AAU - Business Mathematics I

Problem set #3, Due April 10, 2010 - Suggested Solution

1. Solve the following inequalities:

$(a)\quad x^2+4x+10 \le 0$ $\qquad (b)\quad -x^2-4x-10<0$

$(c)\quad \dfrac{2}{x-5}>0$ $\qquad (d)\quad \dfrac{2x}{x-10}\le 0$

Solution:

$$\begin{aligned}(a)\quad & x^2+4x+10 \le 0\\ & x^2+4x+10=0\\ & D=b^2-4ac=16-40<0\end{aligned}$$

Discriminant is negative, this means that quadratic equation does not have any solution or, in other words, parabola has no intercepts with horizontal axis - x. Since "a"=1 is positive, the whole parabola lies above the horizontal axis x. So there are no such values of x for which $x^2+4x+10 \le 0$. There is no solution to this quadratic inequality.

$$\begin{aligned}(b)\quad & -x^2-4x-10<0\\ & -x^2-4x-10=0\\ & D=b^2-4ac=16-40<0\end{aligned}$$

Discriminant is negative, this means that quadratic equation does not have any solution or, in other words, parabola has no intercepts with horizontal axis - x. Since "a"=-1 is negative, the whole parabola lies below the horizontal axis x. So for all values of x, $-x^2-4x-10<0$. There are infinitely many solutions to this quadratic inequality: $x \in R$.

$$(c)\quad \frac{2}{x-5}>0$$

This fraction will be positive only if $x-5$ is positive; i.e $x>5$. Solution is $x \in (5,\infty)$.

$$\begin{aligned}(d)\quad & \frac{2x}{x-10}\le 0\\ (i)\quad & 2x\le 0 \ \text{ and } \ x-10>0\\ & x\le 0 \ \text{ and } \ x>10 \ \Rightarrow \ \text{no solution}\\ (ii)\quad & 2x\ge 0 \ \text{ and } \ x-10<0\\ & x\ge 0 \ \text{ and } \ x<10 \ \Rightarrow \ x\in[0,10)\end{aligned}$$

Solution to this rational inequality is $x \in [0, 10)$. Note: numerator can be 0, but denominator can never be zero - that is why the inequality is strict in the second condition in both (i) and (ii).

2. Solve the following equations and inequalities with absolute value:

$$
\begin{aligned}
&(a) \quad |x-4| = 2 \\
&\qquad x - 4 = \pm 2 \quad \Rightarrow \quad x = 2, 6 \\
&(b) \quad |2x-2| + |x+3| = 4 \\
&\qquad (i) \quad x \in (-\infty, -3] \quad \text{both 2x-2 and x+3 are negative} \\
&\qquad (ii) \quad x \in [-3, 1] \quad \text{2x-2 negative, x+3 positive} \\
&\qquad (iii) \quad x \in [1, \infty) \quad \text{both 2x-2 and x+3 are positive} \\
&\qquad (i) \quad x \in (-\infty, -3]: \quad -(2x-2) - (x+3) = 4 \quad \Rightarrow \quad x = -\frac{5}{3} \notin (-\infty, -3] \quad \Rightarrow \\
&\qquad \Rightarrow \quad \text{not a solution} \\
&\qquad (ii) \quad x \in [-3, 1]: \quad -(2x-2) + (x+3) = 4 \quad \Rightarrow \quad x = 1 \in [-3, 1] \quad \Rightarrow \\
&\qquad \Rightarrow \quad \text{solution} \\
&\qquad (iii) \quad x \in [1, \infty): \quad (2x-2) + (x+3) = 4 \quad \Rightarrow \quad x = 1 \in [1, \infty) \quad \Rightarrow \\
&\qquad \Rightarrow \quad \text{solution} \\
&\qquad \text{There is one solution to this equation with absolute value: } x = 1 \\
&(c) \quad |x-3| < 1 \\
&\qquad -1 < x - 3 < 1 \quad \Rightarrow \quad 2 < x < 4 \quad \Rightarrow \quad x \in (2, 4) \\
&(d) \quad |x+2| \geq 1 \\
&\qquad -1 \geq x + 2 \geq 1 \quad \Rightarrow \quad -3 \geq x \geq -1 \quad x \in (-\infty, -3] \cup [-1, \infty)
\end{aligned}
$$

Solution:

4. Solve the following exponential and logarithmic equations:

(a) $7^{3x+1} = 49^x$

(b) $2^{x^2-7x+10} = 2^{2x-10}$

(c) $\log_2 1 = \log_2 3x - 4$

Solution:

$$
\begin{aligned}
(a) \qquad & 7^{3x+1} = 49^x \\
& 7^{3x+1} = 7^{2x} \\
& 3x + 1 = 2x \\
& x = -1
\end{aligned}
$$

(b) $$2^{x^2-7x+10} = 2^{2x-10}$$
$$x^2 - 7x + 10 = 2x - 10$$
$$x^2 - 9x + 20 = 0$$
$$D = b^2 - 4ac = 81 - 80 = 1$$
$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{9 \pm 1}{2} = 4, 5$$

(c) $$\log_2 1 = \log_2 3x - 4$$
$$0 = \log_2 3x - 4$$
$$4 = \log_2 3x$$
$$2^4 = 3x$$
$$x = \frac{16}{3}$$