AAU - Business Mathematics I

Problem set #2, Due March 18, 2010 - Suggested Solution

1. Solve the following linear equations:

$(a)\ \ 10x + 5 = 8x + 11$ $\qquad$ $(b)\ \ -2x - 4 = -3x + 2$

Solution:

(a)

$10x + 5 = 8x + 11 \ \ / - 8x$

$10x - 8x + 5 = 11 \ \ / - 5$

$2x = 6 \ \ / \div 5$

$x = 3$

(b)

$-2x - 4 = -3x + 2 \ \ / + 3x$

$-2x + 3x - 4 = 2 \ \ / + 4$

$x = 2 + 4 = 6$

2. Solve the following systems of linear equations:

$(a)\ \ 3x + 2y = 7$

$\quad\ \ 7x - y = 5$

$(b)\ \ 2x + 3y = 0$

$\quad\ \ -2/3x - y = 1$

Solution: We will use elimination method for (a) and substitution method for (b).

(a)

$3x + 2y = 7$

$\underline{7x - y = 5} \ \ / \times 2$

$3x + 2y = 7$

$\underline{14x - 2y = 10}$ add second equation to the first one

$3x + 2y + 14x - 2y = 7 + 10$

$17x = 17$

$x = 1$ Plugging 1 for x to one of the two original equations gives us that $y = 2$

$\Rightarrow x = 1, \ y = 2$

(b)

$2x + 3y = 0$

$\underline{-2/3x - y = 1} \quad \Rightarrow \quad y = -2/3x - 1$ plug this to the first equation

$2x + 3(-2/3x - 1) = 0 \quad \Rightarrow \quad 2x - 2x - 3 = 0 \quad \Rightarrow \quad -3 = 0$

This is never true, i.e. this system of equations does not have solution

3. Solve:

$(a)\;\; 2x^2 - 3x + 1 = 0$ $\qquad$ $(b)\;\; x^2 - 2x - 8 = 0$

$(c)\;\; 2x^2 - 32 = 0$ $\qquad$ $(d)\;\; (x-3)(x+1) = 0$

Solution:

$(a)\;\; 2x^2 - 3x + 1 = 0 \quad a = 2,\; b = -3,\; c = 1$

$D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 1 = 9 - 8 = 1$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{3 \pm \sqrt{1}}{4} = 1, \frac{1}{2}$$

$(b)\;\; x^2 - 2x - 8 = 0 \quad a = 1,\; b = -2,\; c = -8$

$D = b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-8) = 4 + 32 = 36$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} = -2, 4$$

$(c)\;\; 2x^2 - 32 = 0 \quad a = 2,\; b = 0,\; c = -32$

$2x^2 = 32 \quad \Rightarrow \quad x^2 = 16 \quad \Rightarrow \quad x = \pm 4$

$(d)\;\; (x-3)(x+1) = 0$ the product is 0 iff at least one expression in the product is 0

$(x-3) = 0$ or $(x+1) = 0 \quad \Rightarrow \quad x = -1,\; 3$

4. Solve the following inequalities:

$(a)\;\; 2x - 3 < x + 1 \leq 6$ $\qquad$ $(b)\;\; -x^2 - x + 2 > 0$

$(c)\;\; x^2 - 3x + 2 > 0$

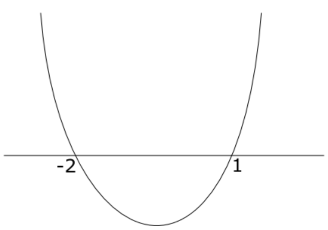
Solution:

$$\begin{aligned}
(a)\quad 2x-3<x+1 &\quad \text{and} \quad x+1\le 6\\
-3-1<x-2x &\quad \text{and} \quad x\le 6-1\\
-4<-x &\quad \text{and} \quad x\le 5\\
4>x &\quad \text{and} \quad x\le 5\\
\Rightarrow x\in(-\infty,4)
\end{aligned}$$

$$\begin{aligned}
(b)\quad & -x^2-x+2>0\\
& x^2+x-2<0
\end{aligned}$$

First, we solve the corresponding quadratic equation:

$$\begin{aligned}
& x^2+x-2=0\\
& D=b^2-4ac=1-4\times 1\times(-2)=9\\
& x_{1,2}=\frac{-1\pm 3}{2}=-2,1
\end{aligned}$$

