AAU - Business Mathematics I

Problem set #2, Due March 27, 2010 - Suggested Solution

**1.** Solve:

$(a)$ $x^2 - 3x + 2 = 0$ $(b)$ $x^2 - 2x - 8 = 0$

$(c)$ $3x^2 - 12 = 0$ $(d)$ $(x-2)(x+5) = 0$

**Solution:**

$(a)$ $x^2 - 3x + 2 = 0 \quad a = 1,\ b = -3,\ c = 2$

$D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times 2 = 9 - 8 = 1$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{3 \pm \sqrt{1}}{2} = 1, 2$$

$(b)$ $x^2 - 2x - 8 = 0 \quad a = 1,\ b = -2,\ c = -8$

$D = b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-8) = 4 + 32 = 36$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} = -2, 4$$

$(c)$ $3x^2 - 12 = 0 \quad a = 3,\ b = 0,\ c = -12$

$3x^2 = 12 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$

$(d)$ $(x-2)(x+5) = 0$ the product is 0 iff at least one expression in the product is 0

$(x-2) = 0$ or $(x+5) = 0 \quad \Rightarrow \quad x = -5,\ 2$

**2.** Solve the following inequalities:

$(a)$ $2x - 3 < x + 1 \leq 6$ $(b)$ $-x^2 + 3x + 10 > 0$

$(c)$ $x^2 - 6x + 12 > 0$ $(d)$ $\dfrac{1-x}{2-2x} > 1$

**Solution:**

$(a)$ $2x - 3 < x + 1$ and $x + 1 \leq 6$

$-3 - 1 < x - 2x$ and $x \leq 6 - 1$

$-4 < -x$ and $x \leq 5$

$4 > x$ and $x \leq 5$

$\Rightarrow x \in (-\infty, 4)$

$(b)$ $\quad -x^2 + 3x + 10 > 0$

First, we solve the corresponding quadratic equation:

$$-x^2 + 3x + 10 = 0$$
$$D = b^2 - 4ac = 9 - 4 \times (-1) \times 10 = 49$$
$$x_{1,2} = \frac{-3 \pm 7}{-2} = 5, -2$$

Now, we get back to the original inequality. We look for such values of $x$ that $-x^2 + 3x + 10 > 0$, i.e. for such values of $x$ for which the graph of the quadratic function is above the axis x. Therefore the solution to our quadratic inequality is $x \in (-2, 5)$.

$(c)$ $\quad x^2 - 6x + 12 > 0$

First, we solve the corresponding quadratic equation:

$$x^2 - 6x + 12 = 0$$
$$D = b^2 - 4ac = 36 - 4 \times 1 \times 12 < 0$$

There are no intercepts of parabola and horizontal axis (equation has not solution). This means that the whole parabola lies above the horizontal axis and hence all real numbers are solution to this inequality. (Note: if D<0 and a<0, then the whole parabola lies below the horizontal axis x).

$(d)$
$$\frac{1-x}{2-2x} > 1$$
$$\frac{1-x}{2-2x} - 1 > 0$$
$$\frac{1-x}{2-2x} - \frac{2-2x}{2-2x} > 0$$
$$\frac{1-x-2+2x}{2-2x} > 0$$
$$\frac{x-1}{2-2x} > 0$$
$$\frac{-(1-x)}{2(1-x)} > 0$$
$$-\frac{1}{2} > 0$$

This is never true, therefore this inequality does not have any solution.

**3.** Sally solves 4 mathematical problems per hour. How many hours did she work if we know that she solved either at most 48 or at least 72 problems?

**Solution:** Let's denote number of hours Sally works as $x$. Than we know that the number of problems she solves is $4x$. So we have:

either $4x \leq 48 \Leftrightarrow x \leq 12$

or $4x \geq 72 \Leftrightarrow x \geq 18$

Sally worked at most 12 or at least 18 hours.

**4.** Greg consumes cheese and wine. Two weeks ago he spent all his pocket money on wine and he enjoyed 8 bottles. Last week he split the money and got 4 bottles of wine and 7 packages of cheese. His weekly income is 560CZK. What is the price of wine and price of cheese?

**Solution:** 8 bottles of wine cost 560CZK, that means that one bottle costs $560/8 = 70$CZK. Four bottles of wine cost 280CZK and remaining 560-280=280CZK is spent on 7 packages of cheese which means that one package of cheese costs 40CZK