AAU - Business Mathematics I

Problem set #4, Due April 24, 2010 - Suggested Solution

1. Find the following determinants:

(a) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{vmatrix}$

Solution:

(a)

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \times 1 - 0 \times 0 = 1$$

(b)

$$\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 3 \times 4 - 1 \times 2 = 10$$

(c)

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{vmatrix} = 1(-1)^{1+1} \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} + 2(-1)^{1+2} \begin{vmatrix} -2 & 0 \\ 3 & 1 \end{vmatrix} + 3(-1)^{1+3} \begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix} =$$

$$= 1 + 4 - 3 = 2$$

2. Solve the following systems using (i) matrix method, and (ii) Cramer's rule:

(a) $3x - 2y = -1$
$x + y = 3$

(b) $-2x + 3y = 2$
$x + y = 4$

(c) $x - 2y + z = 7$
$3x - y - z = -2$
$x - y + 2z = 6$

Solution:

(a)

Matrix method:

$$\left(\begin{array}{cc|c} 3 & -2 & -1 \\ 1 & 1 & 3 \end{array}\right) \begin{array}{c}\nwarrow\!\searrow \\ \swarrow\end{array} \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 3 & -2 & -1 \end{array}\right) \begin{array}{c}\times(-3) \\ \swarrow\end{array} \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -5 & -10 \end{array}\right) \div(-5) \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 2 \end{array}\right) \begin{array}{c}\nwarrow \\ \times(-1)\end{array} \sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right) \Rightarrow \begin{array}{c} x = 1 \\ y = 2 \end{array}$$

Cramer's rule:

$$x = \frac{\begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{5}{5} = 1$$

$$y = \frac{\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{10}{5} = 2$$

(b)

Matrix method:

$$\left(\begin{array}{cc|c} -2 & 3 & 2 \\ 1 & 1 & 4 \end{array}\right) \begin{array}{c}\nwarrow\!\searrow \\ \swarrow\end{array} \sim \left(\begin{array}{cc|c} 1 & 1 & 4 \\ -2 & 3 & 2 \end{array}\right) \begin{array}{c}\times(2) \\ \swarrow\end{array} \sim \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 5 & 10 \end{array}\right) \div(5) \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 1 & 2 \end{array}\right) \begin{array}{c}\nwarrow \\ \times(-1)\end{array} \sim \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 2 \end{array}\right) \Rightarrow \begin{array}{c} x = 2 \\ y = 2 \end{array}$$

Cramer's rule:

$$x = \frac{\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}}{\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{-10}{-5} = 2$$

$$y = \frac{\begin{vmatrix} -2 & 2 \\ 1 & 4 \end{vmatrix}}{\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{-10}{-5} = 2$$

(c)

Matrix method:

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & -1 & -1 & -2 \\ 1 & -1 & 2 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 5 & -4 & -23 \\ 0 & 1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 1 & -1 \\ 0 & 5 & -4 & -23 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -9 & -18 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

Cramer's rule:

$$x = \frac{\begin{vmatrix} 7 & -2 & 1 \\ -2 & -1 & -1 \\ 6 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & -1 & -1 \\ 1 & -1 & 2 \end{vmatrix}} = \frac{-9}{9} = -1, \quad y = \frac{\begin{vmatrix} 1 & 7 & 1 \\ 3 & -2 & -1 \\ 1 & 6 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & -1 & -1 \\ 1 & -1 & 2 \end{vmatrix}} = \frac{-27}{9} = -3$$

$$z = \frac{\begin{vmatrix} 1 & -2 & 7 \\ 3 & -1 & -2 \\ 1 & -1 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & -1 & -1 \\ 1 & -1 & 2 \end{vmatrix}} = \frac{18}{9} = 2$$