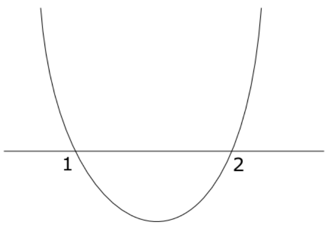


Now, we get back to the original inequality. We look for such values of x that $x^2+x-2<0$, i.e. for such values of x for which the graph of the quadratic function is below the axis x. Therefore the solution to our quadratic inequality is $x\in(-2,1)$.

$$(c)\quad x^2-3x+2>0$$

First, we solve the corresponding quadratic equation:

$$\begin{aligned}
& x^2-3x+2=0\\
& D=b^2-4ac=9-4\times 1\times(2)=1\\
& x_{1,2}=\frac{3\pm 1}{2}=1,2
\end{aligned}$$



Now, we get back to the original inequality. We look for such values of x that $x^2 - 3x + 2 > 0$, i.e. for such values of x for which the graph of the quadratic function is above the axis x. Therefore the solution to our quadratic inequality is $x \in (-\infty, 1) \cup (2, \infty)$.

5. Ben can drink 3 beers per hour. How many hours did he spend in the pub yesterday night if he drank either at most 12 or at least 15 beers?

Solution: Let's denote number of hours spent in the pub as x. Than we know that the number of beers is $3x$. So we have:

either $3x \leq 12 \Leftrightarrow x \leq 4$

or $3x \geq 15 \Leftrightarrow x \geq 5$

Ben spent in the pub at most 4 or at least 5 hours.

6. Solve:

$(a)\quad |x-3| - 2 = 0$ $\qquad (b)\quad |x-2| - |x+1| + 3 = 0$

$(c)\quad |x+1| < 4$ $\qquad (d)\quad \dfrac{2-2x}{1-x} \leq 1$

Solution:

(a) $|x-3| - 2 = 0$

$x - 3 = \pm 2$

$x = 1, 5$

(b) $|x-2| - |x+1| + 3 = 0$

- $x - 2 > 0 \quad \Leftrightarrow \quad x > 2$
- $x - 2 < 0 \quad \Leftrightarrow \quad x < 2$
- $x + 1 > 0 \quad \Leftrightarrow \quad x > -1$
- $x + 1 > 0 \quad \Leftrightarrow \quad x < -1$

We divide this problem into three subcases:

1. $x \in (-\infty, -1] \quad -x + 2 + x + 1 + 3 = 0$

$$6 = 0$$

This is not true for any value of x. This means that there is no solution in this subcase.

2. $x \in [-1, 2]$ $\quad -x + 2 - x - 1 + 3 = 0$

$$-2x + 4 = 0 \Rightarrow x = 2$$

$2 \in [-1, 2]$ so this is a solution.

3. $x \in [2, \infty)$ $\quad x - 2 - x - 1 + 3 = 0$

$$0 = 0$$

This holds for all values of x, however, this equation is valid only for $x \in [2, \infty)$. So solution in the last subcase is $x \in [2, \infty)$.

(c)
$$\begin{aligned}
&|x + 1| < 4 \\
&-4 < x + 1 < 4 \\
&-4 - 1 < x < 4 - 1 \\
&-5 < x < 3 \\
&x \in (-5, 3)
\end{aligned}$$

(d)
$$\begin{aligned}
&\frac{2 - 2x}{1 - x} \leq 1 \\
&\frac{2 - 2x}{1 - x} - 1 \leq 0 \\
&\frac{2 - 2x}{1 - x} - \frac{1 - x}{1 - x} \leq 0 \\
&\frac{2 - 2x - (1 - x)}{1 - x} \leq 0 \\
&\frac{1 - x}{1 - x} \leq 0 \\
&1 \leq 0
\end{aligned}$$

This is never true, therefore this inequality does not have any solution.