Problem: Find the inverse of the function y if:

$$(c) \qquad y = \sqrt{x+1}$$

$$(d) \qquad y = \ln \frac{x}{2}$$

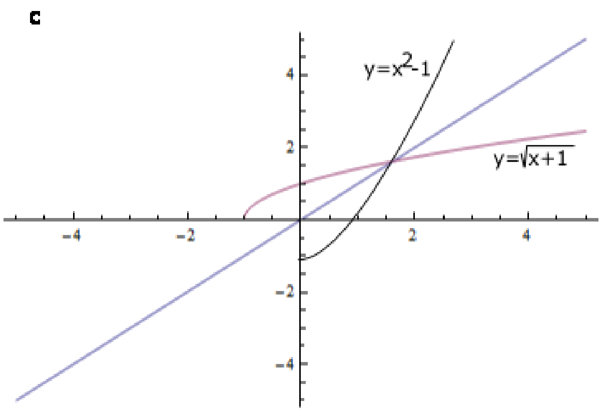
$$(e) \qquad y = e^{\left(\frac{1}{3}x-2\right)}$$

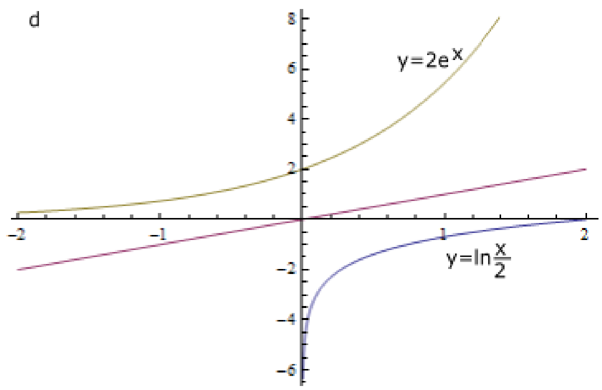
Solution: We find inverse functions in two steps: first, we switch x for y and second we rearrange terms in the equation such that again y is on the left hand side and everything else on the right hand side.

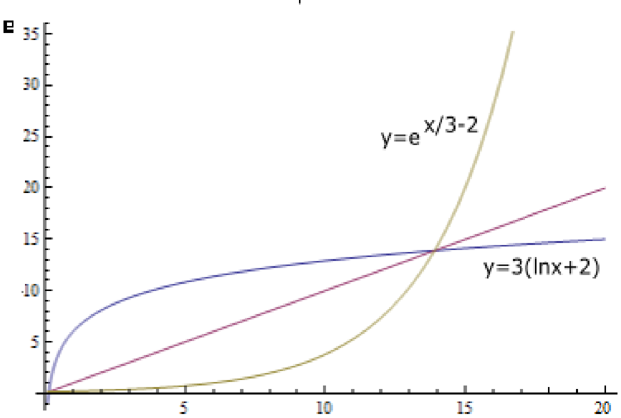
$$(c) \qquad y = \sqrt{x+1} \quad \longrightarrow \quad x = \sqrt{y+1} \quad \Rightarrow \quad y = x^2 - 1$$

$$(d) \qquad y = \ln \frac{x}{2} \quad \longrightarrow \quad x = \ln \frac{y}{2} \quad \Rightarrow \quad y = 2e^x$$

$$(e) \qquad y = e^{\left(\frac{1}{3}x-2\right)} \quad \longrightarrow \quad x = e^{\left(\frac{1}{3}y-2\right)} \quad \Rightarrow \quad y = 3(\ln x + 2)$$







10.3 Function composition

The function composition of two or more functions takes the output of one or more functions as the input of others. The functions $f : X \to Y$ and $g : Y \to Z$ can be composed by first applying f to an argument x to obtain $y = f(x)$ and then applying g to y to obtain $z = g(y)$. The composite function formed in this way from general f and g may be written $x \to g(f(x))$.

The function on the right acts first and the function on the left acts second, reversing English reading order. We remember the order by reading the notation as g of f. The order is important, because rarely do we get the same result both ways. For example, suppose $f(x) = x^2$ and $g(x) = x + 1$. Then $g(f(x)) = x^2 + 1$, while $f(g(x)) = (x + 1)^2$, which is $x^2 + 2x + 1$, a different function.

Problem: If $f(x) = x^2 + 3$ and $g(x) = 3x - 1$ then find the following: $(f + g)(x)$; $(f + g)(3)$; $f(g(x))$; $g(f(x))$; $f(g(2))$ and $g(f(3))$

Solution:

$$
\begin{aligned}
&(f+g)(x) = f(x) + g(x) = x^2 + 3 + 3x - 1 = x^2 + 3x + 2\\
&(f+g)(3) = x^2 + 3x + 2|_{x=3} = 3^2 + 3 \times 3 + 2 = 20\\
&f(g(x)) = f(3x-1) = (3x-1)^2 + 3 = 9x^2 - 6x + 1 + 3 = 9x^2 - 6x + 4\\
&g(f(x)) = g(x^2+3) = 3(x^2+3) - 1 = 3x^2 + 8\\
&f(g(2)) = 9x^2 - 6x + 4|_{x=2} = 36 - 12 + 4 = 28\\
&g(f(3)) = 3x^2 + 8|_{x=3} = 27 + 8 = 35
\end{aligned}
$$

Problem: Find $f(f(f(x)))$ if:

$$f(x) = \frac{1}{1-x}$$

Solution:

$$f(x) = \frac{1}{1 - \frac{1}{1 - \frac{1}{1-x}}}$$

Problem: Decompose the following functions:

$$
\begin{aligned}
&y = 2^{x+1}\\
&y = (x+2)^2 - 3\\
&y = (2x-5)^{10}\\
&y = \ln \frac{1}{\sqrt{x+2}}
\end{aligned}
$$

Solution:

$$y = g(f(x)) \quad \text{where} \quad f(x) = x + 1; \; g(x) = 2^x$$
$$y = h(g(f(x))) \quad \text{where} \quad f(x) = x + 2; \; g(x) = x^2; \; h(x) = x - 3$$
$$y = h(g(f(x))) \quad \text{where} \quad f(x) = 2x; \; g(x) = x - 5; \; h(x) = x^{10}$$
$$y = i(h(g(f(x)))) \quad \text{where} \quad f(x) = x + 2; \; g(x) = \sqrt{x}; \; h(x) = \frac{1}{x}; \; i(x) = \ln x$$