frac{y-1}{3} \Rightarrow y = 3e^x + 1$



5. Decompose the following functions to the most elementary functions.

(a) $y = (x + 1)^2$

(b) $y = \frac{1}{x-2}$

(c) $y = \frac{1}{(x+1)^2}$

Solution:

(a) $y = f(g(x)), \quad f(x) = x^2, \quad g(x) = x + 1$

(b) $y = f(g(x)), \quad f(x) = \frac{1}{x}, \quad g(x) = x - 2$

(c) $y = f(g(h(x))), \quad f(x) = \frac{1}{x}, \quad g(x) = x^2, \quad h(x) = x + 1$

6. $f(x) = e^x, g(x) = 3x, h(x) = \frac{1}{x}$. Find the following composite functions:

(a) $f(g(x))$

(b) $h(h(x))$

(c) $h(g(f(x)))$

Solution:

(a) $f(g(x)) = f(3x) = e^{3x}$

(b) $h(h(x)) = h\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$

(c) $h(g(f(x))) = h(g(e^x)) = h(3e^x) = \frac{1}{3e^x}$