AAU - Business Mathematics I

Problem set #1, Due March 4, 2010 - Suggested Solution

1. Do the following equalities hold? Graphically justify your answer.

(a) $A \cap (A \cup B) = A$

(b) $\left(A \cap B^C\right) \cap B = A \cap B$

(c) $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$

(d) $(A \cup B)^C = (A^C) \cap (B^C)$

Solution:

(a) $A \cap (A \cup B) = A$	TRUE
(b) $\left(A \cap B^C\right) \cap B = A \cap B$	FALSE
(c) $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$	TRUE
(d) $(A \cup B)^C = (A^C) \cap (B^C)$	TRUE

To see that these equalities hold, draw Venn diagrams.

2. Find negations of following expressions:

(a) Every car is red.

(b) There are two banks with low charges in the CR.

(c) More than five students do not have homework.

(d) There are no firms producing Sony computers in the Czech Republic.

(e) It snowed at least four times this January.

(f) The equation has at least one solution.

Solution:

(a) At least one car is not red.

(b) There is either at most one or at least three banks with low charges in the CR.

(c) At most five (less than six) students do not have homework.

(d) There is at leas one firm producing Sony computers in the Czech Republic.

(e) It snowed at most three (less than four) times this January.

(f) The equation has no solution.

3. Travel agency sold 100 vacation packages during one day. Flight vacations were sold two times more often than vacations to Croatia. There were 5 less flight vacations to Croatia than non-flight vacations to Croatia. Vacations that were not to Croatia nor flight vacations were sold 10 less than vacations to Croatia that are not flight vacations. How many vacations to Croatia were sold? How many flight vacations were to other destinations than Croatia? It is enough to draw Venn diagram with corresponding system of equations solving of which would lead to the result. For finding actual numbers of vacations you can get extra points.

Solution: We will use the following notation:

x - flight vacations outside Croatia

y - flight vacations to Croatia

z - non-flight vacations to Croatia

w - non-flight vacations outside Croatia

Then the set-up of the problem says:

- $(x+y) = 2(y+z)$
- $z = y+5$
- $w+10 = z$
- $x+y+z+w = 100$

This is true for $x = 55, y = 15, z = 20, w = 10$. Therefore, 35 vacations were sold to Croatia and 55 flight vacations outside Croatia were sold.

4. Find intersection and union of the following sets:

(a) $A = [-5,-2]$, $B = (3,\infty)$

(b) $A = (-\infty, 14)$, $B = (-1,5)$, $C = [3,25]$

(c) $A = (-\infty,-2]$, $B = [-2,3/2)$, $C = [0,10)$

Solution:

(a) $A \cap B = [-5,2] \cap (3,\infty) = \emptyset$

$A \cup B = [-5,2] \cup (3,\infty)$

(b) $A \cap B \cap C = [3,5)$

$A \cup B \cup C = (-\infty, 25]$

(c) $A \cap B \cap C = \emptyset$

$A \cup B \cup C = (-\infty, 10)$

5. Simplify (factorize) the following algebraic expressions:

$(a)\quad \dfrac{7x-1}{2x^2+4x}+\dfrac{5-3x}{x^2-4}$ $\qquad (b)\quad 2a^3-a^2-8a+4$

$(c)\quad \dfrac{x}{1-x}-\dfrac{1}{x}$ $\qquad (d)\quad \dfrac{a-3}{1-a}(a^2-2a+1)$

$(e)\quad \dfrac{3x-1}{2x+2}-\dfrac{2x+2}{2x+1}$ $\qquad (f)\quad \dfrac{\sqrt{x^2-8x+16}}{x^2-16}-\dfrac{x}{x+4}$

$(g)\quad \dfrac{3xy^3z^{-2}}{5x^{-3}y^{-2}z}$ $\qquad (h)\quad \dfrac{(a^{3/2}b^{1/2})^2}{(ab)^{1/4}}$

Solution:

$$(a)\quad \frac{7x-1}{2x^2+4x}+\frac{5-3x}{x^2-4}=\frac{7x-1}{2x(x+2)}+\frac{5-3x}{(x+2)(x-2)}=\frac{(7x-1)(x-2)+(5-3x)(2x)}{2x(x+2)(x-2)}=$$
$$=\frac{7x^2-15x+2+10x-6x^2}{2x(x+2)(x-2)}=\frac{x^2-5x+2}{2x(x+2)(x-2)}$$

$$(b)\quad 2a^3-a^2-8a+4=a^2(2a-1)-4(2a-1)=(2a-1)(a+2)(a-2)$$

$$(c)\quad \frac{x}{1-x}-\frac{1}{x}=\frac{x^2+x-1}{x(1-x)}$$

$$(d)\quad \frac{a-3}{1-a}(a^2-2a+1)=\frac{a-3}{1-a}(a-1)^2=\frac{a-3}{-(a-1)}(a-1)^2=(3-a)(a-1)$$

