AAU - Business Mathematics I

Problem set #1, Due March 13, 2010 - Suggested Solution

1. Find intersection and union of the following sets:

(a) $A = (-\infty, 5)$; $B = (2, 8)$

(b) $A = [1, 7]$; $B = (7, 11)$

(c) $A = (-\infty, 2)$; B=[0,4); C=(1,102]

Solution:

(a) $A \cap B = (2, 5)$

$A \cup B = (-\infty, 8)$

(b) $A \cap B = \emptyset$

$A \cup B = [1, 11)$

(c) $A \cap B \cap C = (1, 2)$

$A \cup B \cup C = (-\infty, 102]$

2. Simplify (factorize) the following algebraic expressions:

(a) $\dfrac{7x-1}{2x^2+6x} + \dfrac{5-3x}{x^2-9}$ (b) $a^3 - 2a^2 - 4a + 8$

(c) $\dfrac{1}{1-x} + \dfrac{1}{x}$ (d) $\dfrac{3x-1}{2x+2} - \dfrac{2x+2}{2x+1}$

Solution:

(a) $\dfrac{7x-1}{2x^2+6x} + \dfrac{5-3x}{x^2-9} = \dfrac{7x-1}{2x(x+3)} + \dfrac{5-3x}{(x+3)(x-3)} = \dfrac{(7x-1)(x-3)+2x(5-3x)}{2x(x+3)(x-3)} =$

$= \dfrac{7x^2-22x+3+10x-6x^2}{2x(x+3)(x-3)} = \dfrac{x^2-12x+3}{2x(x+3)(x-3)}; \quad x \neq 0, \pm 3$

(b) $a^3 - 2a^2 - 4a + 8 = a^2(a-2) - 4(a-2) = (a-2)(a^2-4) = (a-2)(a-2)(a+2)$

(c) $\dfrac{1}{1-x} + \dfrac{1}{x} = \dfrac{x+1-x}{x(1-x)} = \dfrac{1}{x(1-x)}; \quad x \neq 0, 1$

(d) $\dfrac{3x-1}{2x+2} - \dfrac{2x+2}{2x+1} = \dfrac{(3x-1)(2x+1)-(2x+2)(2x+2)}{(2x+2)(2x+1)} =$

$= \dfrac{6x^2+x-1-(4x^2+8x+4)}{(2x+2)(2x+1)} = \dfrac{2x^2-7x-5}{(2x+2)(2x+1)}; \quad x \neq -1/2, -1$

3. Use substitution method to solve the following system of equations:

$x + 3y = 3$

$-x + 3y = 1$

Solution:

$x + 3y = 3$

$-x + 3y = 1 \Rightarrow x = 3y - 1$ - plug this to the first equation:

$(3y - 1) + 3y = 3 \Rightarrow 6y = 4 \Rightarrow y = 2/3$

$x = 3y - 1 = 1$

4. Use elimination method to solve the following system of equations:

$2x + 3y = 13$

$2x - y = 1$

Solution:

$2x + 3y = 13$

$2x - y = 1 \ /\cdot(-1)$

$2x + 3y = 13$

$-2x + y = -1$

Add these two equations together:

$4y = 12 \Rightarrow y = 3$

$2x - y = 1 \Rightarrow 2x - 3 = 1 \Rightarrow x = 2$

5. The demand for apples is $Q = 80 - P$ and the supply is $Q = 2P + 50$, where P is the price measured in dollars and Q is the quantity.

(a) On one graph, draw the demand curve and the supply curve for apples.

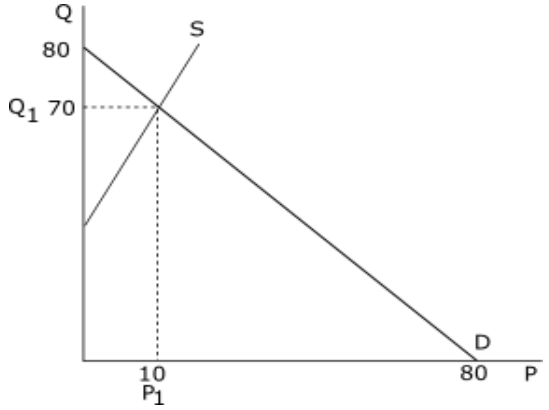
(b) What is the equilibrium price of apples? What is the equilibrium quantity? Show the equilibrium price and quantity on the graph and label them P_1 and Q_1.

(c) Due to the bad weather conditions there are less apples on trees than usual. The supply schedule shifts to $Q = 2P + 20$. The demand schedule remains as before. Draw the new supply schedule.

(d) What is the new equilibrium price of apples? What is the new equilibrium quantity? Show the equilibrium price and quantity on the graph and label them P_2 and Q_2.

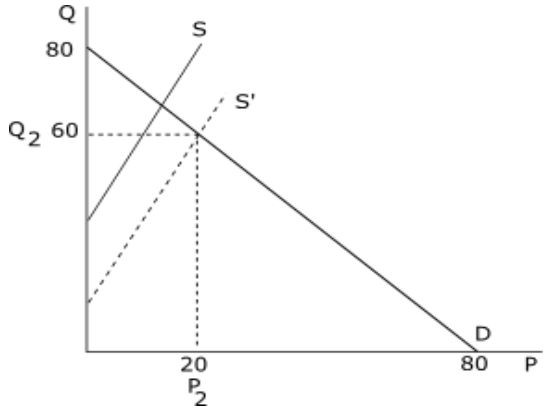
Solution:

(a) Demand curve and the supply curve for apples:



(b) $80 - P_1 = 2P_1 + 50 \Rightarrow P_1 = 10$. And hence, $Q_1 = 80 - P_1 = 70$.

(c) Demand curve and the supply curve for apples:



(d) $80 - P_2 = 2P_2 + 20 \Rightarrow P_2 = 20$. And $Q_2 = 80 - P_2 = 60$.