$$(e)\quad \frac{3x-1}{2x+2}-\frac{2x+2}{2x+1}=\frac{(3x-1)(2x+1)-(2x+2)(2x+2)}{(2x+2)(2x+1)}=\frac{(6x^2+x-1)-(4x^2+8x+4)}{(2x+2)(2x+1)}=$$
$$=\frac{2x^2-7x-5}{(2x+2)(2x+1)}$$

$$(f)\quad \frac{\sqrt{x^2-8x+16}}{x^2-16}-\frac{x}{x+4}=\frac{\sqrt{(x-4)^2}}{(x+4)(x-4)}-\frac{x}{x+4}=\frac{1}{x+4}-\frac{x}{x+4}=\frac{1-x}{x+4}$$

$$(g)\quad \frac{3xy^3z^{-2}}{5x^{-3}y^{-2}z}=\frac{3}{5}x^4y^5z^{-3}$$

$$(h)\quad \frac{(a^{3/2}b^{1/2})^2}{(ab)^{1/4}}=\frac{(a^{3/2})^2(b^{1/2})^2}{a^{1/4}b^{1/4}}=\frac{a^3b}{a^{1/4}b^{1/4}}=a^{11/4}b^{3/4}$$

6. The demand for oranges is $q=100-p$ and the supply is $q=3p+20$, where p is the price measured in dollars per hundred pounds and q is the quantity measured in hundred pound units.

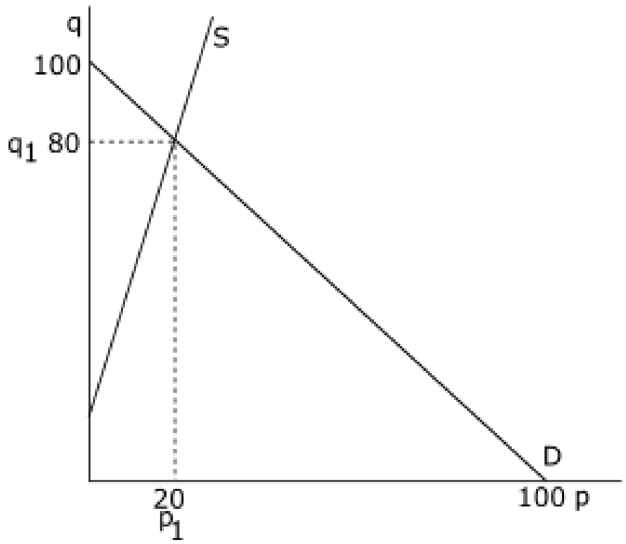
(a) On one graph, draw the demand curve and the supply curve for oranges.

(b) What is the equilibrium price of oranges? What is the equilibrium quantity? Show the equilibrium price and quantity on the graph and label them p_1 and q_1.

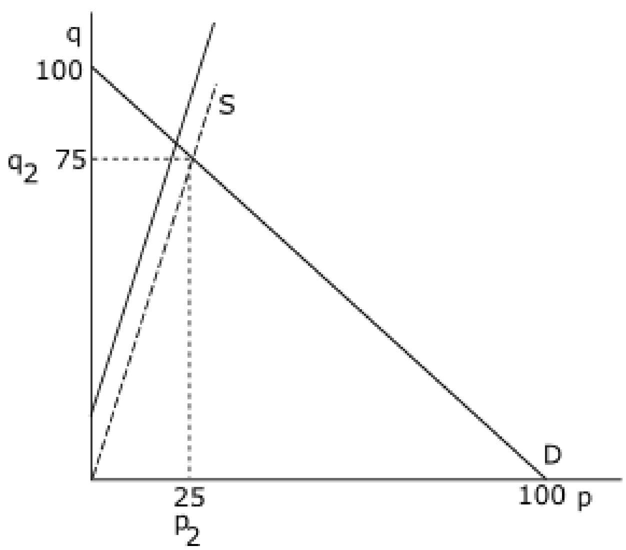
(c) A terrible drought strikes California, traditional homeland of oranges. The supply schedule shifts to $q=3p$. The demand schedule remains as before. Draw the new supply schedule.

(d) What is the new equilibrium price of oranges? What is the new equilibrium quantity? Show the equilibrium price and quantity on the graph and label them p_2 and q_2.

Solution:

(a) 

(b) $100 - p = 3p + 20 \Rightarrow 4p = 80$ and hence $p_1 = 20$. Plug this price into demand equation to find the equilibrium quantity. $q_1 = 100 - p = 100 - 20 = 80$.

(c) 

(d) $100 - p = 3p \Rightarrow 4p = 100$ and hence $p_2 = 25$. Plug this price into demand equation to find the equilibrium quantity. $q_2 = 100 - p = 100 - 25 = 75$.